Toward Brane anti-brane inflation: Holographic MQCD

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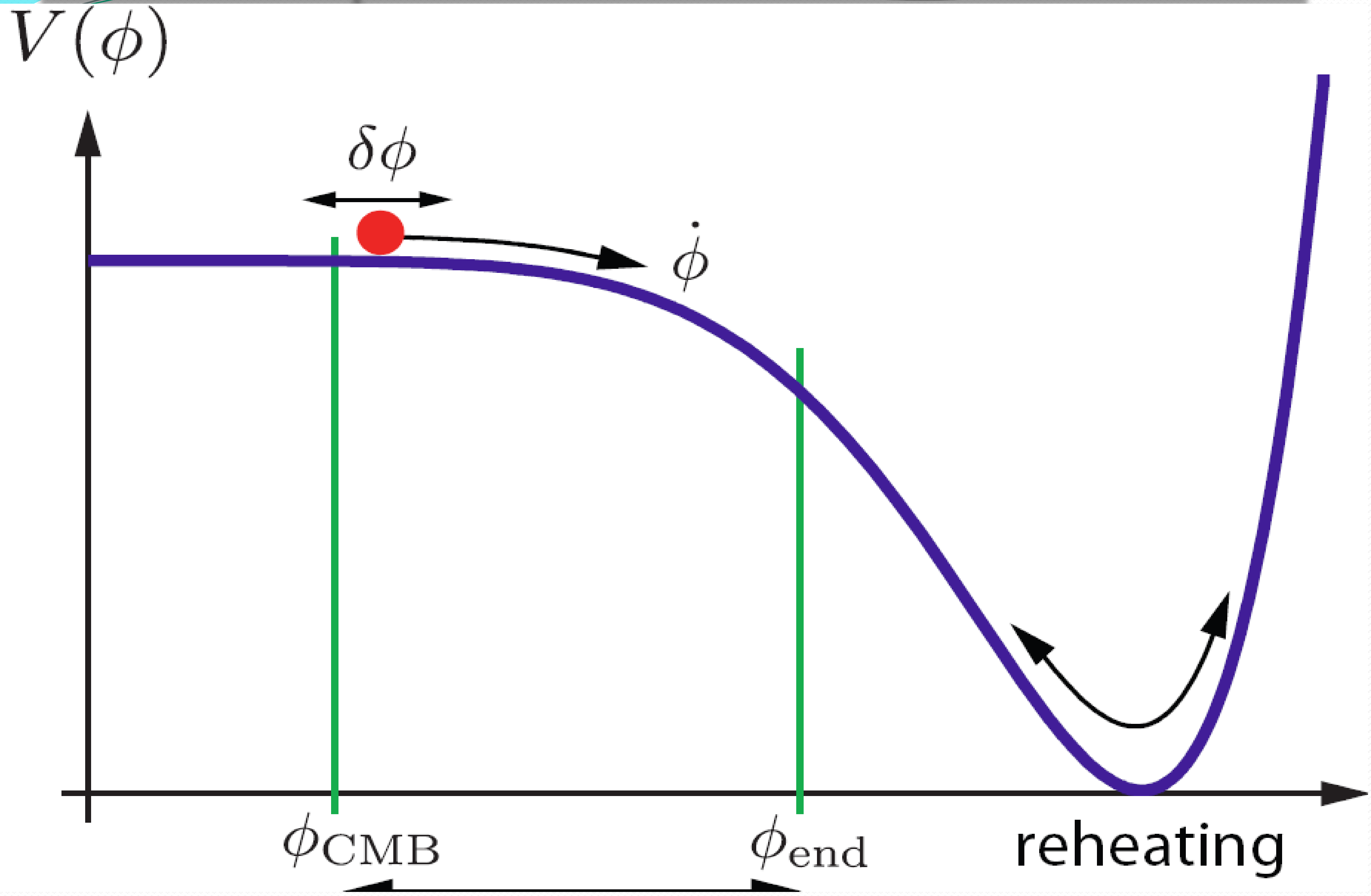
O. Aharony, D. Kutasov, O. Lunin, S. Yankielowicz

H. Nastase

Introduction: Brane-anti brane inflation models

- The inflationary scenario is by far the most successful cosmological model.
- It explains the large-scale homogeneity and the small-scale inhomogeneities required for galaxy formation
- Inflation models are sensitive to the uv physics.
- String theory may provide a uv complete theory that can realize an inflation period in the universe evolution.
- Several string inflation models were proposed.
- It is clear that these are not the ultimate models and there is room to additional proposals. That's the goal of our project

The inflaton potential



The goals of the project

- The goals of the project are:
- To find a consistent stringy scenario.
- To identify a scalar field that will play the role of the inflaton.
- To check that the potential of the inflaton admits the slow role parameters that obey the phenomenological requirements.
- To discuss the stabilization of the moduli of the model
- To provide an appropriate passage to the reheating era

The KKLT model-

- The most known string inflation models are the KKLT (Kachru, Kallosh, Linde Trevedi) and KKLMMT (+Maldacena, McAllister) models.

- KKLT is a model based on cutting off the Klebanov Strassler deformed connifold and gluing a CY manifold

- An anti-brane is added at the tip of the KS cone, breaking supersymmetry and lifting the AdS minimum into a dS minimum

