PROCEEDINGS OF SCIENCE

Heterotic gauged linear sigma Models with a worldsheet Green–Schwarz mechanism

Stefan Groot Nibbelink*
Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, 80333 München, Germany
E-mail: `Groot.Nibbelink@physik.uni-muenchen.de`

We report on recent progress in heterotic string compactifications using gauged linear sigma models (GLSMs). (2,0) GLSMs are proposed to smoothly interpolate between non–compact heterotic orbifold models and large volume Calabi–Yau compactifications. The charges of these GLSMs are determined from the shifted momenta that characterize the twisted states that take VEVs which generate the blow–up. To allow for more flexibility to obtain solutions to the worldsheet anomaly cancellation conditions, we consider a worldsheet version of the Green–Schwarz mechanism involving field dependent Fayet–Iliopoulos terms.

Proceedings of the Corfu Summer Institute 2011 School and Workshops on Elementary Particle Physics and Gravity
September 4-18, 2011
Corfu, Greece

*Speaker.

1. Approaches to heterotic string phenomenology

Heterotic strings [1, 2] provide promising candidates for unifying descriptions of gravity and particle physics. There has been a huge effort over the past years in identifying Minimal Supersymmetric Standard Model (MSSM) candidates within this framework. There are two traditional approaches which have been vigorously pursued to find interesting heterotic string compactifications:

a. Calabi–Yau model building

A general approach for heterotic string model building is the compactification on smooth Calabi–Yau (CY) threefolds [3] with stable vector bundles [4, 5]. Given that smooth CY spaces with stable bundles are difficult to obtain, finding the MSSM–like models has proven extremely laborious, especially because of the issue of bundle stability [6]. Ongoing efforts of refs. [7 – 11] have resulted in MSSM-like candidates [12 – 15].

b. Orbifold model building

The other possibility of heterotic model building is the compactification on toroidal orbifolds, i.e. discrete quotients of six dimensional tori [16 – 18]. In the so–called "mini–landscape" study on the $T^6/\mathbb{Z}_{6\text{–II}}$ orbifold a large number of models has been identified, which upon switching on a certain number of vacuum expectation values (VEVs) lead to MSSM–like spectra [19 – 23].

Model building on orbifold resolutions

This means that both approaches become dynamically related: The orbifold theory is driven away from the orbifold point resulting in a (partially) smoothed out CY geometry. In order to understand the properties of the resulting CY and the gauge backgrounds it can support, one needs to systematically study the blow–up process. In refs. [24 – 27] the topology of the smooth CY resulting from orbifold resolutions was discussed using toric geometry methods [28, 29]. Abelian gauge fluxes (line bundles) on both non-compact and compact resolutions have been obtained in [30 – 33]; sometimes they can even be constructed explicitly [34 – 36].

For the "mini–landscape" models these results imply that this blow–up process can never lead to a completely smooth CY geometry since the hypercharge (or the weak $SU(2)$) would be broken [33]. To avoid this in full blow–up another interesting MSSM realization was constructed on a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold (see e.g. [37]) as an $SU(5)$ GUT that was subsequently broken down to the MSSM using a freely acting $\mathbb{Z}_{2,\text{free}}$ involution [38]. This model can in principle be blown up completely without breaking the hypercharge [39].

2. Heterotic orbifold resolutions as GLSMs

Why GLSMs?

Unfortunately, the above description of the blow-up procedure has some severe limitations. The main problem is that one uses different frameworks in different regimes: On the one hand, one has an orbifold theory which can be studied using conformal field theory (CFT) techniques. To

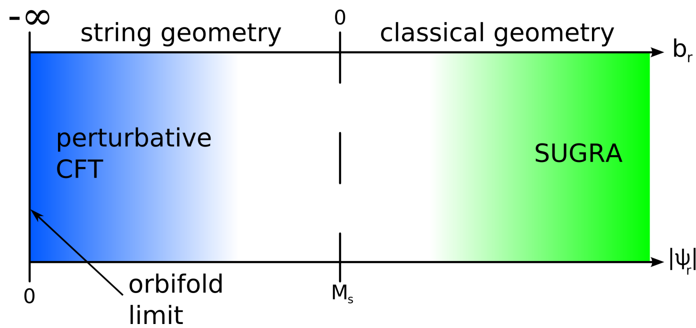

Figure 1: Matching of orbifold and resolution models is hampered at least by the fact that we are comparing different regions in the moduli space.

