Orthogonal Wilson loops in flavor backreacted confining gauge/gravity duality

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In this note we mainly review certain results of the flavor backreacted confining gauge/gravity dualities. We focus more on the static potential and force in presence of perturbatively backreacting flavor branes in a confining theory and find them to be screened compared to the quenched approximation. We comment on the methods used, and on the properties of the minimal surfaces that lead to the screening. We also make qualitative comparisons of our holographic and the lattice field theory results in presence of flavors. Finally, we present an additional comparison of the Coulomb's constant for $SU(N)$ gauge theories in five dimensions, in a limit of the non-flavored version of the confining model we use, and in the corresponding lattice mean-field expansion.

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Since the original Maldacena conjecture [1] has been found, there has been a lot of effort to understand the duality better. An other direction that was taken is to find new gauge/gravity dualities that are closer to the QCD. There is a significant progress in this direction. By now it is well understood that the finite temperature limit of the theories can be achieved by considering a black hole in the external *AdS* space. Moreover the supersymmetry of the duality can be reduced either by deforming the original one, eg. using the β deformations, or by finding new solutions for the internal space instead of the sphere, ie. using Sasaki-Einstein manifolds. The conformal invariance of the theory can be also broken and even confining gauge/gravity dualities have been found, like the Witten D4 model[2]. Additionally to that one can introduce matter in the fundamental representation, by introducing in the gravity side, flavor branes that extend to the boundary giving new degrees of freedom. These branes in the case of the initial AdS/CFT are D7 [3], while for the Witten's D4 model are D8 [4]. As in the field theory side, there are two ways to introduce in gravity these flavor degrees of freedom. The first and simpler one is in the quenched or probe approximation, where the branes are introduced in the background in a consistent way, but the space itself does not feel their backreaction. In the field theory side this is equivalent to neglecting the fermionic determinant, in the lattice calculations for example. In the probe approximation one can study how the mesons are influenced by the gluons of the theory, but not how the glueballs are affected by the dynamical quarks. Therefore, at this approximation the static potential is not affected by the screening due virtual quark-antiquark pairs. The more demanding way to introduce the flavor branes is to take into account their backreaction on the space. This corresponds to the dual gauge theory having the dynamics of glueballs and mesons influence each other. The accurate way to take into account the flavor branes backreaction is to consider them localized. However this is a demanding computational problem so there are some approximate approaches that have been developed too. In some cases like the case of the Sakai-Sugimoto model [4], one can calculate the effects of the backreaction perturbatively [5]. An other approximate approach to consider is the smearing of the flavor branes instead of a localization [6]. This approximation simplifies the differential equations, but the smearing breaks the $U(N_f)$ flavor group to $U(1)^{N_f}$ and hence theories with $U(N_f)$ flavor groups can not be examined exactly.

In this note, motivated by the lattice approach and the gauge/gravity computational difficulties away from the probe limit, we review certain results of the flavor backreacted gauge/gravity dualities. We focus more on the perturbatively flavor backreacted Sakai-Sugimoto model and the relevant results of the static potential and force [7]. As a second application, we present a certain quantity that we call Coulomb's constant in five dimension, we argue that it is weakly *N*-dependent and then we proceed by computing it in the Witten D4 background on one hand and in a lattice Mean-Field (MF) expansion for $SU(2)$ on the other [15].

1. Perturbatively flavor backreacted Sakai-Sugimoto model

To study the static potential in presence of dynamical quarks we use the Sakai-Sugimoto background which is the closest model so far to the holographic description of large N_c QCD. We are taking advantage of the fact that the backreaction of the localized branes has been found near the tip of the geometry for the D8 and D̄8 branes placed on the antipodal points of the compactified circle. Then we argue that the leading corrections to the static quark potential for the particular

Wilson loop we consider come from this region. We have checked this assumption numerically in several cases.