The KKLMMT model- Brane-anti brane inflation

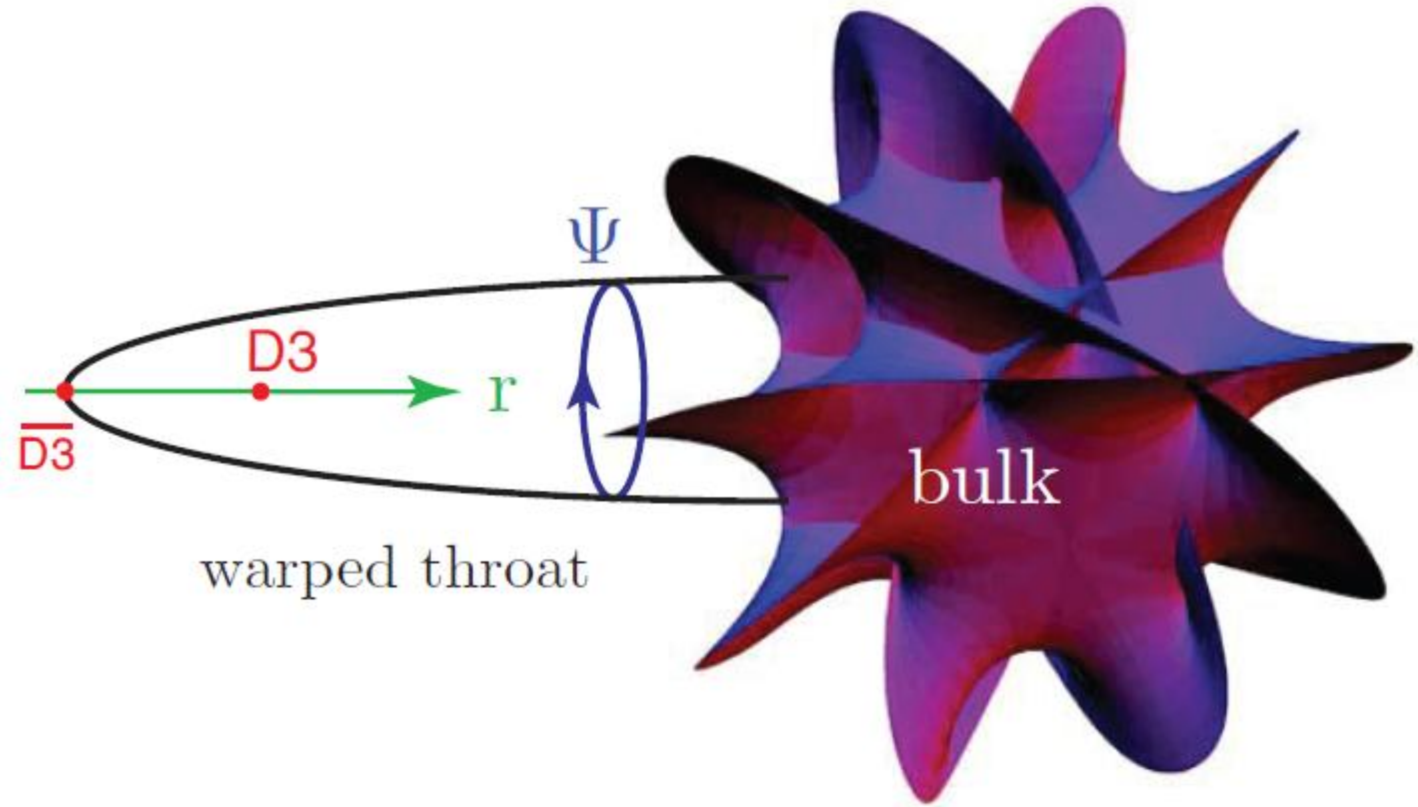
- KKLMMT modified the KKLT model by introducing an additional D3 brane sliding along the radial direction

Inflaton = Separation distance between brane and anti-brane

- Using the explicit metric and fluxes, the inflaton action is determined using the non-compact background approximation.
- Corrections in the form of Planck suppressed terms where also included both perturbatively and non peturbatively.

- Requiring volume stabilization generically spoils some of the nice features of the brane anti-brane inflation

The KKLMMT model- Brane-anti brane inflation

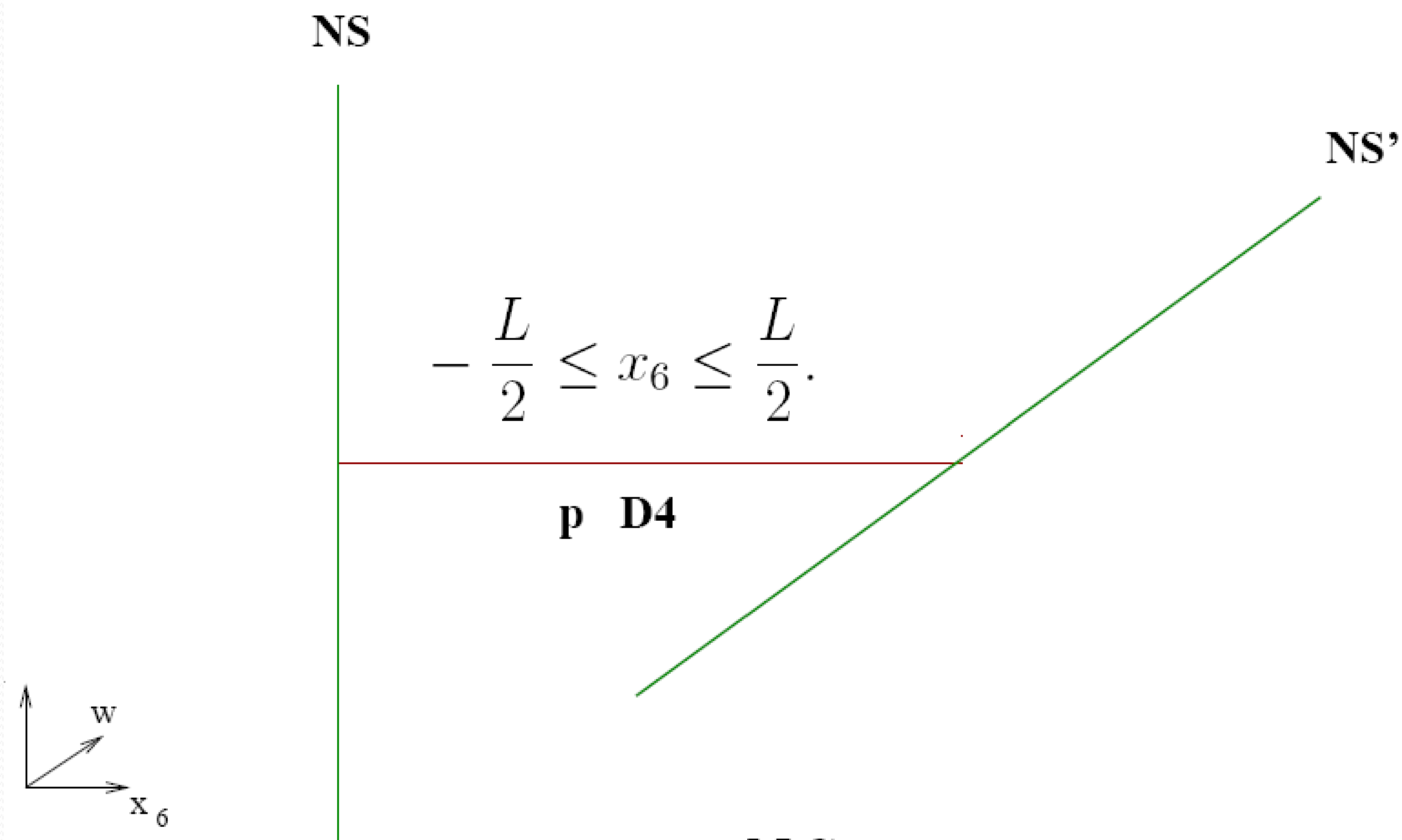


Outline

- Introduction: Brane anti-brane inflation models
- What are the MQCD models
- Holographic MQCD models
- Adding a sliding probe D4 brane (anti-brane)
- Holographic MQCD as inflation model
- Extracting the inflaton potential
- Cosmological properties of the models
- Summary and outlook

Basic brane construction

- A brane configuration that corresponds to N=1 SYM in 4d



$$-\frac{L}{2} \le x_6 \le \frac{L}{2}.$$

- The Ns Ns' stretch

$$NS: \qquad v = x_4 + i x_5,$$
$$NS': \qquad w = x_8 + i x_9,$$

Classical and Q.M pictures

- The fields apart from the gluons and gluinos acquire mass of the order 1/L
- The classical 4d gauge coupling
$$g_{YM}^2 = \frac{g_s l_s}{L}$$

The 4d ‘t Hooft coupling
$$\lambda_p^{(4)} = g_{YM}^2 p = \frac{\lambda_p}{L}; \qquad \lambda_p \equiv g_s l_s p$$

- QM, the 4d coupling runs with the scale. One can view 1/L as the UV cutoff of this theory, and $\lambda^{(4)}{}_p$ as the value of the coupling at the UV cutoff scale.

