

Longitudinal Correlation as a Probe for the Local P_L Distribution

Shin MUROYA*

Matsumoto University

Matsumoto, Japan 390-1295

E-mail: muroya@matsu.ac.jp

HBT effect of identical particle correlation is a well established tool to investigate the spatial extent of produced particle source. In heavy-ion experiment, with respect to the direction of the momenta difference, three-types of length, R_{side} , R_{out} and R_L are investigated. R_{side} and R_{out} provide us with both geometrical and dynamical information of the source in the transverse plane, i.e., spatial extent and the history of freeze-out hyper-surface. On the other hand, in the ultra-relativistic collisions, the dynamics in the longitudinal direction quite differs from the one in the transverse plane. Boost invariance and scaling property of the produced system are well established, therefore, R_L is not a simple geometrical extent of source in the collision direction.

If we adopt a hydrodynamical picture, we can regard R_L as $\tau_f \sqrt{T/m_T}$ with τ_f being freeze-out time and m_T being transverse mass, respectively. T is the temperature of the fluid and it works as a slope parameter in the longitudinal momentum, P_L , distribution in the local system of the fluid. Because expanding dynamics in longitudinal direction and the one in the transverse plane are completely different, the isotropic property of local system is not obvious and the slope parameter of P_L may not be equal to the one of transverse momentum, P_T .

By virtue of the scaling property of the system, P_L distribution in the local system is converted to the spatial distribution in longitudinal direction which can be estimated by the two-body correlation of identical particles. In this paper, we will investigate the local particle distribution of P_L and P_T through the two-body correlation functions in longitudinal direction.

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1. Introduction

The local equilibrium is the key concept to understand the multiple production phenomena in the high energy nuclear collision. Quark-Gluon Plasma (QGP) phase, phase transition, temperature, etc., such thermodynamical terminology can be available only for the system which is, at least locally, in equilibrium. The success of hydrodynamical model strongly suggests the achievement of local thermalization. However the local equilibrium is the basic assumption of the model and it is not a derived result. Hydrodynamical model is only a circumstantial evidence.

In order to justify the starting of the hydrodynamical evolution shortly (less than 1 fm/c) after the collision, a great effort has been made to derive the so-called early thermalization. In these discussions, the central issue is the conversion of the longitudinal momenta into the transverse direction which possibly causes the local isotropy. In relativistic nuclear collisions, global dynamics in the longitudinal direction differs completely from the one in the transverse direction. Hence, even in the later stage, the achievement of the local isotropy is not trivial.

Hanbury-Brown Twiss effect (HBT) of identical particle correlation is a well-established method to investigate the spatial extent of the produced particle source[1, 2, 3]. In the case of Heavy-ion experiment, opening the momentum difference into the three directions, q_L , q_{out} and q_{side} , we can obtain three parameter, R_L , R_{out} and R_{side} , respectively. Source sizes estimated through the Gaussian fit of two-particle correlation functions are called HBT radii. Both R_{out} and R_{side} provide us with the source size in the transverse plane and the difference between R_{out} and R_{side} has been expected to be related to the time duration of the particle emission[4] and the transverse expansion[5].

At RHIC and LHC, the above three HBT radii are almost the same and the analyses based on the spherical harmonic function seems to be de facto standard[6]. However, because of the quite different mechanism in the longitudinal expansion and the transverse expansion, the physical meanings of HBT radii are completely different to each other. In this paper, we focus our discussion to R_L and investigate its special role.

2. R_L of Boost Invariant Source

Two particle correlation function is the ratio of two particle distribution function $W(k_1, k_2)$ to single particle distribution functions $W(k_1)$ and $W(k_2)$,

$$C(q, K) \equiv \frac{W(k_1, k_2)}{W(k_1)W(k_2)}, \quad (2.1)$$

denoting momentum difference as $q = k_1 - k_2$ and average momentum as $K = \frac{1}{2}(k_1 + k_2)$. The two-particle correlation function is essentially given as an weighted average of the plane wave,

$$C(q, K) \simeq 1 + |\langle e^{-iq \cdot x} \rangle|^2. \quad (2.2)$$

The weight of the average $\langle e^{-iq \cdot x} \rangle$ is given by the source function, $S(x, K)$ [7],

$$\langle e^{-iq \cdot x} \rangle(K) \equiv \frac{\int d^4x e^{-iq \cdot x} S(x, K)}{\int d^4x S(x, K)}. \quad (2.3)$$

The source function $S(x, K)$ is given as the sum of the volume elements $d\sigma_v(x)$ on freeze-out hypersurface,

$$S(x, K) \sim \sum_{\text{F-O surface}} k^Y d\sigma_v f(x, k) \quad (2.4)$$

with $f(x, k)$ being the particle distribution in the local volume element. If the source is static, supposing that the particle distribution in a volume element $f(x, k)$ is independent of position x and that the volume elements are in the spherical Gaussian distribution,

$$\sim \exp\left(\frac{-r^2}{R^2}\right), \quad (2.5)$$

the two-particle correlation function becomes the Fourier transform of the Gaussian distribution,

$$C(q, K) \sim \exp(-q^2 R^2). \quad (2.6)$$

So, HBT radius corresponds to the width of the Gaussian distribution of the volume elements.