The Sakai-Sugimoto background is the extension of the D4 Witten model where in order to realize the flavor symmetry, N_f D8 branes and N_f $\bar{\text{D}}8$ branes transverse to a compactified direction and localized have been introduced. The non-flavored background is the near horizon limit of N_c D4 color branes, where one spatial direction is compactified on S^1 with radius ρ. The submanifold spanned by the radial coordinate u and the spatial x_4 has a topology of a cigar which ends at $u = u_k$, where to avoid the singularity at the tip, one requires that

$$x^4 \sim x^4 + 2\pi\rho , \qquad \rho := \frac{2}{3}\frac{R^{3/2}}{u_k^{1/2}} . \tag{1.1}$$

In the probe approximation, where $N_c \gg N_f$, the effect of the flavor branes on the geometry is negligible and for our case this is not the interesting limit. The metric of the background in that case reads:

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)dx_4^2\right) + \left(\frac{R}{u}\right)^{3/2}\left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \tag{1.2}$$

$$e^\phi = g_s\left(\frac{u}{R}\right)^{3/4} , \quad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\varepsilon_4 , \quad f(u) = 1 - \frac{u_k^3}{u^3} , \quad R^3 = \pi g_s N_c l_s^3 ,$$

where $\mu = 0,..,3$ and $d\Omega_4^2$, ε_4 and $V_4 = 8\pi^2/3$ are the line element, the volume form and the volume of a unit S^4, respectively.

In the Sakai-Sugimoto model the D4 branes extend along x^μ, x_4 while the D8 branes along the coordinates x^μ, Ω_4 and u. At the boundary the branes are localized at $x_4 = 0$ (D8) and at $x_4 = L_8$ ($\bar{\text{D}}8$). In the bulk the flavor branes meet naturally and the chiral symmetry breaks since the group $U(N_f)_L \times U(N_f)_R$ which exists close to the boundary, breaks to the diagonal $U(N_f)$ inside the bulk. Taking the maximum distance for the favor branes at the boundary to be $L_8 = \pi\rho$, they meet at the lowest possible radial point $u = u_k$.

This theory is dual to the 5-d pure Yang-Mills theory and comes from a D4 brane metric solution. The dual theory in this limit is a $4+1$ dimensional theory with gauge group $SU(N_c)$, compactified on a circle with antiperiodic boundary conditions for the fermions. It is coupled to N_f left handed fermions and N_f right handed fermions in the fundamental representation of $SU(N_c)$. At energies lower than $M_{KK} = 1/\rho$ the dual gauge theory is effectively four dimensional confining. At the limit $\lambda_5 \ll \rho$, the theory is $3+1$ dimensional and approaches a pure Yang-Mills theory where the mass gap is exponentially suppressed compared to $1/\rho$.

In order to introduce backreaction in this model in the strong coupling regime one needs to study the D4-D8 system in the Romans massive type IIA supergravity. The corresponding equations for the dilaton and the graviton have been found [5], and can be solved perturbatively. The small flavor expansion parameter q_f used is taken as

$$q_f = g_s \kappa_{10}^2 K_8 = \frac{g_s N_f}{4\pi l_s} \tag{1.3}$$

where K_8 is proportional to the 8-dim brane tension and the κ_{10} is proportional to the 10-dim Newton gravitational constant and q_f has dimension of inverse length.

The solution when $u \to u_k$ is parameterized as

$$ds^2 = e^{2A_1(u,x_4)}\left(-dt^2 + dx_i^2\right) + e^{2A_2(u,x_4)}dx_4^2 + e^{2A_3(u,x_4)}du^2 + e^{2A_4(u,x_4)}d\Omega_4^2 \qquad (1.4)$$

$$\phi(u,x_4) = \frac{1}{2}\hat{\phi}(u,x_4) + 2A_1(u,x_4) + 2A_4(u,x_4) , \quad F_{(4)} = Q_c V_4 ,$$

where $Q_c = 3R^3/g_s = 3\pi N_c l_s^3$, and the functions in the exponentials are expanded as

$$A_i(u,x_4) = A_{u,i}(u) + q_f A_{d,i}(u,x_4) , \quad \phi(u,x_4) = \phi_u(u) + q_f \phi_d(u,x_4) ,$$

where the u subscripted functions are the usual undeformed solutions of the $q_f = 0$ equations, i.e. their exponents are equal to the metric elements of (1.2) and the index i runs from one to four. The functions determining the backreaction around u_k turn out to be

$$A_{d,1} = \frac{\sqrt{u^3 - u_k^3}\sin\left(\frac{x_4}{\rho}\right)\left(u_k^3\left(7 + 2\cos\left(\frac{2x_4}{\rho}\right)\right) + 4\sin\left(\frac{x_4}{\rho}\right)^2 u^3\right)}{54 u_k^{7/2}} ,$$

