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The Effective Action of Branes in Calabi-Yau Orientifolds

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We discuss the four-dimensional $\mathcal{N}=1$ supersymmetric effective actions of a single space-time filling D6-brane in general Type IIA Calabi-Yau orientifold compactifications. The $\mathcal{N}=1$ Kähler potential, the gauge-coupling function, the superpotential and the D-terms are determined as functions of fields describing brane deformations and U(1) fields on the brane. In general, there will be an infinite number of independent moduli for the deformations, but this moduli space reduces to a finite-dimensional space of special Lagrangian submanifolds upon imposing F- and D-term supersymmetry conditions. Via mirror symmetry we map the data obtained to single brane setups in Type IIB theory.

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1. Introduction

In the last decade our understanding of effective actions of Type IIA and IIB string theory has considerably increased. Knowing what features a field theory can or cannot have if derived from string theory is a important step in predictability of the latter. Effective actions arising from setups with D-Branes are of great interest for phenomenology, since stacks of D-Branes give rise to effective non-Abelian Yang-Mills theories living on the brane, and intersection of D-Branes give a natural way of constructing bi-fundamental chiral matter (for example, see [3]).

The present work, based on [1] (also studied independently by [2]), describes the $\mathcal{N} = 1$ supersymmetric four-dimensional low-energy effective action of a Type IIA string theory compactified on a general Calabi-Yau orientifold with a single D6-Brane. That is, we consider the supergravity limit of Type IIA string theory, and we add to it light modes of the degrees of freedom living on the Brane (the open-string fields). We then work to write the resulting action in the standard $\mathcal{N} = 1$ form and extract the characteristic data, namely the gauge coupling functions, superpotential, D-term-inducing gaugings, and the corresponding moduli space Kähler potential and chiral coordinates.

With the data in hand, we look for connections via mirror symmetry to the known works for single D-Branes in Type IIB theory compactified on Calabi-Yau orientifolds, [4, 5, 6]. Mirror symmetry is a well established duality in Type II string theories. It states that Type IIA string theory compactified on a Calabi-Yau Y is dual to Type IIB string theory compactified on another Calabi-Yau $\tilde{Y}$, the *mirror* of the first, with even and odd cohomologies exchanged from one another. The closed string sector effective action on Calabi-Yau orientifold compactifications was already discussed in [7]. In the present work we extend the analysis to include the data from single D-Branes.

We thus proceed as follows. In section 2 we perform the dimensional reduction of the Calabi-Yau by Kaluza-Klein expansions of the fields and keeping, in a first moment, only the massless modes. This restriction on the brane moduli will also preserve supersymmetry conditions. From the reduction we extract the gauge coupling functions, and compute corrections coming from mixings with closed string gauge fields. We identify the correct chiral coordinates and write an expression for the Kähler potential of the moduli space of open-string fields. In section 3, we allow for brane modes that can break the supersymmetry conditions. These modes give rise to a scalar potential in the effective action, that can be independently obtained by D-terms and a suitable Superpotential. Finally in section 4 we discuss how to relate our setup to the Type IIB setups known in the literature. The SYZ conjecture [17], that says that mirror symmetry can be understood as T-Duality on a T^3 torus of a T^3 fibred Calabi-Yau, will be useful to understand the precise mirror mapping of the brane modes. Results here obtained can be extended to richer and phenomenologically more interesting scenarios. We refer the reader to [1], for a more detailed analysis.

2. The dimensional reduction of the action

The first step in our analysis to obtain the four-dimensional effective action is to specify the background. We will work in a direct product of a 4D Minkowski space $\mathbb{R}^{1,3}$ and a general Calabi-Yau orientifold Y. We introduce in our setup space-time filling D6-branes.

PoS(CORFU2011)037

Requiring that we have $\mathcal{N}=1$ supersymmetry in 4d spacetime, this fixes the orientifold projection to obey [7, 8]

$$\mathcal{O}=(-1)^{F_L}\Omega_p\sigma^* \ , \qquad \sigma^*J=-J \ , \qquad \sigma^*C\Omega=\overline{C\Omega} \ , \tag{2.1}$$

Here Ω_p is the world-sheet parity reversal, F_L is the space-time fermion number in the left-moving sector, σ is an anti-holomorphic and isometric involution of the compact Calabi-Yau, J the Kähler form, and C is a function that contains the dilaton and normalizes the holomorphic three-form Ω as

$$e^{2\phi}C\Omega\wedge\overline{C\Omega}=\frac{1}{6}J\wedge J\wedge J \ . \tag{2.2}$$

Supersymmetry also implies conditions on the orientifold plane and the 3-cycle L_0 wrapped by the brane, namely [9]

$$J|_{\text{O6-plane}}=0 \ , \qquad \text{Im}(C\Omega)|_{\text{O6-plane}}=0 \qquad \text{and} \tag{2.3}$$

$$J|_{L_0}=0 \ , \qquad \text{Im}(C\Omega)|_{L_0}=0 \ , \qquad 2\,\text{Re}(C\Omega)|_{L_0}=e^{-\phi}\text{vol}_{L_0} \ . \tag{2.4}$$

The first two conditions in (2.4) are just the statement that the brane must wrap *Special Lagrangian cycles*, while the last one states that the L_0 has minimum volume. If we allow for fluxes in our setup, it was shown in [10] that supersymmetry implies

$$F_{\text{D6}}-B_2|_{L_0}=0 \ , \tag{2.5}$$

where F_{D6} is $U(1)$ flux on the brane and $B_2|_{L_0}$ is the NS-NS two-form restricted to the wrapped cycle.

2.1 The four-dimensional Kaluza-Klein spectrum

The action in four dimensions will be obtained from compactification of the full action, that is the action on the world-volume of the brane and the 10D Type IIA supergravity action. For that, we perform a Kaluza-Klein expansion of the fields and keep only the massless states (as we take the compactification energy scale high enough). That is, in the internal space the fields will be expanded in harmonic forms of the Calabi-Yau.

Closed string sector

The closed string spectrum [7] consists of the 10d metric, the dilaton, the NS-NS B-Field and the R-R fields C_1,C_3,C_5,C_7. After the orientifold projection, the surviving fields must obey (2.1), $\sigma^*B_2=-B_2$ and $\sigma^*C_p=(-1)^{(p+1)/2}C_p$.