describe a resolution one then investigates the consequences of presumably marginal operators to characterize the blow–up procedure. As we have seen above these operators correspond to VEVs of twisted states, hence only when they are small, a perturbative treatment of the orbifold resolution process is possible. On the other hand, in a CY compactification the metric is Ricci flat at leading order in the α' expansion. Hence, in order to be able to work with these theories one has to study the supergravity limit of heterotic string theory where higher order α' corrections are negligible. This only allows for partial access to the low energy data of the theory. Seen as a perturbation theory in the string scale, the validity of the supergravity approach can only be guaranteed in the large volume limit. If one instead ones to consider an orbifold limit, one has to include an infinite set of higher order α' corrections. These difficulties when compare orbifold CFTs with VEV deformations and smooth heterotic CY compactifications are illustrated in Figure 1.

In addition, there is the practical complication that often a single orbifold can be related to many smooth CYs which are distinguished by their intersection numbers: Even though a toric resolution of an orbifold singularity is well understood mathematically, it often does not offer an uniquely determined geometry, because the resolution allows for various triangulations. For the $T^6/\mathbb{Z}_{6\text{-II}}$ orbifold this gives us millions of smooth CYs to consider [33]; and there are many orders of magnitude more resolutions of the orbifold $T^6/\mathbb{Z}_2\times\mathbb{Z}_2$ [39]. In the light of this, it would be extremely useful to have a framework that treats all these resolutions of a single orbifold on equal footing.

Two dimensional gauged linear sigma models (GLSMs) provide a formalism that can deal with all these issues [40]. In this proceedings we are only concerned with (2,0) GLSMs. An important ingredient of such a GLSM is the Fayet-Iliopoulos (FI) parameter, which has the geometrical interpretation of a Kähler parameter. When this parameter tends to minus infinity, the worldsheet gauge symmetry is dynamically broken to a finite discrete subgroup, i.e. in this limit the GLSM describes an orbifold [40, 41]. The opposite limit leads to a large volume description, which can be compared with a supergravity treatment. GLSMs are also capable of describing vector bundles and their deformations [42 – 45]. Interest in the subject of (2,0) GLSMs have recently been revived, see e.g. [46 – 49].

superfield type	superfield notation	dimension	charge	bosonic DOF on	bosonic DOF off	fermionic DOF on	fermionic DOF off
chiral	Ψ^a	0	q_I^a	z^a	-	ψ^a	-
chiral-Fermi	Λ^α	1/2	Q_I^α	-	h^α	λ^α	-
gauge	$(V,A)^I$	(0,1)	0	$A^I_\sigma, A^I_{\bar\sigma}$	$\widetilde{D}^I$	ϕ^I	-

Table 1: The superfield content of a generic (2,0) GLSM and their physical on- and off-shell degrees of freedom (DOF).

Twisted states VEVs on orbifolds

From the target space perspective the VEVs of twisted states of the orbifold CFT generate the blow-up; i.e. we expect that the twisted state VEVs specifies which GLSM should be used. Therefore, we briefly recall some basics of twisted states in heterotic orbifolds, details can be found in the refs. [16 – 18, 50].

We consider non-compact orbifolds $\mathbb{C}^3/\mathbb{Z}_N$ defined by the action

$$z^a \to e^{2\pi i v^a}\, z^a ~, \qquad \psi^a \to e^{2\pi i v^a}\, \psi^a ~, \tag{2.1}$$

for $a = 1,2,3$ on the coordinates z^a of $\mathbb{C}^3$ and their superpartners ψ^a with $N\, v^a \in \mathbb{Z}$ and $\sum_i v^a/2 \equiv 0$. The $\mathbb{Z}_N$ action on the gauge fermions λ^I parameterized by the shift vector $V = (V^1,\ldots,V^{16})$ as

$$\lambda^\alpha \to e^{2\pi i V^\alpha}\, \lambda^\alpha ~, \qquad \alpha = 1,\ldots,16 ~. \tag{2.2}$$

These entries are required to satisfy $N\,V \in \Lambda_{16}$, with Λ_{16} the $\text{Spin}(32)/\mathbb{Z}_2$ lattice.