However, if the source is not static, it is impossible to separate the configuration space distribution of the volume elements and the momentum distribution of the particle in a volume element. For simplicity, let us suppose 1+1 dimensional scaling expansion of the fluid. The boost invariant property stands for the uniform distribution of the volume element in the longitudinal direction. That is to say, the spatial extent of the source in the longitudinal direction is infinite. On the other hand, the scaling property of the solution of hydrodynamics gives strong correlation between configuration space and momentum space. The thermal distribution in the volume element of which four velocity $U_v(x)$ is given as,

$$f(x, k) = \frac{1}{e^{(k^v U_v(x) - Q\mu(x))/T(x)} \pm 1}, \quad (2.7)$$

with $\mu(x)$ being chemical potential and $T(x)$ being temperature, respectively. Taking Boltzmann approximation, we can obtain,

$$f(x, k) \sim \exp\left(\frac{-m_T Y^2}{T}\right), \quad (2.8)$$

with Y being rapidity and m_T being transverse mass of the particle, respectively. The scaling solution of the 1+1 dimensional hydrodynamical model is given as,

$$Y^2 \sim \left(\frac{z}{\tau}\right)^2. \quad (2.9)$$

Hence, in eq(2.8), the thermal distribution of the scaling fluid imitates the Gaussian source distribution

$$\exp\left(\frac{-m_T Y^2}{T}\right) \sim \exp\left(\frac{-z^2}{\left(\tau_f \sqrt{\frac{T}{m_T}}\right)^2}\right). \quad (2.10)$$

This is the R_L we can obtain as a HBT radius in the longitudinal direction[8],

$$R_L \sim \tau_f \sqrt{\frac{T_L}{m_T}}. \quad (2.11)$$

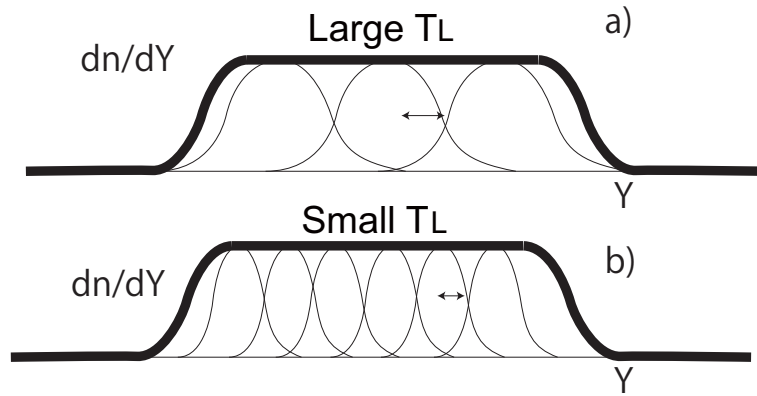


Figure 1: $\frac{dn}{dY}$ distribution and T_L .

Here, in eq(2.11), T_L is a slope parameter in the longitudinal direction (P_L slope parameter). The existence of the T_L is not trivial. Experimental data of the particle distribution of rapidity is the bold line in Figure 1. T_L obtained through longitudinal HBT radius is the width of the Gaussian distribution drawn by thin lines in Figure 1. The size of T_L is not trivial. Though the existence of T_L suggests the achievement of the thermalization in a local system but T_L may not be the thermal temperature. The P_L slope T_L is a independent quantity of the P_T slope, T_T , which can be estimated from P_T distribution. Coincidence of the P_L slope and the P_T slope is the important evidence of the local thermal equilibrium, $T_L = T_T = T_{thermal}$.

Unfortunately, in eq (2.11) we can obtain only the combination of $\tau_f \sqrt{\frac{T_L}{m_T}}$, direct comparison with P_T slope is still difficult without fixing τ_f . However, from the m_T dependence of R_L we can obtain some informations on the local particle distribution.