One can split the de Rham cohomologies into even and odd eigenspaces under the involution $\sigma*$, $H^n_\pm$. Since J and B_2 obey the same condition under the anti-holomorphic involution, we can define the complex field $J_c=B_2+iJ$ and expand it in $H^2_-(Y,\mathbb{R})$,

$$J_c=B_2+iJ=(b^a+iv^a)\,\omega_a=t^a\omega_a \ , \tag{2.6}$$

where $a=1,\dots,h_-^{(1,1)}$ labels a basis ω_a of $H^2_-(Y,\mathbb{R})$. The 4d scalar fields t^a are in $\mathcal{N}=1$ chiral multiplets [7]. Similarly we can define

$$\Omega_c=2\,\text{Re}(C\Omega)+iC_3^{\text{sc}}=N'^k\alpha_k-T'_\lambda\beta^\lambda \ , \tag{2.7}$$

where $k = 1,\dots,n_-, \lambda = 1,\dots,n_+$ label a basis $(\alpha_k, \beta^\lambda)$ of $H^3_+(Y,\mathbb{R})$. Here C_3^{sc} is the component of the R-R three-form which is also a three-form on the Calabi-Yau manifold Y and hence descends to scalars in four dimensions,

$$C_3^{\text{sc}} = \xi^k \wedge \alpha_k + \xi_\lambda \wedge \beta^\lambda \ . \tag{2.8}$$

In $\mathcal{N}=2$ compactifications we can expand $\Omega = X^K\alpha_K - \mathcal{F}_K\beta^K$ with (α_K,β^K) a symplectic basis of $H^3(Y,\mathbb{R})$. Here X^K are the Special Geometry coordinates and $\mathcal{F}_K$ the X^K-derivative of the prepotential $\mathcal{F}$. The involution σ^* splits this basis into (α_k,β^λ) of $H^3_+(Y,\mathbb{R})$ and a dual basis (α_λ,β^k) of $H^3_-(Y,\mathbb{R})$, so we can identify

$$N'^k = 2\,\text{Re}(CX^k) + i\,\xi^k \ , \qquad T'_\lambda = 2\,\text{Re}(C\mathcal{F}_\lambda) + i\,\tilde{\xi}_\lambda \ . \tag{2.9}$$

Additionally, the R-R three-form also leads to $U(1)$ vectors in four space-time dimensions via the expansion terms

$$C_3^{\text{vec}} = A^\alpha \wedge \omega_\alpha \ \ , \tag{2.10}$$

where ω_α is a basis of $H^2_+(Y,\mathbb{R})$. Their holomorphic gauge coupling functions $f_{\alpha\beta}$ have also been determined in ref. [7]. Denoting by $\mathcal{K}_{\alpha\beta a} = \int_Y \omega_\alpha \wedge \omega_\beta \wedge \omega_a$, the intersection form of two elements of $H^2_-(Y,\mathbb{R})$ with one element of $H^2_+(Y,\mathbb{R})$ one finds that $f_{\alpha\beta} = i\mathcal{K}_{\alpha\beta a}t^a$.

Open string sector

To study the open string spectrum, we consider fixed background Kähler and complex structure moduli. The first fields we analyze are the ones corresponding to brane deformations that do not break $\mathcal{N}=1$ supersymmetry, that is, preserve special Lagrangian conditions (2.4). If we describe the deformation by a vector field η normal to the special Lagrangian cycle L_0, it was shown by McLean [11] that the subset of deformations through special Lagrangian submanifolds are the ones in which the 1-form defined as $\theta_\eta = \eta \lrcorner J$ is harmonic. We can thus expand θ_η in a basis of harmonic one-forms θ_i on L_0 as

$$\theta_\eta = \eta^i\theta_i \tag{2.11}$$

with the basis defined as

$$\theta_i = s_i \lrcorner J|_{L_0} \ , \qquad *\theta_i = -2e^\phi s_i \lrcorner \text{Im}(C\Omega)|_{L_0} \ , \qquad i = 1,\dots,b^1(L_0) \ , \tag{2.12}$$

where s_i is a basis of the real special Lagrangian normal deformations, $\eta = \eta^i s_i$ and the condition for $*\theta_i$ follows from [12].

Additionally to brane deformations, the spectrum also contains Wilson line the $U(1)$ gauge boson A_{D6} on the D6-brane that compactifies to

$$A_{\text{D6}} = A + a^i\,\tilde{\alpha}_i \ , \tag{2.13}$$

where A is a $U(1)$ gauge field and the $a^i(x)$ are $b^1(L_0)$ real scalars in four dimensions, arising from U(1) Wilson lines wrapping non-trivial one-cycles of the D6-brane. The forms $\tilde{\alpha}_i$ provide a basis of $H^1(L_0,\mathbb{Z})$.

To summarize, one finds as massless variations around a supersymmetric vacuum $h_-^{(1,1)} + h^{(2,1)} + 1$ chiral multiplets from the bulk and $b^1(L_0)$ chiral multiplets (η^i, a^i) from the D6-brane. The precise organization of these fields into $\mathcal{N}=1$ complex coordinates will be described in the next section.

2.2 Effective action of D6-branes

Since the closed sector was already treated in details in [7], we calculate the contribution coming from the D6-brane to the four-dimensional low-energy effective action. To do so, we perform the field expansions described in previous section to reduce the D6-Brane action

$$S^{\mathrm{SF}}_{\mathrm{D6}} = -\int_{\mathcal{W}_7} d^7\xi\, e^{-\phi}\sqrt{-\det\left(\iota^*\left(g_{10}+B_2\right)-F_{\mathrm{D6}}\right)} + \int_{\mathcal{W}_7}\sum_{q\ \mathrm{odd}}\iota^*(C_q)\wedge e^{F_{\mathrm{D6}}-\iota^*(B_2)}\ . \tag{2.14}$$

The first term is the Dirac-Born-Infeld (DBI) action that describes the local dynamics of string fields on the brane (deformations and U(1) fields) while the second is the Chern-Simons action, that contains information on brane charges and is important for global consistencies. ι^* is the pullback onto the brane world-volume $\mathcal{W}_7 = \mathcal{M}^{3,1}\times L_0$.

Dirac-Born-Infeld Action

We start with the Dirac-Born-Infeld action. For simplicity, we start considering a zero background B-field. We expand the determinant in (2.14) to quadratic order in the fluctuations around the supersymmetric background, using the normal coordinate expansion of the metric

$$\iota^* g_{10} = \left(e^{2D}\eta_{\mu\nu} + g(\partial_\mu\eta,\partial_\nu\eta)\right)dx^\mu\cdot dx^\nu + \left(\iota^* g + \delta(\iota^* g)\right)_{mn} d\xi^m\cdot d\xi^n\ , \tag{2.15}$$

where g_{mn} is the induced metric on L, and $\delta(\iota^* g)_{mn}$ is the metric variation induced by the variation of the background Kähler and complex structure, set to zero by our assumptions, and the four-dimensional flat space metric $\eta_{\mu\nu}$ is in the Einstein frame.