- In order for the dynamics of the brane configuration to reduce to that of N = 1 SYM at energies well below the cutoff scale, this coupling must be taken to be very small, $\lambda^{(4)}{}_p \ll 1$.

Uplift to M theory

- The classical picture of D4-branes ending on NS5-branes is qualitatively modified by g_s effects .
- To exhibit these effects, it is convenient to view type IIA at finite g_s as M-theory compactified on a circle of radius $R = g_s l_s$.
- Both the D4-branes and the NS5-branes correspond from the 11 d point of view to M5-branes, either wrapping the M-theory circle (D4-branes), or localized on it (NS5-branes).
- The configuration of figure 1 lifts to a single M5-brane, which wraps $R^{3,1}$ and a two dimensional suface in the $R^5 \times S^1$ labeled by (v,w, z).

$$z \equiv x_6 + i x_{11}$$

Uplift to M theory

- The shape of the M5

$$vw = \xi^2, \qquad v = \xi e^{-z/pR} = \xi e^{-z/\lambda_p}$$

- The classical brane configuration lies on the surface $vw = 0$
- The QM shape is deformed away from this surface.
- In particular, the D4-branes, that are classically at $v = w = 0$, are replaced in the quantum theory by a tube of width $\sim \xi$ connecting the (deformed) NS and NS′-branes.
- Indeed, defining the radial coordinate u via

$$u^2 = |v|^2 + |w|^2, \qquad u \geq \sqrt{2}\xi.$$

One can interpret ξ/l^2_s as the brane analog of the dynamically generated scale of the $N = 1$ SYM theory.

Uplift to M theory

- Classically, the brane configuration is confined to the interval , while QM it extends to arbitrarily large $|x_6|$. For example, for large positive x_6, the fivebrane takes the shape

$$z \simeq \lambda_p \ln(w/\xi); \qquad v \simeq 0.$$

- One can think of it as describing an NS′-brane deformed by the D4-branes ending on it from the left.

Running coupling

- At large u, the distance between the NS and NS′-branes goes to infinity; hence the separation L appears to be ill defined.
- This is not surprising, since L relates to the gauge coupling, which changes with the scale.
- To define it, one can take the radial coordinate u to be bounded, $u \leq u_\infty$, and demand that at $u = u_\infty$, $x_6 = \pm L/2$.
- Assuming that the four dimensional 't Hooft coupling $\lambda^{(4)}_p$ is very small, this gives the following relation among the different scales

$$\xi = u_\infty \exp\left(-L/2\lambda_p\right) = u_\infty \exp\left(-1/2\lambda_p^{(4)}\right)$$

Running coupling

- This is the brane analog of the relation between the QCD scale and the gauge coupling in SYM theory, with $u_\infty/l^2{}_s$ playing the role of a UV cutoff.
- As in SYM, we can remove the cutoff, by sending u_∞, $L/\lambda_p \to \infty$

while keeping the

"QCD scale" $\xi/l^2{}_s$ fixed.

The shape of the NS brane

- The shape of this NS5-brane is given by the reduction of the 11-d profile to 10d.
- It is parameterized by two functions of x6, u and α, which are defined (on the fivebrane) by

$$v = ue^{i\phi}\cos(\alpha), \qquad w = ue^{-i\phi}\sin(\alpha)$$

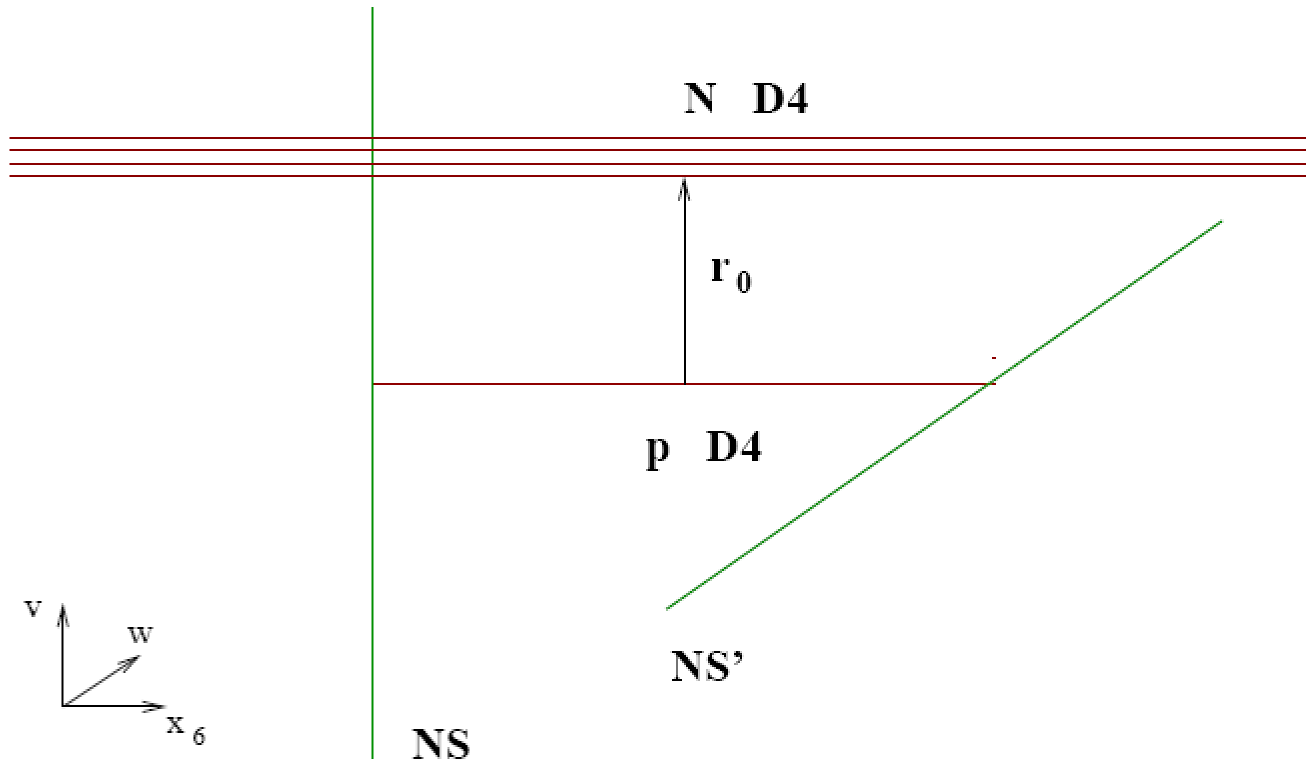
- u and α are given by

$$u = \xi\sqrt{2\cosh\left(\frac{2x_6}{\lambda_p}\right)}, \qquad \tan(\alpha) = \exp\left(2x_6/\lambda_p\right)$$

- As ϕ varies between 0 and 2π, the scalar field winds p times around the circle, giving the NS5-brane its D4-brane charge.

A holographic embedding of MQCD

- We extend the brane configuration to a one which is more amenable to a holographic analysis. We add N infinite D4-branes stretched in the x6 direction



A holographic embedding of MQCD

- These branes do not break any of the supersymmetries preserved by the configuration .

