Toward Brane anti-brane inflation: Holographic MQCD

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O. Aharony, D. Kutasov, O. Lunin, S. Yankielowicz
H. Nastase
Introduction: Brane-anti brane inflation models

The **inflationary scenario** is by far the most successful cosmological model.

It explains the large-scale **homogeneity** and the small-scale **inhomogeneities** required for **galaxy formation**.

Inflation models are sensitive to the **uv physics**.

**String theory** may provide a **uv complete theory** that can realize an inflation period in the universe evolution.

Several string inflation models were proposed.

It is clear that these are not the ultimate models and there is room to additional proposals. That’s the goal of our project.
The inflaton potential

\[ V(\phi) \]

\[ \phi \]

\[ \phi_{\text{CMB}} \quad \phi_{\text{end}} \quad \text{reheating} \]
The goals of the project are:

To find a consistent stringy scenario.

To identify a scalar field that will play the role of the inflaton.

To check that the potential of the inflaton admits the slow role parameters that obey the phenomenological requirements.

To discuss the stabilization of the moduli of the model.

To provide an appropriate passage to the reheating era.
The most known string inflation models are the KKLT (Kachru, Kallosh, Linde Trevedi) and KKLMMT (+Maldacena, McAllister) models.

KKLT is a model based on cutting off the Klebanov Strassler deformed connifold and gluing a CY manifold.

An anti-brane is added at the tip of the KS cone, breaking supersymmetry and lifting the AdS minimum into a dS minimum.
KKLMMT modified the KKLT model by introducing an additional D3 brane sliding along the radial direction.

Inflaton = Separation distance between brane and anti-brane

Using the explicit metric and fluxes, the inflaton action is determined using the non-compact background approximation.

Corrections in the form of Planck suppressed terms where also included both perturbatively and non-perturbatively.

Requiring volume stabilization generically spoils some of the nice features of the brane anti-brane inflation.
The KKLMMT model - Brane-anti brane inflation
Outline

Introduction: Brane anti-brane inflation models

What are the MQCD models

Holographic MQCD models

Adding a sliding probe D4 brane (anti-brane)

Holographic MQCD as inflation model

Extracting the inflaton potential

Cosmological properties of the models

Summary and outlook
A brane configuration that corresponds to N=1 SYM in 4d

\[ -\frac{L}{2} \leq x_6 \leq \frac{L}{2}. \]

The Ns Ns’ stretch

\[ NS : \quad v = x_4 + ix_5, \]
\[ NS' : \quad w = x_8 + ix_9, \]
Classical and Q.M pictures

The fields apart from the gluons and gluinos acquire mass of the order $1/L$.

The classical 4d gauge coupling

$$g_{YM}^2 = \frac{g_s l_s}{L}$$

The 4d ‘t Hooft coupling

$$\lambda_p^{(4)} = g_{YM}^2 p = \frac{\lambda_p}{L}; \quad \lambda_p \equiv g_s l_s p$$

QM, the 4d coupling runs with the scale. One can view $1/L$ as the UV cutoff of this theory, and $\lambda_p^{(4)}$ as the value of the coupling at the UV cutoff scale.

In order for the dynamics of the brane configuration to reduce to that of $N = 1$ SYM at energies well below the cutoff scale, this coupling must be taken to be very small, $\lambda_p^{(4)} \ll 1$. 
Uplift to M theory

The classical picture of D4-branes ending on NS5-branes is qualitatively modified by $g_s$ effects.

To exhibit these effects, it is convenient to view type IIA at finite $g_s$ as M-theory compactified on a circle of radius $R = g_s l_s$.

Both the D4-branes and the NS5-branes correspond from the 11d point of view to M5-branes, either wrapping the M-theory circle (D4-branes), or localized on it (NS5-branes).

The configuration of figure 1 lifts to a single M5-brane, which wraps $R^{3,1}$ and a two dimensional surface in the $R^5 \times S^1$ labeled by $(v, w, z)$.

$$z \equiv x_6 + ix_{11}$$
Uplift to M theory

The shape of the M5

\[ vw = \xi^2, \quad v = \xi e^{-z/p^R} = \xi e^{-z/\lambda_p} \]

The classical brane configuration lies on the surface \( vw = 0 \).

The QM shape is deformed away from this surface.