We also perform the Kaluza-Klein expansion for the Field-Strength from (2.13),

$$F_{\mathrm{D6}} = dA_{\mathrm{D6}} = F + da^i\wedge\tilde{\alpha}_i + f_{\mathrm{D6}}, \tag{2.16}$$

where $f_{\mathrm{D6}}\in H^2(L_0)$ is a brane U(1) flux that we set to zero in most part of this work. The DBI action reduces to

$$S^{(4)}_{\mathrm{DBI}} = -\int \tfrac{1}{2}\mathrm{Re} f_{\mathrm{r}}\, F\wedge *F + e^{2D}\mathcal{G}_{ij}\, da^i\wedge *da^j + e^{2D}\widehat{\mathcal{G}}_{ij}\, d\eta^i\wedge *d\eta^j\ , \tag{2.17}$$

with the metrics

$$\mathcal{G}_{ij} = \tfrac{1}{2}e^{-\phi}\mathcal{G}(\tilde{\alpha}_i,\tilde{\alpha}_j) \qquad \widehat{\mathcal{G}}_{ij} = \tfrac{1}{2}e^{-\phi}\mathcal{G}(\theta_i,\theta_j) \tag{2.18}$$

and $\mathcal{G}(\tilde{\alpha},\tilde{\alpha}') = \int_{L_0}\tilde{\alpha}\wedge *\tilde{\alpha}'$ a "canonical" L^2-metric defined on the L_0 cycle. The real part of the gauge coupling function is given simply by the volume wrapped by the brane,

$$\mathrm{Re}\, f_{\mathrm{r}} = \int_{L_0} 2\,\mathrm{Re}(C\Omega)\ . \tag{2.19}$$

The real fields η^i and a^i are not good coordinates for the $\mathcal{N}=1$ theory since in those theories all the scalar fields must appear in chiral (complex) multiplets. It turns out that one can perform a consistent change of basis from θ_i to $\tilde{\alpha}_i$ via

$$\theta_i = \lambda_i^{\,j}\,\tilde{\alpha}_j\ , \qquad \tfrac{1}{2}e^{-\phi} * \theta_i = \mu_{ji}\,\tilde{\beta}^j\ , \tag{2.20}$$

where $\tilde{\beta}^i$ is a basis of $H^2(L_0,\mathbb{Z})$. We define then the chiral coordinates

$$\xi^i = u^i + ia^i \qquad \text{with} \qquad u^i = \int_{L_0} \eta \lrcorner J \wedge \tilde{\beta}^i, \tag{2.21}$$

and the DBI action can be written in the standard $\mathcal{N}=1$ form

$$S^{(4)}_{\text{DBI}} = -\int \tfrac{1}{2}\text{Re} f_{\text{r}}\, F \wedge *F + e^{2D}\mathcal{G}_{i\bar{j}}\, d\xi^i \wedge * d\bar{\xi}^{\bar{j}} \,, \tag{2.22}$$

that allows the straightforward introduction of the B-field, simply by defining $u_c = \int_{L_0} \eta \lrcorner J_c \wedge \tilde{\beta}^i$, and $\xi^i_c = u^i_c + ia^i$ implies that the introduction of B can be absorbed in a redefinition of a^i.

Chern-Simons Action

The Chern-Simons action also contains pullbacks of forms from the Calabi-Yau to the brane 3-cycle. Instead of performing a normal coordinate expansion as in (2.15), we take a different approach and parameterize the normal variations by introducing a four-chain $\mathcal{C}_4$ which contains the three-cycle L_η in its boundary

$$\partial \mathcal{C}_4 = L_\eta - L_0 \,, \tag{2.23}$$

where L_0 is the reference three-cycle, the supersymmetric background cycle.

Then, the Chern-Simons action can be rewritten as

$$S^{\mathcal{C}_4}_{\text{CS}} = \int_{\mathcal{W}_8} d\left[e^{F-B_2} \wedge (C_3 + C_5 + C_7)\right] \,, \tag{2.24}$$

with $\mathcal{W}_8 = \mathcal{M}^{3,1} \times \mathcal{C}_4$ such that $\mathcal{W}_7 \subset \partial \mathcal{W}_8$. This is in a similar spirit as the constructions in [13].

It is convenient to expand not the R-R forms individually, but rather the combination with the B-field,

$$\begin{aligned} \sum_{p=3,5,7} e^{-B_2} \wedge C_p \;=\; & (\xi^k \alpha_k - \tilde{\xi}_\lambda \beta^\lambda) + (A^\alpha \wedge \omega_\alpha + A_\alpha \wedge \tilde{\omega}^\alpha) \\ & + (C_2^\lambda \wedge \alpha_\lambda - \tilde{C}^2_k \wedge \beta^k) + (C_3^0 + C_3^a \wedge \omega_a + C^3_a \wedge \tilde{\omega}^a) \,, \end{aligned} \tag{2.25}$$

where $(\alpha_\lambda, \beta^k)$ is a basis of $H^3_-(Y,\mathbb{R})$, and ω_a, ω_α, $\tilde{\omega}^a$, $\tilde{\omega}^\alpha$ are respectively bases of $H^2_-(Y,\mathbb{R})$, $H^2_+(Y,\mathbb{R})$, $H^4_+(Y,\mathbb{R})$, $H^4_-(Y,\mathbb{R})$. The four-dimensional two-forms $(C_2^\lambda, \tilde{C}^2_k)$ are dual to the scalars $(\xi^k, \tilde{\xi}_\lambda)$, introduced in (2.9). The vectors A^α have been already introduced in (2.10), and A_α are their four-dimensional duals. The last brackets in (2.25) contains the four-dimensional three-forms (C_3^0, C_3^a, C^3_a) which although non-dynamical, contribute to the scalar potential as in ref. [6].

The four dimensional Chern-Simons action becomes

$$\begin{aligned} S^{(4)}_{\text{CS}} \;=\; & \int \tfrac{1}{2}\text{Im} f_{\text{r}}\, F \wedge F - (\delta_\lambda dC_2^\lambda - \delta^k d\tilde{C}^2_k) \wedge A - (\mathcal{I}_{i\lambda}\, dC_2^\lambda - \mathcal{I}^k_i\, d\tilde{C}^2_k) \wedge da^i \\ & + (a^j \Delta_{j\alpha}) dA^\alpha \wedge F + \tilde{\mathcal{J}}^\alpha dA_\alpha \wedge F + (a^j \Delta_{ja}) dC_3^a + dC^3_a\, \tilde{\mathcal{J}}^a \,, \end{aligned} \tag{2.26}$$