$$A_{d,2} = A_{d,3} = \frac{4}{5}\sqrt{\frac{u^3 - u_k^3}{u_k}}\sin\left(\frac{x_4}{\rho}\right) , \quad \phi_d = \frac{1}{3}\sqrt{\frac{u^3 - u_k^3}{u_k}}\sin\left(\frac{x_4}{\rho}\right) \qquad (1.5)$$

and the corresponding metric elements

$$g_{00} = -g_{xx} = -\exp\left(\frac{q_f \sin\left(\frac{x_4}{\rho}\right)\sqrt{u^3 - u_k^3}\left(u_k^3\left(7 + 2\cos\left(\frac{2x_4}{\rho}\right)\right) + 4\sin\left(\frac{x_4}{\rho}\right)^2 u^3\right)}{27 u_k^{7/2}}\right)\left(\frac{u}{R}\right)^{3/2} ,$$

$$g_{44} = \exp\left(q_f \frac{8}{5}\sin\left(\frac{x_4}{\rho}\right)\sqrt{\frac{u^3 - u_k^3}{u_k}}\right)\left(1 - \frac{u_k^3}{u^3}\right)\left(\frac{u}{R}\right)^{3/2} , \qquad (1.6)$$

$$g_{uu} = \exp\left(q_f \frac{8}{5}\sin\left(\frac{x_4}{\rho}\right)\sqrt{\frac{u^3 - u_k^3}{u_k}}\right)\left(1 - \frac{u_k^3}{u^3}\right)^{-1}\left(\frac{u}{R}\right)^{-3/2} . \qquad (1.7)$$

Therefore we have obtained a confining background, which is a deformation of the (1.3) at certain limits has analytically known flavor perturbatively backreacted metric.

1.1 Static Potential and Force

To calculate the static potential in our theory we are taking advantage of the properties of the Wilson loops and the corresponding minimal surfaces in confining backgrounds. The first fact is that in a confining background it can be shown that the the contribution to the area of the minimal surfaces ending on the polygonal Wilson with one large enough side L corresponding to the $Q\bar{Q}$ distance, comes from the region which is close to the tip of the cigar of the geometry. This is exactly where we know analytically the backreaction of our background. Therefore, it is natural to think that the leading flavor contributions come from this area. When we are calculating the area of the corresponding minimal surface we use the backreacted metric close to the tip of the geometry, where we know that almost the total contribution to the area of the minimal surface comes, and

away of the tip we use the non-backreacted metric. So our results expected to capture the leading flavor contributions.

The second fact that we use is that we argue that the perturbative backreacted background contains information about the breaking of the string and moreover the breaking happens at relatively large lengths. This assumption is supported by several recent results as for example the analysis in the Veneziano limit of the Maldacena-Nunez background where backreaction of the smeared flavor branes have been considered [8][1]. It seems, and it is normal to expect, that the backreacted background contains the information of the string breaking and it appears as a cusp in the Wilson loop calculation. Moreover, as the number of flavor branes is reduced compared to color branes or equivalently the backreaction flavor effects get weaker, the length where the string breaking happens increases. In the model under consideration the backreaction comes as perturbation to the original background and is extremely weak, therefore we expect that the string breaking will appear in the Wilson loop at very large L, where due to the constraints in numerical integrations we could not see it clearly. This is a peculiarity of the particular model. Nevertheless, it allows us to consider long strings in the theory before they break. Configuration that is essential in order to use the contributions to the minimal surface of the tip of the geometry.

In order to find the appropriate minimal surface we use the following string configuration:

$$t = \tau, \qquad x_1 = \sigma, \qquad x_4 = \pi\rho/2, \qquad u = u(\sigma) \ . \tag{1.8}$$

The string between the two heavy quarks, is localized at the boundary in the x_4 dimension at an equal distance from the D8 and $\bar{\text{D}}8$ branes. The string is attached to the boundary and hangs inside the bulk down to a minimum value u_0. In our analysis this minimum is chosen to be close to u_k so that most of the string lies close to the tip of the geometry. Then the leading flavor effects for the particular Wilson loop come from the region of the world-sheet near u_k.