Each twisted state $|T_r\rangle = |p_r,P_r\rangle, \tilde\alpha^a_{-\tilde v_r}|p_r,P_r\rangle$, etc., is characterized by shifted left– and right–moving momenta $p_r = p + v_r$, and $P_r = P + V_r$ in a given twisted sector $r \neq 0$. Here p takes values in the vector lattice Λ_4 of $SO(8)$ and P in the direct sum Λ_{16} of the root and spinor lattices of $SO(32)$. In addition, a number of left-moving oscillators, $\tilde\alpha^a_{-\tilde v_r}$ and $\tilde\alpha^{\underline{a}}_{-\tilde v_r}$, may hit the vacuum state $|p_r;P_r\rangle$. The twisted number operator

$$\widetilde{N}_r = \sum_a (n_r^a - \overline{n}_r^{\underline{a}})\, \tilde v_{r\,a} \tag{2.3}$$

counts the number of such excitations weighted by $\tilde v_{r\,a}$. For example the state $|\tilde\alpha^a_{-\tilde v_r}|p_r,P_r\rangle$ has $\widetilde{N}_r$ equal to $\tilde v_{r\,a}$. The right– and left–moving masses of the twisted state $|T_r\rangle$ are given by

$$M_R^2 = \frac{1}{2}\, p_r^2 - \frac{1}{2} + \delta c_r ~, \qquad M_L^2 = \frac{1}{2}\, P_r^2 - 1 + \delta c_r + \widetilde{N}_r ~, \tag{2.4}$$

where $\delta c_r = \frac{1}{2} - \frac{1}{2}\, \tilde v_r^2$ defines the shift in the vacuum energy in that twisted sector. The formulae above together with the level matching condition $M_R^2 = M_L^2$ implies that

$$P_r^2 = 1 + p_r^2 - 2\,\widetilde{N}_r ~, \qquad p_r^2 = \tilde v_r^2 \tag{2.5}$$

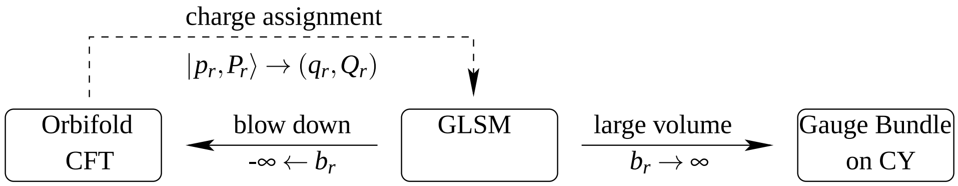

Figure 2: The dashed line schematically indicates that by selecting some blow-up modes $|p_r, P_r\rangle$ within the twisted orbifold CFT spectrum, we can define a specific GLSM with given gauging of the chiral and chiral-Fermi multiplets encoded by the charges (q_r, Q_r). In the blow down limit ($b_r \to -\infty$) of this GLSM we recover the orbifold theory back, while in the large volume limit ($b_r \to \infty$) we obtain a non–compact CY with a line bundle.

(2,0) GLSM Superfields

The field content of a generic (2,0) GLSM can be encoded in a number of superfields which are collected in Table 1. As can be seen from Table 1 a chiral superfield Ψ^a contains a complex scalar z^a and a holomorphic (or right-moving) fermion ψ^a. A chiral-Fermi superfield Λ^α only contains a physical anti-holomorphic (or left-moving) fermion λ^α. The worldsheet theory of the free heterotic string has labels $a = 1,2,3$ and $\alpha = 1,\ldots,16$, respectively. These multiplets are charged w.r.t. gauge multiplets $(V,A)^I$. Their combined action reads

$$S = \int d^2\sigma d^2\theta^+ \left\{ \frac{i}{4}\overline{\Psi}_a \overline{\mathscr{D}}\Psi^a - \frac{1}{4}\overline{\Lambda}_\alpha \Lambda^\alpha + \frac{1}{2e^2}\overline{F}_I F_I \right\} + \int d^2\sigma d\theta^+ \rho_I(\Psi) F_I + \text{h.c.}\,, \tag{2.6}$$

where $\overline{\mathscr{D}} = \bar\partial + 2i(A - i\bar\partial V)_I q_I$ is the gauge covariant derivative and $F_I = -\frac{1}{2}\overline{D}_+(A - i\bar\partial V)$. e_I define the gauge couplings and the complex Fayet–Iliopoulos (FI) parameter $\rho_I = b_I + i\beta_I$. This action results in a scalar potential

$$V = \sum_I \frac{e_I^2}{2}\left(\sum_a q_I^a |z^a|^2 - b_I\right)^2 . \tag{2.7}$$