with couplings between open and closed fields given by

$$\begin{aligned} & \delta_\lambda = \textstyle\int_{L_0} \alpha_\lambda, \qquad \delta^k = \int_{L_0} \beta^k, \qquad \mathcal{I}_{i\lambda} = \int_{\mathcal{C}_4} \tilde{\alpha}_i \wedge \alpha_\lambda, \qquad \mathcal{I}^k_i = \int_{\mathcal{C}_4} \tilde{\alpha}_i \wedge \beta^k \,, \\ & \Delta_{ia} = \textstyle\int_{L_0} \tilde{\alpha}_i \wedge \omega_a, \qquad \Delta_{i\alpha} = \int_{L_0} \tilde{\alpha}_i \wedge \omega_\alpha, \qquad \tilde{\mathcal{J}}^a = \int_{\mathcal{C}_4} \tilde{\omega}^a \,, \end{aligned} \tag{2.27}$$

and the imaginary part of the gauge coupling function $\mathrm{Im} f_{\mathrm{r}} = \int_{L_0} C_3^{\mathrm{sc}}$, that together with (2.19),

$$f_{\mathrm{r}} = \int_{L_0} \Omega_c. \tag{2.28}$$

The coupling of $(C_2^\lambda, \tilde{C}_k^2)$ with A leads, after elimination of the two-forms in terms of their duals $(\xi^k, \tilde{\xi}_\lambda)$, to gaugings under A,

$$D\xi^k = d\xi^k + \delta^k A\,, \qquad D\tilde{\xi}_\lambda = d\tilde{\xi}_\lambda + \delta_\lambda A\,. \tag{2.29}$$

These gaugings will be used in the next section for the general deformation case to derive the scalar potential via D-terms.

The coupling of the brane gauge field A_{D6} with the R-R gauge field A^i appearing in the Chern-Simons action induces a kinetic coupling between the two gauge fields. The duality condition between C_3 and C_5 has to be satisfied, and to be consistent the action must be slightly modified. In the original work [1] we perform a careful treatment of this mixing, and it turns out that the mixing leads to a correction to the gauge coupling function f_{r},

$$f_{\mathrm{corrected}} = f_{\mathrm{r}} + 4\xi^j \Delta_{j\alpha} \tilde{\mathcal{J}}^\alpha. \tag{2.30}$$

2.3 Kähler Potential and $\mathcal{N}=1$ coordinates

To describe the moduli space of $\mathcal{N}=1$ theories, an object of fundamental importance is the Kähler potential. The metric of the moduli space should be obtained as a second derivative of the Kähler potential in terms of the chiral coordinates.

First we look at the open string moduli space. By fixing all the closed moduli, we can treat the open moduli $\xi^i = u^i + ia^i$ as being the only relevant moduli in the theory, and the Kähler potential K_o will be a function of the fields ξ^i that describes the moduli space and gives the metric $\mathcal{G}_{ij} = \partial^2 K_o / \partial\xi^i \partial\bar{\xi}^j$ in (2.22). A natural proposal for K_o that gives the correct metric is

$$K_o = -\frac{1}{2}\int_{\mathcal{C}_4} J \wedge \hat{\beta}^i \int_{\mathcal{C}_4} \mathrm{Im}(C\Omega) \wedge \hat{\alpha}_i. \tag{2.31}$$

In [1] we write similar expressions for the Kähler potential in Type IIB theories.

The closed moduli space in Type IIA orientifold compactifications was studied in [7]. It is locally a direct product of two moduli spaces, $\mathcal{M}^Q \times \mathcal{M}^K$, where $\mathcal{M}^K$ depends on the Kähler moduli and the B-field and $\mathcal{M}^Q$ is described by the complex structure moduli, the dilaton and the moduli from the R-R three-form. In terms of the coordinates introduced in section 2.1, $\mathcal{M}^K$ is described by t^a from $J_c = t^a \omega_a$ with Kähler potential

$$K^K(t-\bar{t}) = -\ln\left[\frac{4}{3}\int_Y J\wedge J\wedge J\right] = -\ln\left[\frac{i}{6}\mathcal{K}_{abc}(t-\bar{t})^a(t-\bar{t})^b(t-\bar{t})^c\right], \tag{2.32}$$

where $\mathcal{K}_{abc}$ is the triple intersection of $H^2_-(Y,\mathbb{R})$ basis elements, $\mathcal{K}_{abc} = \int_Y \omega_a \wedge \omega_b \wedge \omega_c$. The coordinates N'^k and T'_λ of the complex structure moduli space $\mathcal{M}^Q$ get corrected by the open moduli as

$$N^k = U^k - 2\,\partial_{V_k}(e^{2D}K_o) + i\xi^k\,, \qquad T_\lambda = U_\lambda - 2\,\partial_{V^\lambda}(e^{2D}K_o) + i\tilde{\xi}_\lambda, \tag{2.33}$$

where $V_k = 2e^{2D}\text{Im}(C\mathcal{F}_k)$, $V^\lambda = 2e^{2D}\text{Im}(CX_k)$ and D the four-dimensional dilaton. The Kähler potential is still given by

$$K^Q = -2\ln\left[i\int_Y C\Omega\wedge\overline{C\Omega}\right], \tag{2.34}$$

but should be evaluated in terms of $\xi^i + \bar{\xi}^i$, $N^k + \bar{N}^k$ and $T_\lambda + \bar{T}_\lambda$.

3. General deformations and the D- and F-term potential

In the previous section we considered only the harmonic modes of the open string fields, that preserve the supersymmetry conditions. In the following we discuss the inclusion of non-harmonic states, by introducing an infinite basis of fields from the Kaluza-Klein expansion. This infinite field space gives rise to a non-vanishing scalar potential coming from the DBI action. This scalar potential can be independently obtained from a holomorphic Superpotential that gives rise to F terms, and from D terms arising from gaugings of scalar fields.

3.1 An infinite tower of states

When performing a Kaluza-Klein reduction of the D6-brane action to four space-time dimensions now we would like to include all massive modes corresponding to arbitrary deformations of L_0 to L_η. This means that we include sections s_I of NL_0, the vector space normal to L_0, which yield one-forms in the contraction with J

$$\theta_I = s_I \lrcorner J|_{L_0} \quad \in \quad \Omega^1(L_0)\ . \tag{3.1}$$

For a compact L_0 it is possible to label these one-forms by indices $I = 1,\dots,\infty$ by considering the Kaluza-Klein eigenmodes of the Laplacian Δ_{L_0}. The zero modes $\Delta_{L_0}\theta_i = 0$ are precisely the harmonic forms θ_i introduced in (2.12). But since the basis θ_I depends explicitly on the metric inherited form the ambient Calabi-Yau manifold, it will be better to work with a general countable basis $\hat{\alpha}_I$ defined on L_0.