Then the equation of motion and the corresponding energy come from the analysis of the Nambu-Goto action

$$S = -\frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{-g_{00}\left(g_{11} + g_{44}x_4'^2 + g_{uu}u'^2\right)} \ , \tag{1.9}$$

where the metric elements close to u_k are the backreacted ones and away of it are the ones in the probe approximation. We will avoid here the details of the calculations that can be fond is [7]. The method is based on expansions of certain quantities around u_k and with respect to the perturbation backreaction parameter q_f. In the end we manage to find the expression of static potential as a function of L, which is the distance of the Q$\bar{\text{Q}}$ pair, the flavor perturbation backreaction parameter q_f. and the scale of the theory u_k:

$$V = V(L, q_f, u_k) \ . \tag{1.10}$$

The static potential has a constant term that has not a physical interpretation. In order to make comparisons with other theories we need to consider the static force

$$\frac{\partial V}{\partial L} = \frac{\partial V}{\partial L}(L, q_f, u_k) \ , \tag{1.11}$$

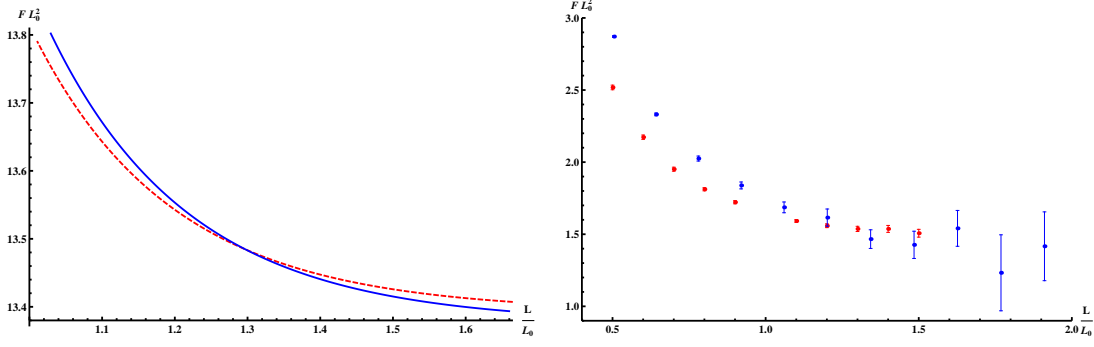[1]Other backreacted models that contain studies of Wilson loops can be found for example in [9].

Figure 1: The plots of the static $Q\bar{Q}$ force, as a function of the heavy quark distance. On the left the results obtained from the gravity dual analysis. On the right, we present indicative lattice results for $N_f = 0$ and $N_f = 2$. With the red dashed line (red points) are the results of the non-flavored case and with the blue solid line (blue points) the results of the flavored case of the Sakai-Sugimoto model (lattice analysis). In the lattice data L_0 is the Sommer scale.

where we get rid off the constant. Our results are plotted on the left of the Figure 1. For large L/L_0, the static force in the flavored background is less than the one in the non-flavored case. This is be due to the fact that in the presence of flavors, there are screening effects caused by the virtual quark-pairs produced between the heavy quarks, and reduce the color force.

On the right of the Figure 1 we plot some indicative lattice QCD results for $N_f = 0$ and $N_f = 2$. The $N_f = 0$ lattice data is taken from [11] and the $N_f = 2$ data from [10] and [12]. In the two sets of data the extrapolated to zero quark mass value $L_0/a = 7.05$ has been kept fixed, with L_0 the Sommer scale and a the lattice spacing. The two plots should not be strictly compared due to several reasons. The most important one is the limitations of the approximations in the holographic calculation when L becomes smaller, and is not certain if the fixed scales in the two approaches have the same physical meaning. However, it is worth noticing the similarities that the plots have.