For consistency such a GLSM has to fulfill at least the following conditions:

1. Cancellation of pure and mixed anomalies:

$$\sum_\alpha Q_I^\alpha Q_J^\alpha = \sum_a q_I^a q_J^a . \tag{2.8}$$

2. Vanishing of sums of chiral superfield charges:

$$\sum_a q_I^a = 0 . \tag{2.9}$$

The first set of conditions reflects the target space condition $c_2(\mathbb{V}) = c_2(TX)$, while the second one the condition $c_1(TX) = 0$ [43].

Characterization of a resolution GLSM

Next we give a precise recipe how to associate a GLSM to an heterotic orbifold model with certain blow–up modes switched on [51]: In a nutshell, the charges of the superfields in the GLSM are determined by the shifted momenta of the twisted blow–up modes. This identification is inspired by our recent findings that the vectors that characterize line bundle embeddings are identical to the shifted momenta of certain twisted states in the orbifold spectrum [33]. The relations between the orbifold CFT, the GLSM and the supergravity descriptions are schematically summarized in Figure 2.

The charges of the chiral superfields are assigned as follows: We promote the shifted right–moving momenta $(p_r)^a$ to charges of an $U(1)_r$ gauge symmetry. Since this gauge symmetry can be used to remove one chiral superfield, one introduces one additional chiral superfield $\Psi^{\text{-}r}$. (We use the index notation -r to denote the additional exceptional chiral superfield; its lowest component we call $x^r = \Psi^{\text{-}r}|$.) Its charge is chosen such that the total sum of charges is zero (2.9):

$$(q_r)^a = (p_r)^a = (\tilde{v}_r)^a \ , \qquad (q_r)^{\text{-}r} = -1 \tag{2.10}$$

The resulting scalar potential

$$V \supset \frac{e_r^2}{2}\Big(q_r^1\,|z^1|^2 + q_r^2\,|z^2|^2 + q_r^3\,|z^3|^2 - |x^r|^2 - b_r \Big)^2 \ , \tag{2.11}$$

has to vanish in order to preserve worldsheet supersymmetry. This leads to two phases: For $b_r < 0$ $z^{\text{-}r}$ takes a non–zero VEV. As the charge of x^r is -1 while the charges q^a of the z^a are fractional, the gauge transformations that preserve $\langle x^r \rangle$ generate a residual $\mathbb{Z}_{n_r}$ gauge action on z^a, with $n_r = N/\gcd(r,N)$ is the order of the twisted sector r. In the opposite case, $b_r > 0$, the zero locus of the potential (2.11) can be written as

$$q_r^1\,|z^1|^2 + q_r^2\,|z^2|^2 + q_r^3\,|z^3|^2 = b_r + |x^r|^2 \ . \tag{2.12}$$

This shows that at least one z^a needs to take a non-vanishing VEV in order that the potential is zero. Moreover, the exceptional divisor $E_r := \{x^r = 0\}$ defines a symplectic quotient $S^5/U(1)$ with an effective radius $\sqrt{b_r}$ for $b_r > 0$. This justifies to interpret b_r as a Kähler parameter (or a blow–up parameter) which measures the size of the exceptional four–cycle E_r.

Inspired by this simple recipe on the right–moving side, we take the charges Q_r of the chiral–Fermi multiplets equal to the left–moving shifted momentum P_r of the twisted state, $|T_r\rangle = |p_r, P_r\rangle$:

$$(Q_r)^\alpha = (P_r)^\alpha \ . \tag{2.13}$$

By the level matching condition (2.5) this satisfies the pure anomaly cancellation condition (2.8):

$$\sum_\alpha (Q_r^\alpha)^2 = P_r^2 = 1 + p_r^2 = \sum_{a=1,2,3,\text{-}r} (q_r^a)^2 \ . \tag{2.14}$$

This assigment can be extended for states with oscillator excitations, see Ref. [51] for details.