The gauge field on the brane, if we allow for massive modes, can be expanded as

$$A_{\text{D6}} = A^J\, h_J + a^I\, \hat{\alpha}_I\ , \tag{3.2}$$

where $h_J \in C^\infty(L_0)$ is a basis of functions on L_0 and $\hat{\alpha}_J \in \Omega^1(L_0)$ is a basis of general one-forms on L_0. The field-strength can be directly obtained as dA_{D6},

$$F_{\text{D6}} = F^J\, h_J - A^J\wedge dh_J + da^I\wedge\hat{\alpha}_I + \tilde{F}\ , \qquad \tilde{F} = a^I\, d\hat{\alpha}_I + f_{\text{D6}}\ , \tag{3.3}$$

where again $f_{\text{D6}} \in H^2(L_0,\mathbb{R})$ is the background flux of F_{D6} on L_0. Via the Hodge decomposition each one-form $\hat{\alpha}_I$ can be uniquely decomposed into a harmonic form, an exact form $d\hat{h}_I$ and an co-exact form $d^*\hat{\gamma}_I$ on L_0 as

$$\hat{\alpha}_I = \mu_I^i\tilde{\alpha}_i + d\hat{h}_I + d^*\hat{\gamma}_I\ , \tag{3.4}$$

where $\tilde{\alpha}_i$ are the $b^1(L_0)$ harmonic forms introduced in (2.13). The field strength can be thus written as

$$\begin{aligned} F_{\text{D6}} &= F^I\, h_I + da^j\wedge\tilde{\alpha}_j + \mathcal{D}\hat{a}^I\wedge dh_I + d\tilde{a}^J\wedge d^*\gamma_I + \tilde{F}\ , \\ \mathcal{D}\hat{a}^I &= d\hat{a}^I - A^I\ , \qquad \tilde{F} = \tilde{a}^I dd^*\gamma_I + f_{\text{D6}}\ . \end{aligned} \tag{3.5}$$

Where we can see that the scalars $\hat{a}^I$ are gauged by A^I.

Similar to what was done in section 2.2, the DBI action can be calculated in terms of the infinite set of fields and gives

$$S^{(4)}_{\rm DBI} = -\int \tfrac{1}{2}\mathrm{Re} f_{\mathrm{r}IJ} F^I \wedge *F^J + e^{2D}\mathcal{G}_{ij}\, da^i \wedge *da^j + e^{2D}\tilde{\mathcal{G}}_{IJ}\, d\tilde{a}^I \wedge *d\tilde{a}^J$$
$$+ e^{2D}\mathcal{G}_{IJ}\, \mathcal{D}\hat{a}^I \wedge *\mathcal{D}\hat{a}^J + e^{2D}\widehat{\mathcal{G}}_{IJ}\, d\eta^I \wedge *d\eta^J + V_{\rm DBI} * 1\ , \tag{3.6}$$

with potential $V_{\rm DBI}$ given by

$$V_{\rm DBI} = \frac{e^{3\phi}}{\mathcal{V}^2}\int_{L_0} d^*\theta_\eta \wedge *d^*\theta_\eta + \frac{e^{3\phi}}{\mathcal{V}^2}\int_{L_0}\Big(d\theta_\eta \wedge *d\theta_\eta + (\tilde{F} - B_2 - d\theta^B_\eta)\wedge *(\tilde{F} - B_2 - d\theta^B_\eta)\Big)\ . \tag{3.7}$$

Note that the two first terms of the scalar potential are precisely the ones that break special Lagrangian condition to the path of deformations, that is, they vanish if we impose deformations η expanded in terms of harmonic forms. The last term contains the condition on the flux (2.5). If the flux does not obey $F - B_2|_{L_0} = 0$, the action also acquires a scalar potential that breaks supersymmetry. There is also an additional term related to the B_2 component along the deformation, $\theta^B_\eta = \eta \lrcorner B_2$.

The next step is to obtain this scalar potential from F and D terms,

$$V_{\rm DBI} = V_F + V_D\ , \tag{3.8}$$

with

$$V_F = \frac{e^{3\phi}}{\mathcal{V}^2}\int_{L_0} d\theta_\eta \wedge *d\theta_\eta + (\tilde{F} - B_2 - d\theta^B_\eta)\wedge *(\tilde{F} - B_2 - d\theta^B_\eta)\ . \tag{3.9}$$

and

$$V_D = \frac{e^{3\phi}}{\mathcal{V}^2}\int_{L_0} d^*\theta_\eta \wedge *d^*\theta_\eta \tag{3.10}$$

F-term and Superpotential

The chiral multiplets can induce a scalar potential written in terms of a holomorphic superpotential W as

$$V_F = e^K \mathcal{G}^{IJ}\partial_{\zeta^I} W \overline{\partial_{\zeta^J} W}. \tag{3.11}$$

To specify W we define an extension of gauge field $A_{\rm D6}$ from L_0 to $\mathcal{C}_4$ (denoted $\mathcal{A}_{\rm D6}$ to distinguish from the one defined on L_0), such that the extension $\mathcal{F}_{\rm D6} = d\mathcal{A}_{\rm D6}$ satisfies

$$\mathcal{F}_{\rm D6}|_{L_0} = f_{\rm D6}\ , \qquad \mathcal{F}_{\rm D6}|_{L_\eta} = f_{\rm D6} + a^I d\hat{\alpha}_I\ . \tag{3.12}$$

One next defines the superpotential functional

$$W = \int_{\mathcal{C}_4} (J_c - \mathcal{F}_{\rm D6}) \wedge (J_c - \mathcal{F}_{\rm D6}) \tag{3.13}$$

depending on the open string data via the chain and $\mathcal{F}_{\rm D6}$ and on closed string data via the complexified Kähler form (2.6). This is an extension of the functional introduced in ref. [14], since we have included the B-field through the complex two-form J_c, and is similar in construction to the Superpotentials from [15].

It is possible to check explictly that W is holomorphic both in the complexified Kähler moduli t^a from (2.6) and the open moduli ξ^I, and from (3.11) this Superpotential yields precisely (3.9).