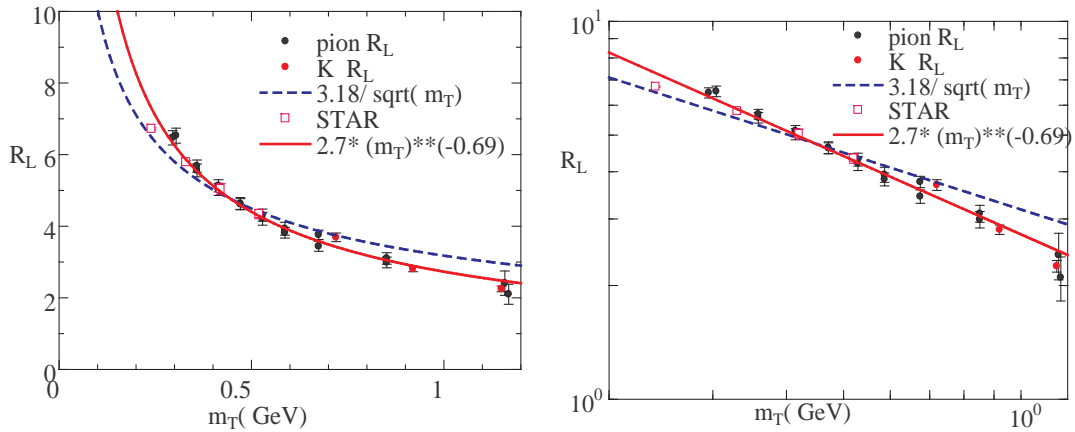
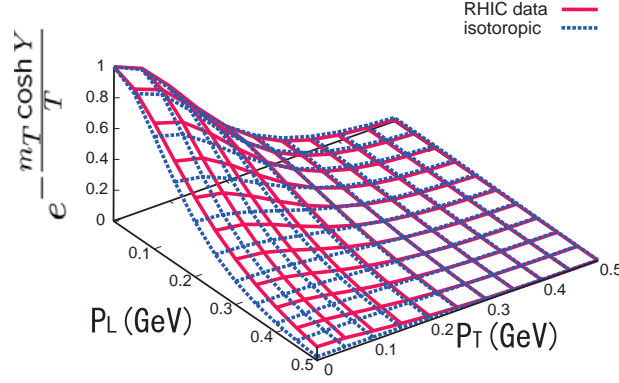
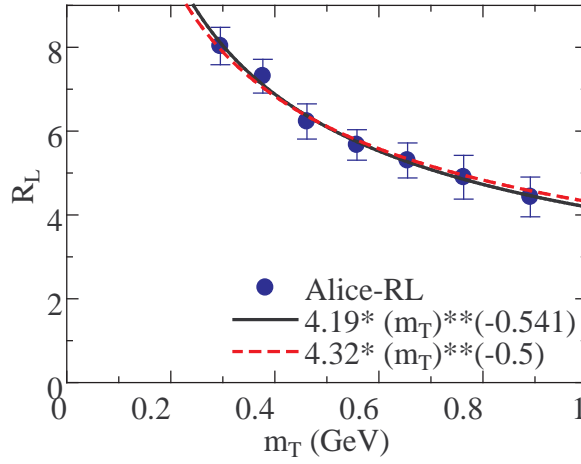


Figure 2: Compiled results of R_L at RHIC.

3. RHIC data

Figure 2 shows compiled data of R_L at RHIC[1, 2]. Data points seem to be on a single curve but show discrepancy from $m_T^{-1/2}$ curve which corresponds to the simple thermal distribution.


Figure 3: Pion distribution as a function of P_L and P_T .

Figure 4: ALICE results.

The right hand panel in fig. 2 shows that the best fit value of the power is -0.69 . With use of this power, we can reproduce particle distribution function as a function of both P_L and P_T . The solid line (red) in fig. 3 shows pion distribution function estimated from R_L in fig. 2. Dashed line (indigo) in fig. 3 is the result of the simple thermal case that corresponds to the power which is equal to -0.5 . Figure 3 indicates that particle distribution of the local system is anisotropic and P_T slope estimated from m_T dependence is larger than the P_L slope. This tendency is opposite to the naive expectation of the effect of the transverse expansion. In figure 3 we assumed that $\tau_f = 10$ fm, but our result is not sensitive to this value.

4. ALICE data

Figure 4 shows R_L result of ALICE[3]. The best fit value of the power of m_T is -0.54 (solid curve). A dashed curve is simple thermal distribution $m_T^{-0.5}$. Both curves seem to be consistent with the experiment within the error bar. It suggests that in the case of heavy ion collisions at the LHC, particle distribution in the local volume element can be isotropic.

5. discussion

In this paper, we discussed P_L and P_T slope parameter in the local volume elements based on the extensive use of boost invariant property of the scaling solution of 1+1 dimensional hydrodynamical model. RHIC experiment indicates the different values for the slopes of P_L and P_T and local particle distribution seems to be anisotropic. The tendency is opposite to the transverse expansion. On the contrary, ALICE R_L data is consistent with the isotropic particle distribution. Our discussion is based on the lowest order of m/T expansion, here m being hadron mass and T being freeze-out temperature, respectively [9, 10]. These quantities change only a little as a function of collision energy and almost the same to RHIC and LHC. Hence, we do not think that the difference between RHIC and LHC will be changed in the higher order approximation.

Our discussion is based on the extensive use of boost invariant scaling solution, but the boost invariance of the system should be also checked by the experimental data. For example, the average rapidity dependence of R_L informs us whether the T_L in the local system (fig. 1), is independent of Y or not (see fig. 3 in [11]). Hence, more detailed analyses on R_L is welcome.

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