  Thus, we can place them anywhere in the R^5 labeled by (v,w, x_7), without influencing the shape of the curved NS5-brane

- This can be seen directly by replacing the N D4-branes by their geometry, and studying the dynamics of the curved NS5-brane in that geometry.

- This description should be valid for $N \gg p$, and we will restrict to this ("probe") regime .

A holographic embedding of MQCD

- Viewing the N D4-branes as M5-branes wrapped around the M-theory circle, their 11d geometry is

$$\begin{aligned} ds^2 &= H^{-1/3}\left(dx_\mu^2 + dx_6^2 + dx_{11}^2\right) + H^{2/3}\left(|dv|^2 + |dw|^2 + dx_7^2\right) \\ C_6 &= H^{-1} d^4x \wedge dx_6 \wedge dx_{11}, \qquad H = 1 + \frac{\pi \lambda_N l_s^2}{|\vec{r} - \vec{r}_0|^3}, \end{aligned}$$

$\lambda_N = g_s l_s N$ is the 5d 't Hooft coupling of the N D4-branes ,

$r = (v, w, x_7)$ labels position in R^5, with $r = 0$ corresponding to the classical position of the p D4-branes , and $r_0 \in R^5$ is the position of the N D4-branes

A holographic embedding of MQCD

- For $g_s > 0$, the NS5 and p D4-branes form a curved M5-brane whose shape may be obtained by plugging the ansatz

$$v = u(x_6)e^{i\phi(x_{11})}\cos(\alpha(x_6)), \qquad w = u(x_6)e^{-i\phi(x_{11})}\sin(\alpha(x_6))$$

into the fivebrane worldvolume action.

- Parameterizing the M5-brane worldvolume by the coordinates (x_μ, x_6, x_{11}), the induced metric

$$ds^2_{ind} = H^{-1/3}\left\{dx_\mu^2 + \left[1 + H\left((u\alpha')^2 + (u')^2\right)\right]dx_6^2 + \left(1 + H(u\dot{\phi})^2\right)dx_{11}^2\right\}$$

- The corresponding Lagrangian is

$$L = H^{-1}\sqrt{1 + H(u\dot{\phi})^2}\sqrt{1 + H((u\alpha')^2 + (u')^2)} - H^{-1}$$

A holographic embedding of MQCD

- The equations of motion imply that $\dot{\phi}$ must be constant; thus $\phi = x_{11}/pR = x_{11}/\lambda_p$.
- The Noether charges J,E associated with the invariances under the shifts of α and x_6, are

$$J = \frac{u^2\alpha'\sqrt{1+Hu^2/\lambda_p^2}}{\sqrt{1+H\left[(u\alpha')^2+(u')^2\right]}},$$

$$E = H^{-1} - \frac{H^{-1}\sqrt{1+Hu^2/\lambda_p^2}}{\sqrt{1+H\left[(u\alpha')^2+(u')^2\right]}}.$$

- Supersymmetric configurations have E = 0. Substituting it we find:

$$\alpha' = J/u^2,$$

$$(u')^2 = \frac{u^2}{\lambda_p^2} - \frac{J^2}{u^2}.$$

A holographic embedding of MQCD

- The solution of the equations

$$u = \xi\sqrt{2\cosh\left(\frac{2x_6}{\lambda_p}\right)}, \qquad \tan(\alpha) = \exp\left(2x_6/\lambda_p\right)$$

With $$J\lambda_p = 2\xi^2.$$

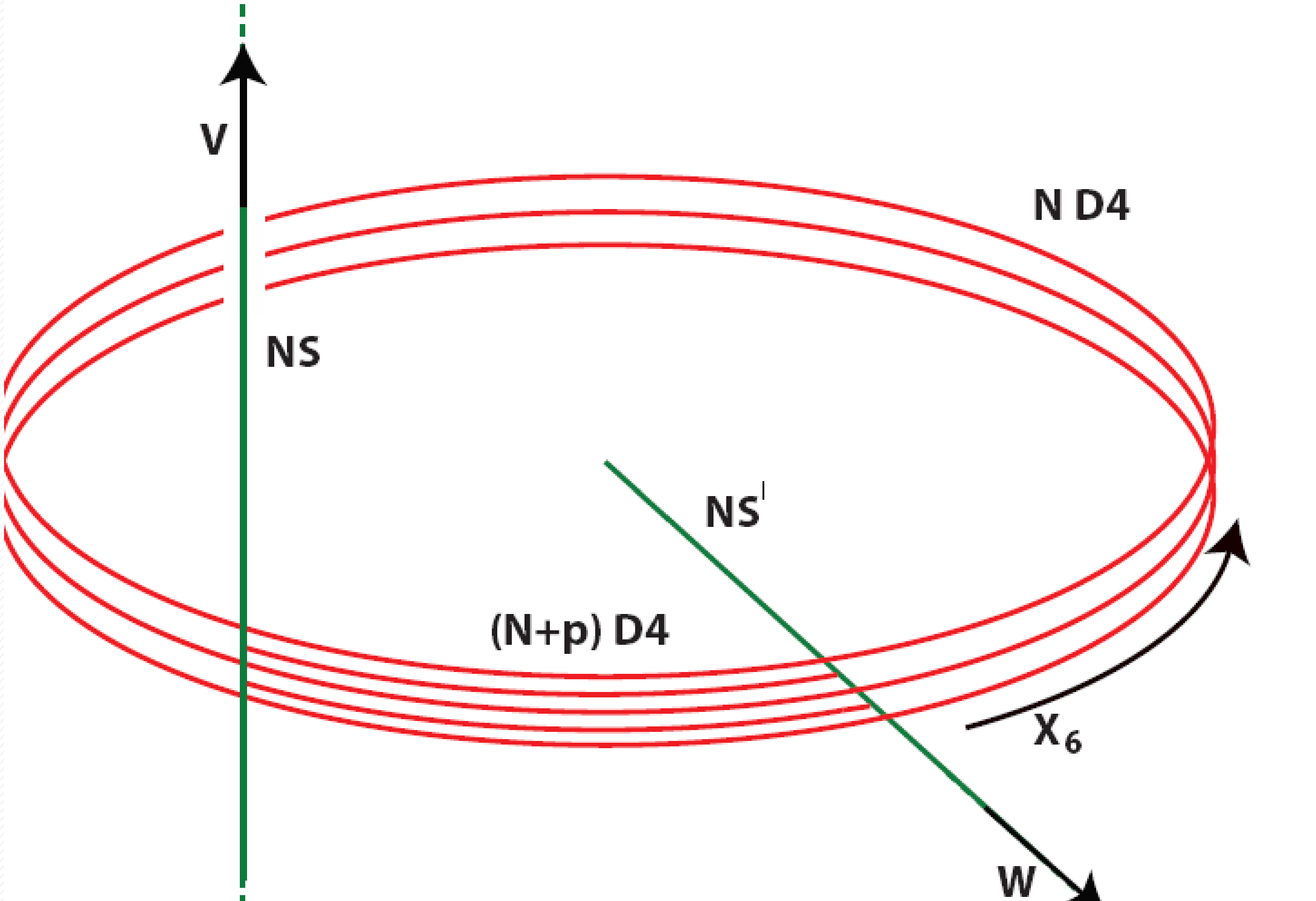
- Note that the equations are H independent and in particular of the positions of the N D4-branes.