Gaugings and D-term

Finally, we also compute the D-term potential in (3.10) induced by the gaugings of the scalars $\hat{a}^I$in (3.5) and $(\xi^k, \tilde{\xi}_\lambda)$ in the generalization of(2.29),

$$D\xi^k = d\xi^k + \delta^k_I A^I \ , \qquad D\tilde{\xi}_\lambda = d\tilde{\xi}_\lambda + \delta_{I\lambda} A^I \ . \tag{3.14}$$

More precisely, these scalars are charged under the gauge transformations $A^I \to A^I + d\Lambda^I$ of the $U(1)$ vectors A^I as

$$\hat{a}^I \to \hat{a}^I - \Lambda^I \ , \qquad (\xi^k, \tilde{\xi}_\lambda) \to (\xi^k - \delta^k_I \Lambda^I, \tilde{\xi}_\lambda - \delta_{\lambda I} \Lambda^I) \tag{3.15}$$

The potential arising from D-terms can be calculated from

$$V_D = \tfrac{1}{2} \mathrm{Re} f^{AB} D_A D_B \ , \qquad \partial_A D_I = K_{A\bar{B}} X^B_I \ , \tag{3.16}$$

where X^B_I are the Killing symmetries appearing in the covariant derivatives (3.14). By direct calculation, the only non-vanishing contribution for the scalar potential is

$$V_D = \frac{e^{3\phi}}{\mathcal{V}^2} \int_{L_0} d * \theta_\eta \wedge * d * \theta_\eta \ . \tag{3.17}$$

yielding the remaining term obtained from dimensional reduction. The vanishing of the D-term potential, which is necessary in a supersymmetric vacuum, happens when the two-form $*\theta_\eta$ is closed.

4. Mirror Symmetry

In this final section we comment on the connection via mirror symmetry of the data obtained so far to what has been known in the literature for the effective actions of D3, D5 and D7 spacetime filling branes in Type IIB string theory. First we review mirror symmetry on general orientifold compactifications, without branes. Then we include brane moduli, and relate each single spacetime filling D-brane configuration using the SYZ conjecture.

4.1 Calabi-Yau Orientifold background without branes

In Type IIB compactifications, there are two possible orientifold action $\mathcal{O}$, corresponding to either O3/O7 or O5/O9 orientifold planes,

$$\begin{aligned} \mathcal{O}_1 &= \Omega_p \sigma_B (-)^{F_L} \ , & \sigma_B^* \Omega &= -\Omega \ , & \text{O3/O7} \ , \\ \mathcal{O}_2 &= \Omega_p \sigma_B \ , & \sigma_B^* \Omega &= \Omega \ , & \text{O5/O9} \ . \end{aligned} \tag{4.1}$$

Here σ_B is a holomorphic (instead of antiholomorphic, as in the Type IIA case) involutive symmetry $\sigma_B^2 = 1$ of the Calabi-Yau target space, and F_L is the space-time fermion number in the left-moving sector. The fields that survive the acting of (4.1) are not the same for both cases.

As in Type IIA orientifold compactifications, the moduli space in Type IIB also splits locally in a direct product $\mathcal{M}^K_B \times \mathcal{M}^Q_B$. Now $\mathcal{M}^K_B$ contains the complex-structure moduli and, up to leading order, is not affected by the inclusion of brane moduli, while $\mathcal{M}_B^Q$ contains the Kähler structure, B-field, dilaton and R-R-forms, and recieve corrections from open moduli. Imposing the symmetries (4.1) to the Calabi-Yau, we expand the surviving fields as

$$\text{O3/O7} \qquad \begin{array}{lll} B_2 = b^k\,\omega_k \ , & C_2 \ = \ c^k\,\omega_k \ , & k = 1,\dots,h_-^{(1,1)}(\tilde{Y}) \ , \\ J = v^\lambda\,\omega_\lambda \ , & C_4 = \rho_\lambda\,\tilde{\omega}^\lambda \ , & \lambda = 1,\dots,h_+^{(1,1)}(\tilde{Y}) \ . \end{array} \tag{4.2}$$

$$\text{O5/O9} \qquad \begin{array}{lll} B_2 = b^\lambda\,\omega_\lambda \ , & C_4 = \rho_\lambda\,\tilde{\omega}^\lambda \ , & \lambda = 1,\dots,h_-^{(1,1)}(\tilde{Y}) \ , \\ J = v^k\,\omega_k \ , & C_2 \ = \ \tilde{C}_2 + c^k \ , & k = 1,\dots,h_+^{(1,1)}(\tilde{Y}) \ . \end{array} \tag{4.3}$$

Note the difference in the definition of the basis labels k and λ. Although it might look confusing, this notation will allow us a direct mapping to the basis $(\alpha^k, \beta_\lambda)$ of $H^3(Y)$ in Type IIA.

In terms of the moduli from (4.2) and (4.3), the chiral coordinates for the moduli space $\mathcal{M}_B^Q$ of the four-dimensional effective action in each orientifold compactification (4.1) are [16]

$$\text{O3/O7} \qquad \begin{array}{l} \tau = C_0 + ie^{-\phi_B} \ , \qquad G^k = c^k - \tau b^k \ , \\ T_\lambda^{\prime B} = e^{-\phi_B}\tfrac{1}{2}\mathcal{K}_{\lambda\rho\sigma}v^\rho v^\sigma + i\rho_\lambda - i\tfrac{1}{2}\mathcal{K}_{\lambda k l}b^k G^l \ . \end{array} \tag{4.4}$$

$$\text{O5/O9} \qquad \begin{array}{l} t^{\prime k} = e^{-\phi_B}v^k - ic^k \ , \quad P_\lambda = \mathcal{K}_{\lambda\rho k}b^\rho t^k + i\rho_\lambda \ , \\ S = e^{-\phi_B}\mathcal{V} + ih - \tfrac{i}{2}\rho_\lambda b^\lambda - \tfrac{1}{2}P_\lambda b^\lambda \ . \end{array} \tag{4.5}$$

Recall that on a Calabi-Yau there is a rescaling invariance of the holomorphic three-form Ω, that allows us to fix one of its periods X^I, that we will name X^0 (and the corresponding cycle α_0). But in orientifold compactifications $H^3(Y)$ splits in H^3_+ and H^3_-. Whether α_0 is in H^3_+ or H^3_- will specify if we will discriminate N^0 or T_0. Consequently, this choice will dictate whether the Type IIA setup with O6 planes will be mirror to Type IIB with O3/O7 or to O5/O9. The mapping between the chiral coordinates can be shown to be, in the large complex structure and large volume limit, [16] (also reviewed in [1])

$$\begin{array}{lccc} & \text{Type IIB} & \leftrightarrow & \text{Type IIA} \\ O3/O7 & (-i\tau, -iG^k, -T_\lambda^{\prime B}) & \leftrightarrow & (N^{\prime 0}, N^{\prime k}, T_\lambda^{\prime A}) \\ O5/O9 & (t^{\prime k}, P_\lambda, S) & \leftrightarrow & (N^{\prime k}, T_\lambda^{\prime A}, T_0^{\prime A}) \ . \end{array} \qquad \begin{array}{r} (4.6) \\ (4.7) \\ (4.8) \end{array}$$

Here we have introduced the label A to T_λ of Type IIA side just to distinguish from the Type IIB coordinate T_λ^B.