1.2 The ultraviolet limit and Coulomb's constant in five dimensions

Apart from the flavor backreacted case, the Witten background offers the opportunity for another non-trivial check of the gauge-gravity duality in the ultraviolet limit, despite the fact that in this limit the dual gauge theory is not close to QCD. It is a five-dimensional pure $SU(N)$ gauge theory compactified on a circle of radius ρ in the large N limit where the radius goes to infinity in the UV limit. The Wilson loop analysis gives the following system of equations

$$L = 3\rho\sqrt{A} \int_1^\infty \frac{dy}{\sqrt{(y^3 - A^3)(y^3 - 1)}} \tag{1.12}$$

and

$$V = \frac{u_k}{(2\pi\alpha')^2} \frac{2}{A} \left\{ \int_1^\infty dy \left[\frac{y^3}{\sqrt{(y^3 - A^3)(y^3 - 1)}} - \frac{1}{\sqrt{1 - \frac{A^3}{y^3}}} \right] - \int_A^1 dy \frac{1}{\sqrt{1 - \frac{A^3}{y^3}}} \right\}, \tag{1.13}$$

where u_k is the minimal length in the radial direction, u_0 is the turning point of the string and the integrals are expressed in terms of the two dimensionless quantities $A = u_k/u_0$ and $y = u/u_0$.

One then has to invert the first of these and substitute in the second so that finally an analytical expression for $V(L)$ is obtained. The result is of the five-dimensional Coulomb form [7]

$$V(L) = \text{const.} - \frac{c_2}{L^2}, \qquad c_2 = \frac{1}{54\pi^2}\left(\frac{\sqrt{\pi}\Gamma(2/3)}{\Gamma(7/6)}\right)^3 \lambda_5. \tag{1.14}$$

Here $\lambda_5 = g_5^2 N$ with g_5 the five-dimensional gauge coupling constant. The additive constant in this case turns out to be zero and this allows us to use the potential directly without having to take its derivative. The interesting fact here is that the expression for c_2/λ_5 is a number that seems to be independent of N, at least in the limit where the computation is valid. Moreover, this ratio is in principle directly computable in the gauge theory. One can therefore test the duality and the degree of the expected weak N-dependence by computing in some way this quantity for $N = 2$. The complication is that since the duality works in a limit where the gauge theory is strongly coupled, one can not do this in the perturbative limit of the gauge theory. All this forces us to use a lattice regularization and then the only analytical handle near the bulk first order phase transition of these theories is the Mean-Field expansion. We remind that $SU(N)$ gauge theories in infinite volume have two phases. At weak coupling there is a Coulomb phase and at strong coupling a confined phase. The duality tells us that the system is in the Coulomb phase and in order that it is strongly coupled it must be near the phase transition that separates the two phases.

To be precise one has to first express the ratio defined above in terms of lattice parameters. Introducing the lattice spacing a and the dimensionless lattice coupling $\beta = 2Na/g_5^2$ we have

$$\bar{c}_2 = \frac{1}{27\pi^2}\left(\frac{\sqrt{\pi}\Gamma(2/3)}{\Gamma(7/6)}\right)^3 \frac{N^2}{\beta}. \tag{1.15}$$

This motivates us to define the quantity [15]

$$k_5 = \bar{c}_2 \frac{\beta}{N^2} \tag{1.16}$$

with $\bar{c}_2 = c_2/a$, which is a sort of analogue to Coulomb's constant in five dimensions. The holographic computation gave us for it the specific prediction $k_5 = \left[\frac{B(2/3,1/2)}{3\pi^{2/3}}\right]^3 = 0.0649\cdots$ with $B(x,y)$ the Euler Beta function.

On the lattice, k_5 depends on β and the size of the lattice. Its dependence on β is useful because it allows us to approach the phase transition on the phase diagram. The finite size of the lattice however must be taken out because the gravitational computation assumes an infinite volume. We denote the size of the lattice by L_l (not to be confused with the string length above), and we compute the static potential in the MF expansion of a five-dimensional pure lattice gauge theory. The result [13] reads

$$aV(r/a) = -2\log(\bar{v}_0) - \frac{1}{\bar{v}_0^2}\frac{1}{L_l^4}\sum_p\sum_{M\neq 0}\delta_{p_0,0}\left\{(\frac{1}{4}\cos(p_M r)+1)K_{00}^{-1}(p,0)\right.$$

$$\left. + \sum_A(\frac{1}{4}\cos(p_M r)-1)K_{00}^{-1}(p,A)\right\}. \tag{1.17}$$