What do we know about holographic MQCD

- We have determined full correspondence with the field theory picture.
- We have computed the mesonic spectrum
- The Wilson line was extracted
- The finite temperature phase diagram

Compactified x_6

- The brane cofiguration which is relevant to our story is achieved by compactifying the x_6 on a circle $x_6 \sim x_6 + 2\pi R$

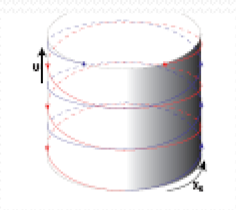


Boundary conditions

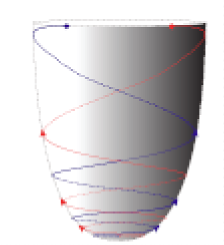
- While the bosons living on the branes must satisfy periodic boundary conditions around the circle.

Fermions

Periodic , supersymmety unbroken

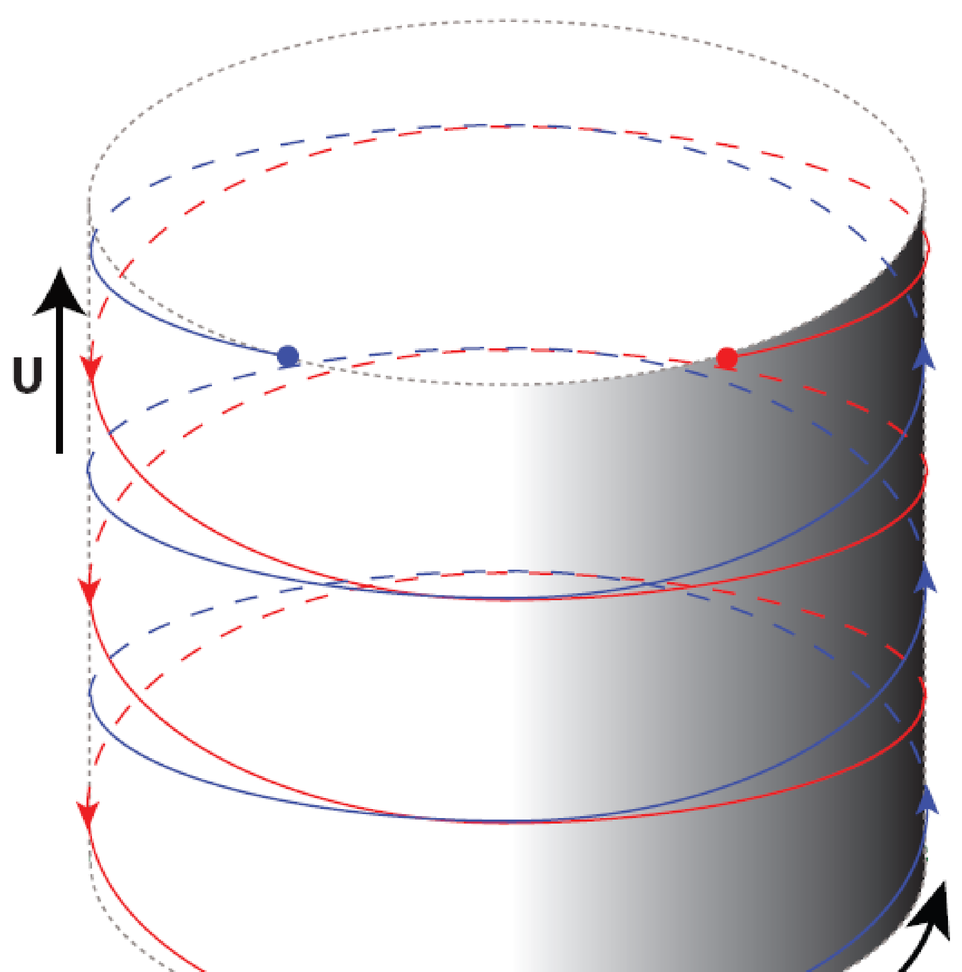


Anti-periodic , supersymmetry broken



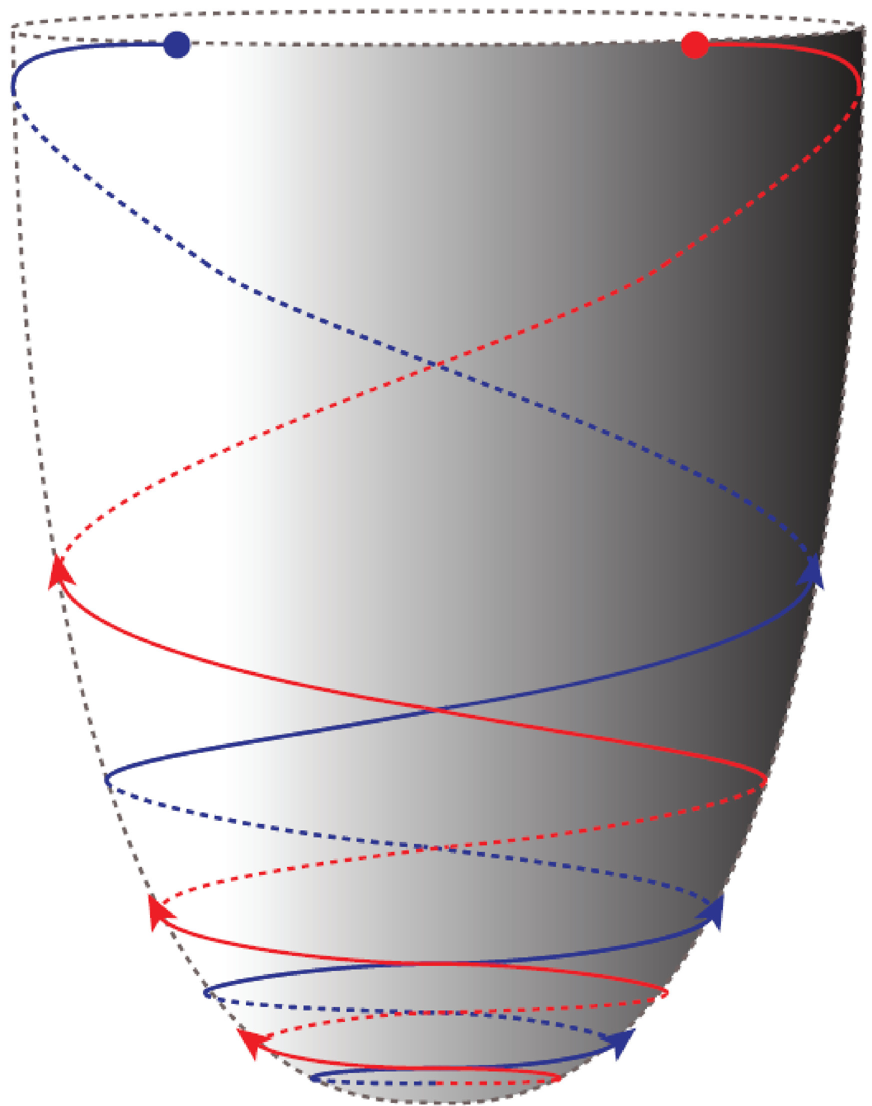
Compact x_6 - spiral brane in cylinder

- For the supersymmetric case the U shape of the probe branes spirals down the cylinder and then climbs back up to the boundary.