3. Anomalies on resolutions of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$

To illustrate these methods we consider the GLSM description of resolutions of the orbifold $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$. The orbifold group contains four elements, apart from the identity we have

$$\theta_a \ : \ \begin{cases} z^a \to z^a \ , \\ z^b \to -z^b \ , \ b \neq a \ , \end{cases} \tag{3.1}$$

divisor	exists when	curve	exists when	curve	exists when
D_1	always	E_1E_2	$b_1,b_2 \geq 0\ ,\ b_3 \leq b_1+b_2$	D_1E_1	$b_1 \geq 0\ ,\ b_1 \geq b_2+b_3$
D_2	always	E_2E_3	$b_2,b_3 \geq 0\ ,\ b_1 \leq b_2+b_3$	D_2E_2	$b_2 \geq 0\ ,\ b_2 \geq b_1+b_3$
D_3	always	E_1E_3	$b_1,b_3 \geq 0\ ,\ b_2 \leq b_1+b_3$	D_3E_3	$b_3 \geq 0\ ,\ b_3 \geq b_1+b_2$
E_1	$b_1 \geq 0$	D_1D_2	$b_3 \leq 0$	$D_1E_{2,3}$	$b_2 \geq 0\ ,\ b_3 \geq 0$
E_2	$b_2 \geq 0$	D_1D_3	$b_2 \leq 0$	$D_2E_{1,3}$	$b_1 \geq 0\ ,\ b_3 \geq 0$
E_3	$b_3 \geq 0$	D_2D_3	$b_1 \leq 0$	$D_3E_{1,2}$	$b_1 \geq 0\ ,\ b_2 \geq 0$

Table 2: This Table indicates under which restrictions of the Kähler parameters b_r the various divisors and curves exist.

for $a = 1,2,3$. There exist six heterotic orbifold models distinguished by the choice of gauge shifts V_a, $a = 1,2,3$; the orbifold standard embedding corresponds to the assignment:

$$V_1 = \frac{1}{2}(0,\text{-}1,\text{-}1,0^{13})\ , \quad V_2 = \frac{1}{2}(\text{-}1,0,\text{-}1,0^{13})\ , \quad V_3 = \frac{1}{2}(\text{-}1,\text{-}1,0,0^{13})\ . \tag{3.2}$$

The $\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold models have three distinct twisted sectors, with shifted right–moving momenta $p_1 = (0,\frac{1}{2},\frac{1}{2})$, $p_2 = (\frac{1}{2},0,\frac{1}{2})$, $p_3 = (\frac{1}{2},\frac{1}{2},0)$, respectively. Hence, we consider the following chiral superfields:

superfield	Ψ^0	Ψ^1	Ψ^2	Ψ^3	Ψ^{-1}	Ψ^{-2}	Ψ^{-3}
q_1	0	0	$1/2$	$1/2$	-1	0	0
q_2	0	$1/2$	0	$1/2$	0	-1	0
q_3	0	$1/2$	$1/2$	0	0	0	-1

This leads to the worldsheet potential

$$V = \frac{e_1^2}{2}\Big(\tfrac{|z^2|^2+|z^3|^2}{2} - |x^1|^2 - b_1\Big)^2 + \frac{e_2^2}{2}\Big(\tfrac{|z^1|^2+|z^3|^2}{2} - |x^2|^2 - b_2\Big)^2 + \frac{e_3^2}{2}\Big(\tfrac{|z^1|^2+|z^2|^2}{2} - |x^3|^2 - b_3\Big)^2. \tag{3.3}$$

Phases of $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold resolutions

By studying the phases defined by this potential we can get a lot of information about the topologies of the corresponding geometries. In particular the existence of the divisors $D_a := \{z^a = 0\}$ and $E_r := \{x^r = 0\}$ and the curves obtained by their intersections depend on the parameters b_r, see Table 2. Using this information one concludes that there are 14 phases in total. These different phases can be represented by toric diagrams: Dots in a toric diagram represent the existing divisors, lines existing curves and cones existing triple intersections. The 14 toric diagrams are depicted in Figure 3. Only four phases correspond to complete smooth geometries and therefore can be treated using supergravity methods. All the other phases correspond to singular geometries: Apart from the orbifold phase, where the $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ geometry is recovered, there are nine partially resolved phases in which not all three possible exceptional divisors exist.