In particular, the D4-branes, that are classically at \( v = w = 0 \), are replaced in the quantum theory by a tube of width \( \sim \xi \) connecting the (deformed) NS and NS' branes.

Indeed, defining the radial coordinate \( u \) via

\[ u^2 = |v|^2 + |w|^2, \quad u \geq \sqrt{2} \xi. \]

One can interpret \( \xi/l_s^2 \) as the brane analog of the dynamically generated scale of the \( N = 1 \) SYM theory.
Classically, the brane configuration is confined to the interval, while QM it extends to arbitrarily large $|x_6|$. For example, for large positive $x_6$, the fivebrane takes the shape

$$z \approx \lambda_p \ln(w/\xi); \quad v \approx 0.$$  

One can think of it as describing an NS'-brane deformed by the D4-branes ending on it from the left.
Running coupling

At large $u$, the distance between the NS and NS’-branes goes to infinity; hence the separation $L$ appears to be ill defined.

This is not surprising, since $L$ relates to the gauge coupling, which changes with the scale.

To define it, one can take the radial coordinate $u$ to be bounded, $u \leq u_\infty$, and demand that at $u = u_\infty$, $x_6 = \pm L/2$.

Assuming that the four dimensional ‘t Hooft coupling $\lambda^{(4)}_p$ is very small, this gives the following relation among the different scales

$$\xi = u_\infty \exp\left(-\frac{L}{2\lambda_p}\right) = u_\infty \exp\left(-\frac{1}{2\lambda^{(4)}_p}\right)$$
Running coupling

This is the brane analog of the relation between the QCD scale and the gauge coupling in SYM theory, with $u_\infty/l^2_\tau$ playing the role of a UV cutoff.

As in SYM, we can remove the cutoff, by sending $u_\infty, L/\lambda_p \to \infty$

while keeping the “QCD scale” $\xi/l^2_\tau$ fixed.
The shape of the NS brane

- The shape of this NS5-brane is given by the reduction of the 11-d profile to 10d.
- It is parameterized by two functions of \( x_6, u \) and \( \alpha \), which are defined (on the fivebrane) by

\[
 v = u e^{i\phi} \cos(\alpha), \quad w = u e^{-i\phi} \sin(\alpha)
\]

- \( u \) and \( \alpha \) are given by

\[
 u = \xi \sqrt{2 \cosh \left( \frac{2x_6}{\lambda_p} \right)}, \quad \tan(\alpha) = \exp \left( \frac{2x_6}{\lambda_p} \right)
\]

- As \( \phi \) varies between 0 and \( 2\pi \), the scalar field winds \( p \) times around the circle, giving the NS5-brane its D4-brane charge.
We extend the brane configuration to a one which is more amenable to a holographic analysis. We add $N$ infinite D4-branes stretched in the $x_6$ direction.
These branes do not break any of the supersymmetries preserved by the configuration. Thus, we can place them anywhere in the $R^5$ labeled by $(v,w, x_7)$, without influencing the shape of the curved NS5-brane.

This can be seen directly by replacing the $N$ D4-branes by their geometry, and studying the dynamics of the curved NS5-brane in that geometry.

This description should be valid for $N \gg p$, and we will restrict to this (“probe”) regime.
A holographic embedding of MQCD

Viewing the N D4-branes as M5-branes wrapped around the M-theory circle, their 11d geometry is

\[
\begin{align*}
\text{ds}^2 &= H^{-1/3} \left( dx_\mu^2 + dx_6^2 + dx_{11}^2 \right) + H^{2/3} \left( |dv|^2 + |dw|^2 + dx_7^2 \right), \\
C_6 &= H^{-1} d^4x \wedge dx_6 \wedge dx_{11}, \\
H &= 1 + \frac{\pi \lambda_N l_s^2}{|\mathbf{r} - \mathbf{r}_0|^3},
\end{align*}
\]

\( \lambda_N = g_s l_s N \) is the 5d \('t Hooft coupling of the N D4-branes, \( \mathbf{r} = (v, w, x_7) \) labels position in \( \mathbb{R}^5 \), with \( \mathbf{r} = 0 \) corresponding to the classical position of the p D4-branes, and \( \mathbf{r}_0 \in \mathbb{R}^5 \) is the position of the N D4-branes.
A holographic embedding of MQCD

For $g_s > 0$, the NS5 and p D4-branes form a curved M5-brane whose shape may be obtained by plugging the ansatz

$$v = u(x_6)e^{i\phi(x_{11})}\cos(\alpha(x_6)), \quad w = u(x_6)e^{-i\phi(x_{11})}\sin(\alpha(x_6))$$

into the fivebrane worldvolume action.