spiral brane in cigar

- The same happens for the non-susy cigar case

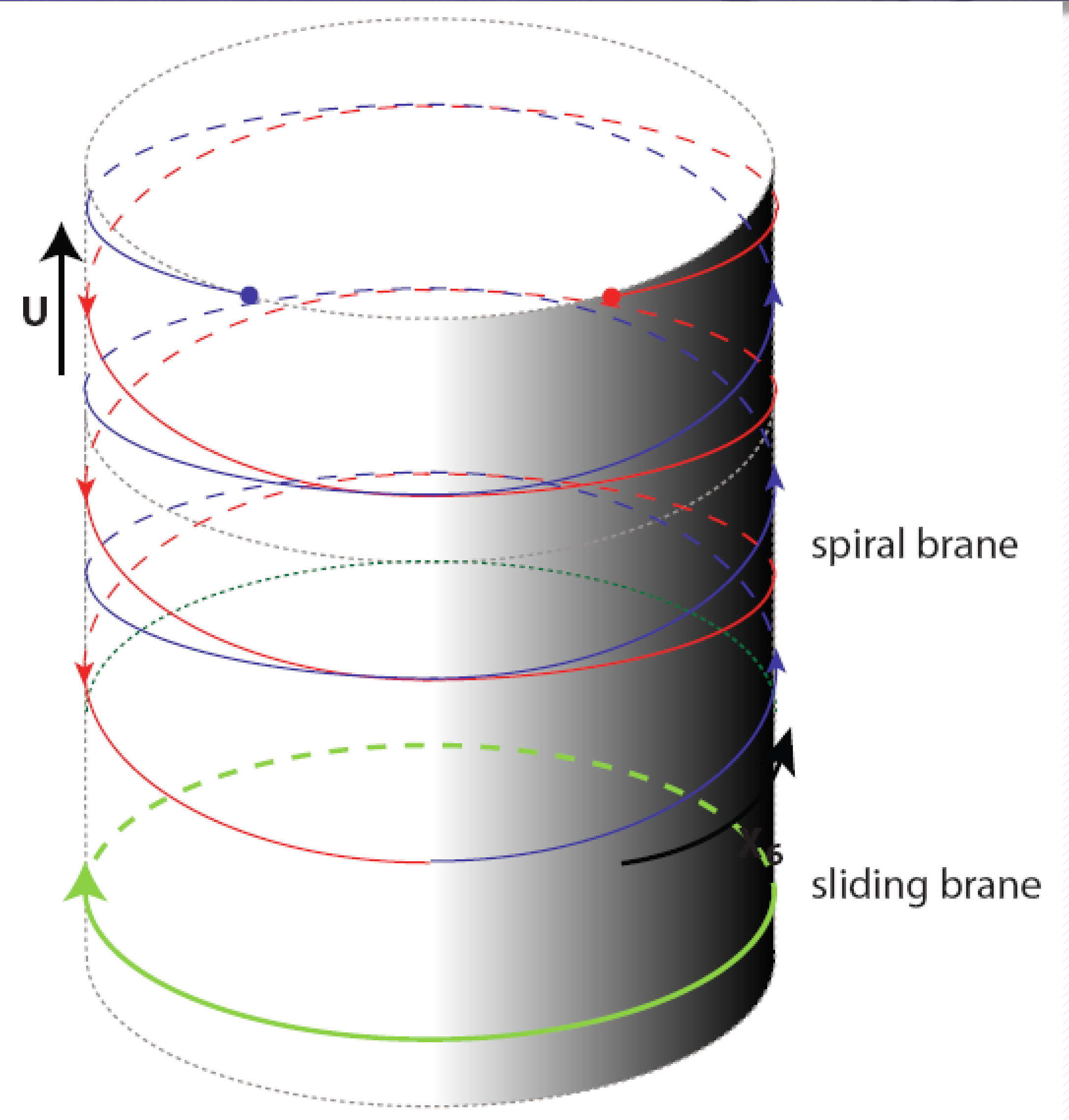


From MQCD to inflation models

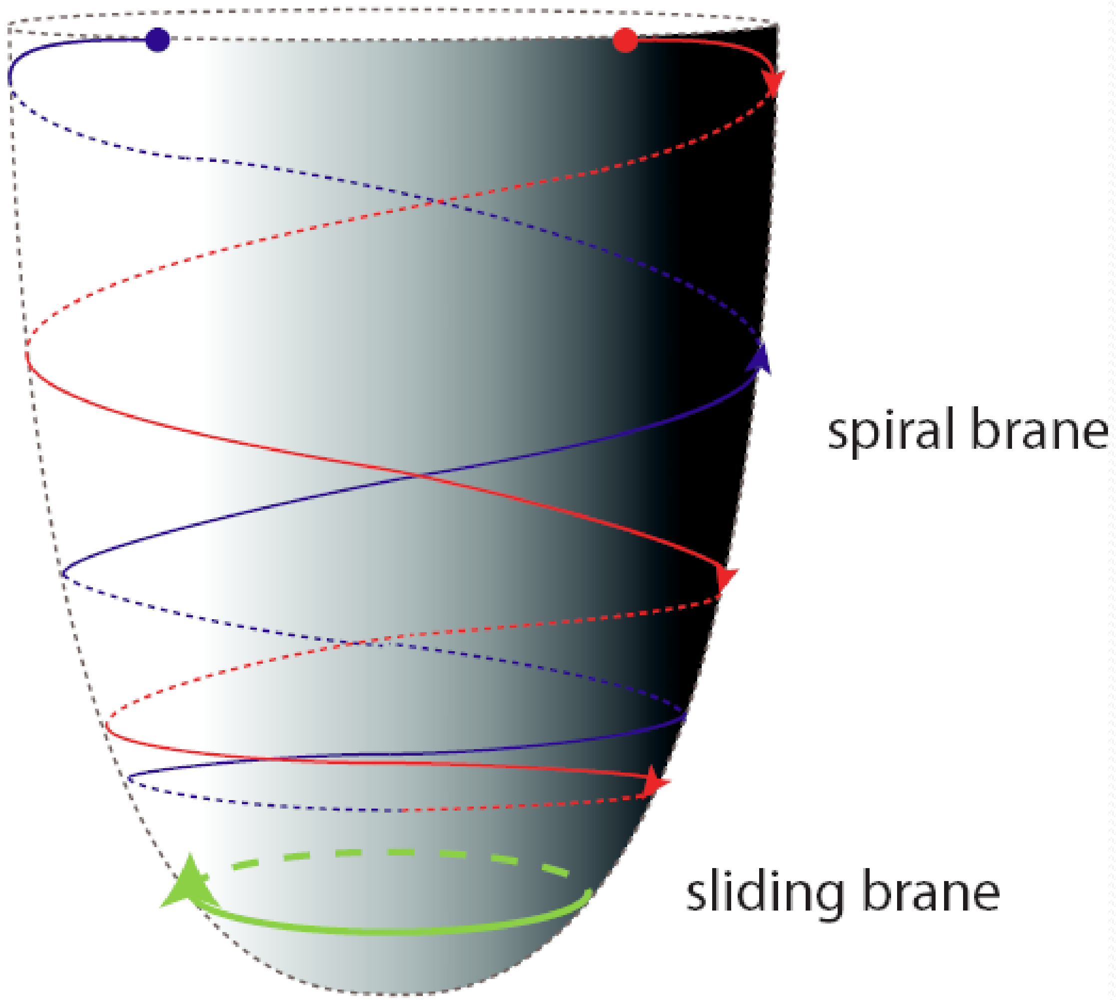
- The cylinder model is a T-dual of the IIB Klebanov Strassler model (in the $R \to 0$ limit)
- In a similar manner to the KKLMMT model to get an inflation model we add on the cylinder a sliding anti-D4 brane.
- For the non-susy cigar model we add instead a sliding D4 brane
- The inflaton is identified with