Pure and mixed GLSM anomalies

There are three pure and three mixed anomaly conditions for the GLSM that describes resolutions of a $\mathbb{C}^3/\mathbb{Z}_2\times\mathbb{Z}_2$ orbifold model:

$$Q_1^2 = Q_2^2 = Q_3^2 = \frac{3}{2}\ , \qquad Q_1\cdot Q_2 = Q_2\cdot Q_3 = Q_3\cdot Q_1 = \frac{1}{4}\ . \tag{3.4}$$

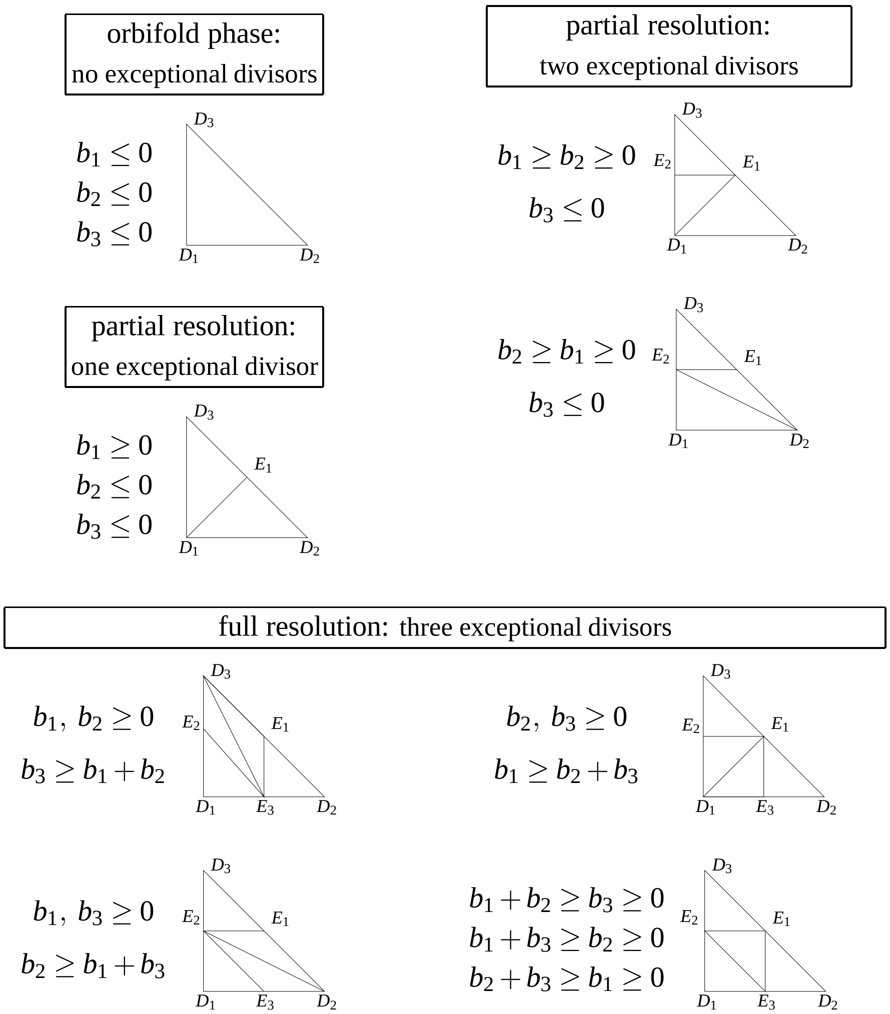

Figure 3: This table gives the 14 phases of the GLSM associated with the $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold. We have not displayed all the partial resolutions: The others are obtained by cyclic permute of the labels 1,2,3.

The orbifold standard embedding model can be blown up by a single blow–up mode in each of the three twisted sectors without oscillators. This can be realized by the left–moving shifted momenta: $P_1 = (0, \frac{1}{2}, \frac{1}{2}, \text{-}1, 0, 0, 0^{10})$, $P_2 = (\frac{1}{2}, 0, \frac{1}{2}, 0, \text{-}1, 0, 0^{10})$, $P_3 = (\frac{1}{2}, \frac{1}{2}, 0, 0, \text{-}1, 0^{10})$, respectively. Taking the charges Q_1, Q_2, Q_3 of the chiral–Fermi multiplets equal to these shifted momenta, one sees that they fulfill all pure and mixed anomaly cancellation conditions (3.4) simultaneously.