Parameterizing the M5-brane worldvolume by the coordinates $(x_\mu, x_6, x_{11})$, the induced metric

$$ds_{ind}^2 = H^{-1/3} \left\{ dx_\mu^2 + \left[ 1 + H \left( (u\alpha')^2 + (u')^2 \right) \right] dx_6^2 + \left( 1 + H(u\dot{\phi})^2 \right) dx_{11}^2 \right\}$$

The corresponding Lagrangian is

$$L = H^{-1}\sqrt{1 + H(u\dot{\phi})^2}\sqrt{1 + H((u\alpha')^2 + (u')^2)} - H^{-1}$$
The equations of motion imply that $\dot{\phi}$ must be constant; thus $\phi = x_{11}/pR = x_{11}/\lambda_p$.

The Noether charges $J, E$ associated with the invariances under the shifts of $\alpha$ and $x_6$, are

$$J = \frac{u^2 \alpha' \sqrt{1 + Hu^2/\lambda_p^2}}{\sqrt{1 + H [(u\alpha')^2 + (u')^2]}}$$

$$E = H^{-1} - \frac{H^{-1} \sqrt{1 + Hu^2/\lambda_p^2}}{\sqrt{1 + H [(u\alpha')^2 + (u')^2]}}.$$ 

Supersymmetric configurations have $E = 0$. Substituting it we find:

$$\alpha' = \frac{J}{u^2},$$

$$(u')^2 = \frac{u^2}{\lambda_p^2} - \frac{J^2}{u^2}.$$
A holographic embedding of MQCD

The solution of the equations

\[ u = \xi \sqrt{2 \cosh \left( \frac{2x_6}{\lambda_p} \right)}, \quad \tan(\alpha) = \exp \left( \frac{2x_6}{\lambda_p} \right) \]

With

\[ J\lambda_p = 2\xi^2. \]

Note that the equations are \textbf{H independent} and in particular of the positions of the N D4-branes.
What do we know about holographic MQCD

- We have determined full correspondence with the *field theory* picture.

- We have computed the *mesonic spectrum*

- The *Wilson line* was extracted

- The *finite temperature* phase diagram
The brane configuration which is relevant to our story is achieved by compactifying the $x_6$ on a circle $x_6 \sim x_6 + 2\pi R$. 

![Compactified $x_6$](image)
While the ***bosons*** living on the branes must satisfy **periodic** boundary conditions around the circle.

**Fermions**

- **Periodic**, supersymmetry unbroken
- **Anti-periodic**, supersymmetry broken
For the supersymmetric case the U shape of the probe branes spirals down the cylinder and then climbs back up to the boundary.
The same happens for the non-susy *cigar* case.
From MQCD to inflation models

The cylinder model is a T-dual of the IIB Klebanov Strassler model (in the \(R \to 0\) limit)

In a similar manner to the KKLMMT model to get an inflation model we add on the cylinder a sliding anti-D4 brane.

For the non-susy cigar model we add instead a sliding D4 brane.

The inflaton is identified with

\[ \text{Infalton} = \text{location of the sliding (anti) brane} \]
Inflaton as the location of a sliding anti-brane
Inflaton as the location of the sliding brane
Extraction of the inflaton potential - cigar geometry

The background of the near extremal D4 brane

\[ ds^2 = H_4^{-1/2}(r)(-dt^2 + dx_3^2 + f(r)dx_6^2) + H_4^{1/2}\left(\frac{dr^2}{f} + r^2d\Omega_4^2\right) \]

\[ e^{2\phi} = g_s^2 H_4^{-1/2} \]

\[ C_4 = \frac{1}{g_s} H_4^{-1} dt \wedge dx^1 \ldots \wedge dx^4 \]

\[ f = 1 - \left(\frac{r_H}{r}\right)^3 \]

\[ H_4 = 1 + \alpha_4 \left(\frac{r_4}{r}\right)^3 \]