Infalton= location of the sliding (anti) brane

Inflaton as the location of a slidinding anti-brane



Inflaton as the location of the sliding brane



Extraction of the inflaton potential- cigar geometry

- The background of the near extremal D4 brane

$$
\begin{aligned}
ds^2 &= H_4^{-1/2}(r)(-dt^2 + d\vec{x}_3^2 + f(r)dx_6^2) + H_4^{1/2}(\frac{dr^2}{f} + r^2 d\Omega_4^2) \\
e^{2\phi} &= g_s^2 H_4^{-\frac{1}{2}} \\
C_4 &= \frac{1}{g_s} H_4^{-1} dt \wedge dx^1 \ldots \wedge dx^4 \\
f &= 1 - \left(\frac{r_H}{r}\right)^3 \\
H_4 &= 1 + \alpha_4 \left(\frac{r_4}{r}\right)^3
\end{aligned}
$$

- The action of a Dp brane that resides in this background includes the DBI and CS terms

$$
S_p = -T_p \int e^{-\phi}\sqrt{\det G_{ab}} + \mu_p \int C_{p+1} \quad = -\frac{T_p}{g_s}\int H_p^{-1}(r)\left[\sqrt{f + \frac{H_p(r)}{f_p} g^{\mu\nu}\partial_\mu r \partial_\nu r} - 1\right]
$$

Inflaton potential:cigar

- The corresponding potential for D4 brane is

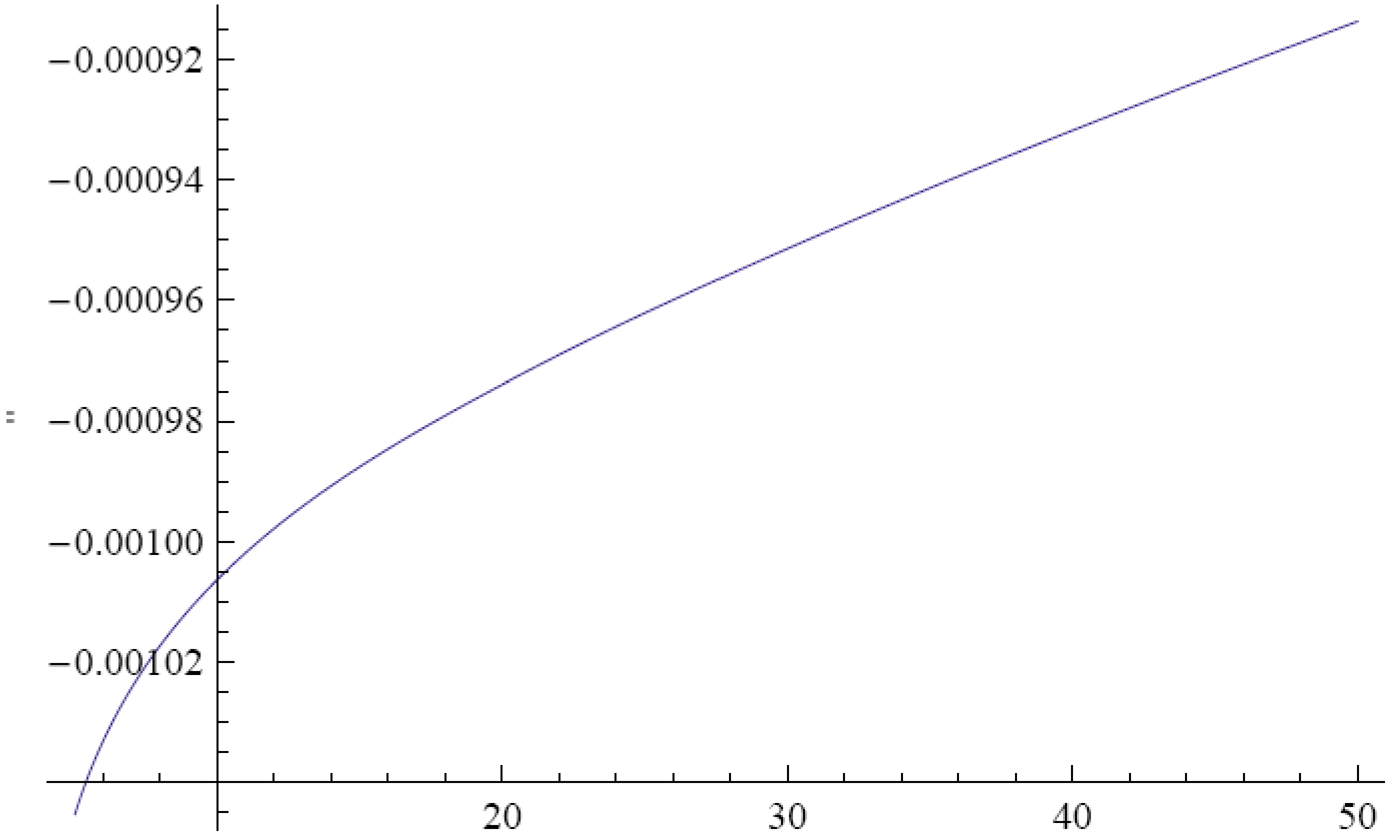
$$V_4(r) = +\frac{T_4 R}{g_s} H^{-1}(r)[\sqrt{f(r)} - 1] = +\frac{T_4 R}{g_s} \frac{1}{1 + \alpha_4 (\frac{r_4}{r})^3} \left[\sqrt{1 - \frac{r_H^3}{r^3}} - 1 \right] < 0$$

- In the regime of $r_H \ll r \ll r_4$, it is approximately

$$V(r) \simeq -\frac{T_4 R}{2 g_s \alpha_4} \left(\frac{r_H}{r_4} \right)^3 \left[1 + \frac{1}{4} \frac{r_H^3}{r^3} - \frac{r^3}{\alpha_4 r_4^3} \right]$$