Under mirror symmetry the odd and even cohomologies of a mirror pair of Calabi-Yau manifolds are interchanged. The basis $H^3(Y)$ in Type IIA can be therefore mapped to $H^{even}(\tilde{Y})$ as

4.2 Inclusion of Brane Moduli

We now comment on the brane moduli of Type IIB. We do not treat the D9 Brane case, since it corresponds to Type I String Theory.

A setup with a single D3 brane [4] adds 6 real fields ϕ^I corresponding to the possible movements of the brane in $\tilde{Y}$, since in the internal space the brane is a point. Using the inherited complex structure of the Calabi-Yau, one can combine the six deformations into three complex scalars ϕ^i, $i = 1,2,3$.

O3/O7

$H^3(Y)$	$H^{\text{even}}(\tilde{Y})$
$\alpha_0 \in H^3_+(Y)$	1
$\alpha_k \in H^3_+(Y)$	$\omega_k \in H^2_-(\tilde{Y})$
$\alpha_\lambda \in H^3_-(Y)$	$\omega_\lambda \in H^2_+(\tilde{Y})$
$\beta^k \in H^3_-(Y)$	$\tilde{\omega}^k \in H^4_-(\tilde{Y})$
$\beta^\lambda \in H^3_+(Y)$	$\tilde{\omega}^\lambda \in H^4_+(\tilde{Y})$
$\beta^0 \in H^3_-(Y)$	$\mathcal{V}^{-1}\text{vol}_{\tilde{Y}}$

O5/O9

$H^3(Y)$	$H^{\text{even}}(\tilde{Y})$
$\alpha_0 \in H^3_-(Y)$	1
$\alpha_k \in H^3_-(Y)$	$\omega_k \in H^2_+(\tilde{Y})$
$\alpha_\lambda \in H^3_-(Y)$	$\omega_\lambda \in H^2_-(\tilde{Y})$
$\beta^k \in H^3_-(Y)$	$\tilde{\omega}^k \in H^4_+(\tilde{Y})$
$\beta^\lambda \in H^3_+(Y)$	$\tilde{\omega}^\lambda \in H^4_-(\tilde{Y})$
$\beta^0 \in H^3_+(Y)$	$\mathcal{V}^{-1}vol_{\tilde{Y}}$

Table 4.1: The mirror mapping from the basis of $H^3(Y)$ to the basis of even cohomologies of the mirror Calabi-Yau $\tilde{Y}$ in $O3/O7$ and $O5/O9$ orientifold setups.

In a D7-Brane setup [5], the brane wraps a divisor S_+ of the Calabi-Yau, and has deformation moduli χ as well as Wilson line moduli a, since the brane U(1) field A can wrap non-trivial one-cycles. The "+" in S_+ indicates that the brane must wrap an even divisor under the orientifold projection. In other words, its volume form must be in $H^4_+(\tilde{Y})$. One can define a basis s_A of NS_+ for the deformations that can be mapped to (2,0)-forms $\mathcal{S}_A$ using the holomorphic 3-form, $\mathcal{S}_A = s_A \lrcorner \Omega$. Also, one define a basis γ_I of (0,1)-forms, with which we decompose $\chi = \chi^A s_A + \bar{\chi}^{\bar{A}} \bar{s}_{\bar{A}}$ and $a = a^I \gamma_I + \bar{a}^{\bar{I}} \bar{\gamma}_{\bar{I}}$.

On a Calabi-Yau with O5 plane, we include a D5 brane that wraps a curve Σ_+ in $\tilde{Y}$ [6]. Similarly as the D7 brane, the D5-brane has volume form in $H^2_+(\tilde{Y})$, the deformations are described by moduli ξ^A expanded in a basis of normal vectors s_A of $N\Sigma_+$, and non-trivial configurations of the U(1) field are described by Wilson line moduli a^I expanded in a basis of one-forms γ_I, with $I = 1, \ldots, \dim H_0^{(0,1)}(\Sigma_+)$.

Following [4, 5, 6], the corrections to the $\mathcal{N} = 1$ chiral coordinates (4.4) and (4.5) are

$$
\begin{aligned}
D3: &\quad (\tau'\ ,\ G^k\ ,\ T'^B_\lambda) \to (\tau\ ,\ G^k\ ,\ T^B_\lambda) \equiv (\tau'\ ,\ G^k\ ,\ T'^B_\lambda + i\omega_{\lambda i\bar{j}}\phi^i\phi^{\bar{j}}); \\
D7: &\quad (\tau'\ ,\ G^k\ ,\ T'^B_\lambda) \to (\tau\ ,\ G^k\ ,\ T^B_\lambda) \equiv (\tau' + \mathcal{L}_{A\bar{B}}\chi^A\bar{\chi}^{\bar{B}}\ ,\ G^k\ ,\ T'^B_\lambda + iC_{\lambda I\bar{J}}a^I\bar{a}^{\bar{J}}); \\
D5: &\quad (t'^k\ ,\ P_\lambda\ ,\ S') \to (t\ ,\ P_\lambda\ ,\ S) \equiv (t'^k + \mathcal{L}^k_{A\bar{B}}\xi^A\bar{\xi}^{\bar{B}}\ ,\ P_\lambda\ ,\ S' + \mathcal{C}_{I\bar{J}}a^I\bar{a}^{\bar{J}})\ ,
\end{aligned}
\tag{4.9}
$$

with couplings

$$
\mathcal{L}_{A\bar{B}} = \frac{\int_{S_+}\mathcal{S}_A \wedge \bar{\mathcal{S}}_{\bar{B}}}{\int_{\tilde{Y}}\Omega\wedge\bar{\Omega}}\ ,\quad \mathcal{C}_{\lambda I\bar{J}} = \int_{S_+}\omega_\lambda \wedge \gamma_I \wedge \bar{\gamma}_{\bar{J}}\ ,\quad \mathcal{L}^k_{A\bar{B}} = -i\int_{\Sigma_+} s_A \lrcorner \bar{s}_{\bar{B}} \lrcorner \tilde{\omega}^k\ ,\quad \mathcal{C}_{I\bar{J}} = i\int_{\Sigma_+}\gamma_I \wedge \bar{\gamma}_{\bar{J}}\ . \tag{4.10}
$$

Let us now turn to the discussion of mirror symmetry. One question we might ask is, to which particular D-brane will the D6-Brane be mapped in the mirror Calabi-Yau? The answer comes easily if we understand mirror symmetry via the SYZ description [17]. It was conjectured by Strominger, Yau and Zaslow that a Calabi-Yau manifold can be viewed as a three-torus T^3 fibration over a basis. Mirror symmetry would be T-duality along all the three-torus directions. Naturally, the fibration must contain singularities, since a smooth fibration would imply a manifold with $h^1(Y) \neq 0$ from the global 1-cycles on the torus, breaking the Calabi-Yau condition. In the following analysis we will however look far from the singular points.