The action of a Dp brane that resides in this background includes the DBI and CS terms

\[ S_p = -T_p \int e^{-\phi} \sqrt{\det G_{ab}} + \mu_p \int C_{p+1} = -\frac{T_p}{g_s} \int H_p^{-1}(r) \left[ \sqrt{f + \frac{H_p(r)}{f_p} g^{\mu\nu} \partial_\mu r \partial_\nu r} - 1 \right] \]
The corresponding potential for D4 brane is

\[
V_4(r) = \frac{T_4 R}{g_s} H^{-1}(r) \left[ \sqrt{f(r)} - 1 \right] = \frac{T_4 R}{g_s} \frac{1}{1 + \alpha_4 \left( \frac{r_4}{r} \right)^3} \left[ \sqrt{1 - \frac{r_H^3}{r^3}} - 1 \right] < 0
\]

In the regime of \( r_H \ll r \ll r_4 \), it is approximately

\[
V(r) \approx -\frac{T_4 R}{2g_s \alpha_4} \left( \frac{r_H}{r_4} \right)^3 \left[ 1 + \frac{1}{4} \frac{r_H^3}{r_4^3} - \frac{r^3}{\alpha_4 r_4^3} \right]
\]
Inflaton potential: cigar

The potential due to the background looks like
Vacuum energy

- The potential is **negative** but it has to be **positive**
- There is an additional **vacuum energy potential** since the background is **non-supersymmetric**
- One can get the near extremality by adding an extra mass with no extra charge
- Therefore we have

\[
\frac{r_H^3}{r_4^3} = \frac{\delta M}{M} = \frac{2\delta N}{N}
\]

- The resulting **vacuum energy** is

\[
E_0 = \frac{T_4 R}{g_s} 2\delta N = \frac{T_4 R N}{g_s} \frac{r_H^3}{r_4^3}
\]
Inflaton potential due to the spiral brane

To calculate the interaction potential between the spiral and the sliding brane, we calculate the change in V due to the variation of the Harmonic function.

The Harmonic function corresponds to N original branes at r=0 and the sliding one at r=r_0 thus

\[ H \rightarrow H + \delta H; \quad \frac{\delta H}{H} \approx \frac{r^3}{N(r - r_0)^3} \]

The corresponding change of the potential

\[ \delta V(r_0) = \frac{2T_4 \lambda_p}{g_s} \int_0^\infty dx_6 \frac{r^3}{N(r - r_0)^3} H^{-1} \left\{ 1 - \frac{H^{-1}f}{(H^{-1} - E)^2} \left[ H^{-1} - E(2 + H u^2) \right] \right\} \]

The upshot is that this is \( g_s \rho / N \) suppressed.
Inflaton potential: The cylinder

The calculation of the **inflaton potential** is similar.

Now the background is **extremal** and hence \( r_h = 0 \) \( f = 1 \).

It is a sliding **anti-brane** and not brane which means that the **CS term changes sign** so instead of cancellation we get twice the DBI contribution.

\[
V_4(r) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{r^3}{r_s^3}}
\]

Again the contribution from the spiral is **suppressed**.

\[
g_s p / N
\]
We express the **cosmological properties** in terms of:

- 10d Newton constant
- Planck scale
- 4D Planck scale
- Volume of 5d space
- D4 brane tension

Mathematical expressions:

\[ 2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 = m^{-8} \]
\[ M_P = m^4 \sqrt{R V_5} \]
\[ T_4 = \frac{1}{2\sqrt{\pi \alpha'} \kappa_{10}} = \frac{m^4}{\sqrt{2\pi \alpha'}} \]
Characteristics of inflation

In order to get a good model of inflation, the slow role parameters

\[ \epsilon \equiv \frac{M_p^2}{2} \left( \frac{V'}{V} \right)^2 \]
\[ \eta = M_p^2 \frac{V''}{V} \]

\[ << 1 \]

Since the spectral index of the primordial fluctuations

\[ P^s_\delta(k) \sim k^{n_s-1} \]
\[ n_s - 1 \equiv \frac{d \ln P^s_\delta(k)}{d \ln k} = -6\epsilon + 2\eta \]

the spectrum is almost exactly flat \( (n_s=1) \)
Characteristics of inflation

The **number of e-folds** during inflation must be

\[ N \equiv \int \frac{H}{\dot{\phi}} \, d\phi = \int_{\phi_{\text{end}}}^{\phi_i} \frac{V}{M_P^2 V'} \, d\phi = \frac{1}{M_P} \int_{\phi_{\text{end}}}^{\phi_i} \frac{d\phi}{\sqrt{2\epsilon}} \geq 60 \]

In addition one needs a sufficiently large **reheating temperature** at the end of inflation.