Inflaton potential:cigar

- The potential due to the background looks like



Vacuum energy

- The potential is negative but it has to be positive
- There is an additional vacuum energy potential since the background is non-supersymmetric
- One can get the near extremality by adding an extra mass with no extra charge $\delta N = \delta M/2$
- Therefore we have

$$\frac{r_H^3}{r_4^3} = \frac{\delta M}{M} = \frac{2\delta N}{N}$$

- The resulting vacuum energy is

$$E_0 = \frac{T_4 R}{g_s} 2\delta N = \frac{T_4 R N}{g_s} \frac{r_H^3}{r_4^3}$$

Inflaton pontential due to the spiral brane

- To calculate the interaction potential between the spiral and the sliding brane we calculate the change in V due to the variation of the Harmonic function
- The Harmonic function corresponds to N original branes at r=o and the sliding one at r=ro thus

$$H \to H + \delta H; \qquad \frac{\delta H}{H} \simeq \frac{r^3}{N(r - r_0)^3}$$

- The corresponding change of the potential

$$\delta V(r_0) = +\frac{2T_4\lambda_p}{g_s}\int_0^\infty dx_6 \frac{r^3}{N(r-r_0)^3} H^{-1}\left\{1 - \frac{H^{-1}f}{(H^{-1}-E)^2}\left[H^{-1} - E(2 + Hu^2)\right]\right\}$$

- The upshot is that this is $g_s p/N$ suppressed

Inflaton pontential: The cylinder

- The calculation of the inflaton potential is similar
- Now the background is extremal and hence rh=0 f=1
- It is a sliding anti-brane and not brane which means that the CS term changes sign so instead of calcelation we get twice the DBI contribution

$$V_4(r) = \frac{2T_4 R}{g_s} \frac{1}{1 + \frac{r_4^3}{r^3}}$$

- Again the contribution from the spiral is suppressed

$$g_s p/N$$

Cosmological inflation from the models

- We express the cosmological properties in terms of

10d Newton constant

Planck scale

$$2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 = m^{-8}$$
$$M_P = m^4 \sqrt{R V_5}$$

4D Planck scale

volume of 5d space

$$T_4 = \frac{1}{2\sqrt{\pi\alpha'}\kappa_{10}} = \frac{m^4}{\sqrt{2\pi\alpha'}}$$

D4 brane tension

Characteristics of inflation

- In order to get a good model of inflation, the slow role parameters

$$\left.\begin{aligned} \epsilon &\equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \\ \eta &= M_P^2\frac{V''}{V} \end{aligned}\right\} << 1$$

- Since the spectral index of the primordial fluctuations

$$P_\delta^s(k) \sim k^{n_s-1}$$

$$n_s - 1 \equiv \frac{d\ln P_\delta^s(k)}{d\ln k} = -6\epsilon + 2\eta$$

the spectrum is almost exactly flat (n_s=1)

Characteristics of inflation

- The number of e-folds during inflation must be

$$\mathcal{N} \equiv \int \frac{H}{\dot{\phi}} d\phi = \int_{\phi_{end}}^{\phi_i} \frac{V}{M_P^2 V'} d\phi = \frac{1}{M_P} \int_{\phi_{end}}^{\phi_i} \frac{d\phi}{\sqrt{2\epsilon}} \geq 60$$

- In addition one needs a sufficiently large reheating temperature at the end of inflation

- This generically requires that the potential energy at the start of inflation be not too far below Planck scale

Inflation parameters for the cigar model

- We first transfer the kinetic term into a canonical one

$$\phi = r\sqrt{T_4 R/g_s}$$

- For the model of sliding D4 along the cigar we get

$$\epsilon \equiv \frac{1}{2}\left(M_P\frac{V'}{V}\right)^2 \simeq \frac{1}{2}\left[\frac{3M_P}{2N\phi}\left(\frac{1}{4}\frac{\phi_H^3}{\phi^3}+\frac{\phi^3}{\phi_4^3}\right)\right]^2$$

$$\eta \equiv M_P^2\frac{V''}{V} = \frac{3M_P^2}{2N\phi^2}\left(-\frac{\phi_H^3}{\phi^3}+2\frac{\phi^3}{\phi_4^3}\right)$$

- The ε and η are thus small in the region

$$\phi_H \ll \phi \ll \phi_4,$$

which is the plateau region

Inflation parameters for the cigar model

- The number of e-folds

$$\mathcal{N} \simeq \frac{2N}{3} \int_{\phi_{end}}^{\phi_{in}} \frac{d\phi}{M_P} \frac{\phi/M_P}{\frac{\phi_H^3}{4\phi^3} + \frac{\phi^3}{\phi_4^3}} \sim \frac{8N}{15} \frac{\phi_{in}^5 - \phi_{end}^5}{M_P^3 \phi_H^3}$$

which can easily be made large enough

Inflation parameters for the cylinder model

- In terms of the canonical field the potential reads

$$V_4(\phi) = \frac{2T_4R}{g_s}\frac{1}{1+\frac{\phi_4^3}{\phi^3}}$$

- The slow role parameters are

$$\begin{aligned}
\epsilon &= \frac{1}{2}\left[\frac{3M_P}{\phi}\frac{\phi_4^3/\phi^3}{1+\phi_4^3/\phi^3}\right]^2 \\
\eta &= -\frac{M_P^2}{\phi^2}\frac{\phi_4^3/\phi^3(32+30\phi_4^3/\phi^3)}{(1+\phi_4^3/\phi^3)^2}
\end{aligned}$$

Inflation parameters for the cylinder model

η and ε can be small

$\phi \gg M_P$

non generic

$\phi \sim M_P$ → $\phi \gg \phi_4$

↓

$\phi_4/M_P \ll 1$

↓

$V_5 m^4/(\sqrt{\alpha'} N^{2/3}) \gg 1$

non generic

If we allow the non-genericity we get large enough

$$\mathcal{N} = \int_{\phi_{end}}^{\phi_{in}} d\phi \frac{\phi^4(1+\phi_4^3/\phi^3)}{3M_P^2\phi_4^3} \simeq \frac{\phi_{in}^5 - \phi_{end}^5}{3M_P^2\phi_4^3}$$

Reheating in the cylinder model

- The potential at the end of inflation is now

$$V_f \sim \frac{2T_4 R}{g_s} \sim 12\pi\epsilon \times 10^{-10} M_P^4$$

- Reheating is simpler in this model since

$$\phi = 0, \text{ where } V(0) = 0$$

- And moreover

$$\frac{dV_4}{d\phi}(\phi) = \frac{6T_4 R}{g_s \phi} \frac{\phi_4^3/\phi^3}{1+\phi_4^3/\phi^3}$$

Reheating in the cylinder model

- So the slope of the potential blows up at $\phi=0$ and we have the usual reheating scenario.
- In this case due to the supersymmetry of the background this result seems to be robust.

Summary and outlook

- We examined holögrphic MQCD models as inflation models
- The models are technically simpler than the KKLMMT model
- We found out that the leading order contribution to the potential comes from the background and not from the spiral
- For near extremal background, the cigar, with a sliding D4 brane flat enough potential with slow-roll conditions easy to fulfill generically.
- For the susy background (cylinder) the gluing of a CY is more natural. Reheating is also easier, however we need to use non-generic initial conditions