Since T-Duality exchange Neumann and Dirichlet boundary conditions for open strings, it exchanges the dimensionality of the brane for each T-dualized cycle that the brane wraps on the three-torus. For example, a D6-Brane that wraps completely the T^3 torus, after T-duality becomes a point in the dual torus, thus corresponding to a D3 brane. In general, the number of T^3 one-cycles wrapped by the D6-brane specifies the brane configuration in the mirror side as the following table:

	D6	D3	D6	D7	D6	D5	D6	D9
T^3	×			×		×		×
	×			×	×			×
	×		×		×			×
Base							×	×
			×	×			×	×
			×	×	×	×	×	×

Table 4.2: It is summarized how mirror symmetry acts on different brane configurations. The table shows the six dimensions of the Calabi-Yau manifold, split into base and fiber. × indicates the directions wrapped by each brane. Different wrappings of a D6-brane correspond to different branes in the Type IIB side.

The SYZ picture of table (4.2) allow us to understand in a simple manner the relation between the open-moduli-corrected Type IIA chiral coordinates (N^k, T^A_λ), (2.33), and the corrected coordinates in Type IIB, (4.9). For example, the D3 brane induces first order corrections only to the coordinate T'^B_λ. If we expect the mirror map (4.6) to hold, the corrections to the Type IIA coordinates should read

$$\begin{aligned} -2\partial_{V^\lambda}(e^{2D_A}K_o) &\cong -i(\omega_\lambda)_{i\bar{j}}\phi^i\phi^{\bar{j}} \\ \partial_{V_0}(e^{2D_A}K_o) &= \partial_{V_k}(e^{2D_A}K_o) = 0\ , \end{aligned} \tag{4.11}$$

where the $\cong$ indicates a mirror map between the data in the evaluation of the derivative in Type IIA to the left-hand side in Type IIB. We will show in the following how to see easily the vanishing and non-vanishing contributions on the Type IIA side of (4.11).

First let us look at the gauge coupling functions. In the limit of vanishing open string moduli they are given by the analogous to the D6-brane gauge coupling function $f_{\text{D6}} = N^k \int_L \alpha_k - T_\lambda \int_L \beta^\lambda$,

$$f_{\text{D3}} = \tau\ , \qquad f_{\text{D5}} = t^\Sigma \int_{\Sigma_+} \omega_\Sigma\ , \qquad f_{\text{D7}} = T_S \int_{S_+} \tilde{\omega}^S\ , \tag{4.12}$$

where $\Sigma_+(S_+)$ is the curve(divisor) wrapped by the D5(D7)-brane, and they are obtained from a basis of homology by

$$\begin{aligned} [\Sigma_+] &= n^k\,[\Sigma_k]\ , \qquad \Sigma_k \in H_2^+(Y) \quad \text{and} \\ [S_+] &= n_\lambda\,[S^\lambda]\ , \qquad S^\lambda \in H_4^+(Y)\ . \end{aligned} \tag{4.13}$$

Therefore the forms appearing in (4.12) are, in terms of the cohomology basis, $\omega_\Sigma = n^k\omega_k$ and $\tilde{\omega}^S = n_\lambda\tilde{\omega}^\lambda$.

From the four internal dimensions the D7-brane wraps, locally two of them are along the T^3-fiber and the other two on the base, as seen from table 4.2. The mirror D6-brane, on the other hand, wraps one dimension on T^3-fiber and two dimensions on the base. It is also inferred from the gauge coupling function of the D7-brane (4.12) that $\tilde{\omega}^\lambda$ sits on the brane, therefore having two "legs" on the 3-Torus and two on the base. We define thus the notation $\tilde{\omega}^\lambda : (bbtt)$, where b and t correspond to base and torus components. Table 4.1 shows that $\tilde{\omega}^\lambda$ on the Type IIB side is mapped on the Type IIA side to β^λ. Therefore, from table 4.2, since β^λ must sit on the mirror D6-brane, it should satisfy $\beta^\lambda : (bbt)$. β^λ must be dual to α_λ on the Calabi-Yau manifold Y, thus $\alpha_\lambda : (btt)$. A similar analysis can be done for the D5 and D3-Branes, from where we obtain $\alpha_k : (btt)$, $\beta^k : (bbt)$, $\beta^0 : (bbb)$ and $\alpha_0 : (ttt)$.

Back to the D3 matching (4.11), we can use the expression for the open Kähler potential (2.31) and write

$$\partial_{V_0}(e^{2D_A}K_o) = \frac{1}{2}\int_{L_0} \hat{\alpha}_k \wedge \eta \lrcorner \beta^0 \int_{L_0} \hat{\beta}^k \wedge \eta \lrcorner J \; . \tag{4.14}$$

Since the brane wraps the three-torus, both integrands in (4.14) must be of the form (ttt). The normal directions of this D6-brane are all on the base, so $\eta \lrcorner \beta^0 : (bb)$, making the first integral vanish. Therefore there is no correction to $N'^0 = i\tau$ coming from $\partial_{V_0}\widehat{\mathcal{G}}_{ij}$. By repeating the analysis to $\partial_{V^\lambda}\widehat{\mathcal{G}}_{ij}$ and $\partial_{V_k}\widehat{\mathcal{G}}_{ij}$ one shows that only the latter can be non-vanishing, and analysing in the same fashion the D5 and the D7 brane cases we obtain the expected corrections from (4.6) and (4.9) .

5. Conclusions

In this work we described some of the main results of [1], on the four-dimensional $\mathcal{N} = 1$ effective action of Type IIA Calabi-Yau orientifolds with a single space-time filling D6-brane. The local $\mathcal{N} = 1$ moduli space for brane deformations can be nicely described by a Kähler potential, and we use it to describe the open moduli corrections to the Calabi-Yau complex structure Kähler potential. We presented an expression for a holomorphic superpotential that reproduces a F-term scalar potential for supersymmetry-breaking brane deformations, while a D-term potential is induced by gaugings of the scalars. The holomorphic gauge coupling functions for the U(1) brane field recieve holomorphic corrections coming from kinetic mixings with the R-R vector fields.

In the last part of the paper we related our Type IIA results to the $\mathcal{N} = 1$ data for Type IIB orientifold compactifications with D3-, D5-, or D7-branes using mirror symmetry. The SYZ proposal to view the internal manifold as a local T^3 fibration allowed us the match of the $\mathcal{N} = 1$ data for branes and orientifold planes of different dimensionalities with the D6/O6 set-up.

As a possible extension of this work one could incorporate the results here obtained into scenarios with intersecting stacks of branes. Another interesting possibility is to analyse the equivalent of D6-branes in M- and F-Theory compatifications, as the D6-Branes become encoded in the geometry, and compare with the results of this work.

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