This generically requires that the potential energy at the start of inflation be not **too far below Planck scale**.
Inflation parameters for the cigar model

We first transfer the kinetic term into a canonical one

\[ \phi = r \sqrt{T_4 R / g_s} \]

For the model of sliding D4 along the cigar we get

\[
\begin{align*}
\epsilon & \equiv \frac{1}{2} \left( \frac{M_P V'}{V} \right)^2 \approx \frac{1}{2} \left[ \frac{3 M_P}{2 N \phi} \left( \frac{1}{4} \frac{\phi_H^3}{\phi^3} + \frac{\phi^3}{\phi_4^3} \right) \right]^2 \\
\eta & \equiv M_P^2 \frac{V''}{V} = \frac{3 M_P^2}{2 N \phi^2} \left( -\frac{\phi_H^3}{\phi^3} + 2 \frac{\phi^3}{\phi_4^3} \right)
\end{align*}
\]

The \( \epsilon \) and \( \eta \) are thus small in the region

\[ \phi_H \ll \phi \ll \phi_4, \]

which is the plateau region.
Inflation parameters for the cigar model

The number of e-folds

\[ N \approx \frac{2N}{3} \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} \frac{d\phi}{M_P} \frac{\phi}{M_P} \frac{\phi^3}{4\phi^3} + \frac{\phi^3}{\phi_4} \sim \frac{8N}{15} \frac{\phi_{\text{in}}^5 - \phi_{\text{end}}^5}{M_P^3 \phi_H^3} \]

which can easily be made large enough
Inflation parameters for the cylinder model

In terms of the canonical field the potential reads

\[ V_4(\phi) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{\phi_4^3}{\phi^3}} \]

The slow role parameters are

\[ \epsilon = \frac{1}{2} \left[ \frac{3M_P}{\phi} \frac{\phi_4^3/\phi^3}{1 + \phi_4^3/\phi^3} \right]^2 \]
\[ \eta = -\frac{M_P^2}{\phi^2} \frac{\phi_4^3/\phi^3 (32 + 30\phi_4^3/\phi^3)}{(1 + \phi_4^3/\phi^3)^2} \]
Inflation parameters for the cylinder model

\[ \phi \gg M_P \]

\[ \phi \sim M_P \]

\[ \phi \gg \phi_4 \]

\[ \phi_4/M_P \ll 1 \]

\[ V_5 m^4 / (\sqrt{\alpha'} N^{2/3}) \gg 1 \]

\[ N = \int_{\phi_{\text{end}}}^{\phi_{\text{in}}} d\phi \frac{\phi^4 (1 + \phi_4^3/\phi^3)}{3 M_P^2 \phi_4^3} \approx \frac{\phi_{\text{in}}^5 - \phi_{\text{end}}^5}{3 M_P^2 \phi_4^3} \]

\( \eta \) and \( \varepsilon \) can be small

If we allow the non-genericity we get large enough
Reheating in the cylinder model

The potential at the end of inflation is now

\[ V_f \sim \frac{2T_4 R}{g_s} \sim 12\pi \epsilon \times 10^{-10} M_P^4 \]

Reheating is simpler in this model since

\[ \phi = 0, \text{ where } V(0) = 0 \]

And moreover

\[ \frac{dV_4}{d\phi}(\phi) = \frac{6T_4 R}{g_s \phi} \frac{\phi_4^3/\phi^3}{1 + \phi_4^3/\phi^3} \]
Reheating in the cylinder model

So the **slope** of the **potential blows** up at $\phi=0$ and we have the usual reheating scenario.

In this case due to the **supersymmetry** of the background this result seems to be **robust**.
Summary and outlook

We examined holographic MQCD models as inflation models.
The models are technically simpler than the KKLMMT model.
We found out that the leading order contribution to the potential comes from the background and not from the spiral.
For near extremal background, the cigar, with a sliding D4 brane flat enough potential with slow-roll conditions easy to fulfill generically.
For the susy background (cylinder) the gluing of a CY is more natural. Reheating is also easier, however we need to use non-generic initial conditions.