Here $\bar{v}_0$ is the value of a lattice link in the MF background, $p_M, M = 0, \cdots, 4$ are the five dimensional lattice momenta and K_{00} is the 00-component of the lattice propagator $K_{M'M''}$. On Figure 2 we
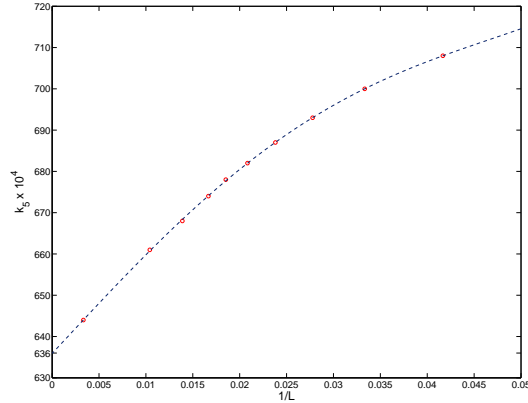
Figure 2: The infinite lattice size L extrapolation of k_5 in the MF expansion of the $SU(2)$ lattice gauge theory.

present the data for k_5 for a set of lattices, while approaching the phase transition along a "line of constant physics", that is along a non-perturbative renormalization group trajectory. The infinite volume extrapolation gives us the result $k_5 = 0.0636$, in a surprisingly good agreement with the gravitational result. The deviation is approximately 2%. While this numerical agreement can not be considered as a proof, it is a strong evidence for the quantity under discussion being indeed weakly N-dependent.

2. Discussions

In this note we have reviewed aspects of the perturbative flavor backreacted Sakai-Sugimoto model. Using the properties of the orthogonal Wilson loops and the corresponding minimal surfaces as well as the peculiarities of a perturbative flavor backreacted model we have studied the static potential. For relatively large Q$\bar{\text{Q}}$ distances where our approximation is more accurate, we have observed screening in the static force compared to the probe approximation approach. We have argued due to the peculiarity of the perturbative model that the string breaking will appear for longer distances than the ones examined here.

From the point of view of minimal surfaces we have noticed some interesting properties. String world-sheets with the same length L, have lower energy in the undeformed background than in the deformed backreacted one at least in the region where $u_0 \sim u_k$. Moreover, string world-sheets that extend same distances inside the bulk have lower energy in the flavored background, and need smaller lengths L to reach the minimum distance u_0. Therefore, to have the same Q$\bar{\text{Q}}$ energy in the flavored and non flavored background, the flavored string is extended less and has smaller length L than the one in the original background. These observations on minimal surfaces are expected to carry on for the full backreacted solution too.

In our calculations the terms coming from the mass of the infinite quark get important contributions no matter how far from u_k the turning point u_0 is. This situation would be different, if for the regularization scheme the Legendre transform was used. However to use this renormalization to subtract the divergence, the conditions for the background derived in [14] must be satisfied. In

our approximation it seems that these are satisfied at least approximately but would be interesting if one could check it for the whole backreacted background and make the relevant analysis there.

There are several interesting ideas following our study. A challenging problem would be to obtain the full backreacted solution of the Sakai-Sugimoto model and even better by including the bifundamental tachyon field in order to have massive pions. If the full solutions will be obtained, for small backreaction where the number of flavored branes compared to the color ones is small, our results should be reproduced. Additionally, in this background one should be able to study more realistically the string breaking.

The screening after some distance L that we have noticed here is expected to be generic feature of all the theories when backreaction of the flavor branes is taken into account. It is interesting to study this effect in presence of flavor backreaction in the finite temperature anisotropic theories, which expected to describe the quark-gluon plasma at the early times after its creation. The interest in the study comes because the strength of the anisotropy might turn out to depend on the density of the fermions, depending on the way that the anisotropy is introduced. In that case the screening effect behavior, could be non-trivial depending on the strength of the anisotropy that the theory has. In the context of anisotropic gauge/gravity dualities the study of the static potential has been done in [16] using the duality [17]. Notice that in the weakly coupled anisotropic theories there are relevant results as for example in [18].

Finally, in this note we have presented an additional comparison of the Coulomb's constant for $SU(N)$ gauge theories in five dimensions in the quenched approximation of the flavor backreacted confining model we used (the D4 Witten's model) using a holographic Wilson loop calculation, and a lattice mean-field expansion computation for $N = 2$. We have argued and found that such a comparison makes sense because we are dealing with a quantity that seems to be weakly N dependent to a good approximation beyond the large N limit. The agreement of the results, numerically was found to be within approximately 2%.

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