Bianchi identities on different triangulations

One can consider heterotic supergravity on one of the four complete resolutions. The fundamental consistency relations for geometrical compactifications are the Bianchi identities. For resolutions of the $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold these conditions strongly depend on the triangulation [31], because the intersection numbers do. In the symmetric triangulation "S" we find three Bianchi identities on the three exceptional divisors E_1, E_2 and E_3

$$Q_1^2 + 2\, Q_2 \cdot Q_3 = 2 \,, \qquad Q_2^2 + 2\, Q_1 \cdot Q_3 = 2 \,, \qquad Q_3^2 + 2\, Q_1 \cdot Q_2 = 2 \,, \tag{3.5}$$

while in the asymmetric triangulation “E_1" we obtain

$$Q_2^2 + Q_3^2 = 3 \,, \qquad Q_2^2 - 2\,Q_1 \cdot Q_3 = 1 \,, \qquad Q_3^2 - 2\,Q_1 \cdot Q_2 = 1 \,. \tag{3.6}$$

(The results in the other two asymmetric triangulations are obtained by cyclic permutations of the labels.) Each of these sets of conditions separately are weaker than (3.4). However, when we combine all four sets of Bianchi identities together, we have a set of equations that are equivalent to the anomaly cancellation conditions (3.4) of the GLSM.

The fact that the anomaly conditions (3.4) of the GLSM contains all the possible Bianchi identities of the supergravity models on the four resolutions, should not come as a surprise: The GLSM formalism allows us to smoothly move in moduli space between the various phases, including the four geometrical phases described by these triangulations. Therefore a consistent string model should at least produce consistent supergravity models in each of these full resolutions. Even though from the supergravity perspective alone, one can argue that the various Bianchi identities that arise in the different triangulations should be superimposed: Even if a flop–transition itself is beyond the range of supergravity validity, in each of the resolutions associated with the different triangulations supergravity should be applicable. Since neither the flux nor any of the exceptional divisors have disappeared during the flop transition, the Bianchi identities on the different resolutions have to be imposed on the same gauge flux.

4. Green–Schwarz mechanism on the worldsheet

We have seen that the GLSM anomaly conditions (2.8) can be very restrictive. One may wonder whether it is possible to alleviate the resulting restrictions. In ten dimension there is a possibility to do: In the so–called Green–Schwarz mechanism certain types of anomalies can be cancelled by an anomalous variation of the Kalb–Ramond two–form. It is therefore natural to ask whether a similar construction is possible in two dimensional GLSMs. This idea has been pursued by Adams et al. [52]: They introduced a chiral superfield which transforms as a shift under Abelian gauge symmetry. To ensure that the target space was compact they assumed that the scalar component of this superfield lives on a torus. For compactification applications this means that one has already lost two real coordinates. However, it is possible to generalize their description by allowing for general field dependent FI–terms.

Field dependent Fayet–Iliopoulos terms

For the gauge superfields we can write down FI–terms

$$W_{\text{FI}} = \frac{1}{2\pi}\,\rho_J(\Psi)\,F^J \,, \tag{4.1}$$

where the FI–parameters ρ_J have become holomorphic functions of chiral superfields Ψ^a. Let us assume that the FI–parameters ρ_I transform as

$$\rho_J \to \rho_J + \mathscr{T}_{IJ}\,\Theta^I \,, \tag{4.2}$$

with $\mathcal{T}_{IJ}$ some constants. These constants are in general not symmetric under the interchange of the gauge indices I and J. (A single ρ^J may be charged under various $U(1)$ gauge symmetries simultaneously.) Consequently, the FI–superpotential transforms as

$$W_{\rm FI} \rightarrow W_{\rm FI} + \frac{1}{2\pi}\sum_I \mathcal{T}_{II}\,\Theta^I F^I + \frac{1}{2\pi}\sum_{I<J}\Big(\mathcal{T}_{IJ}\,\Theta^I F^J + \mathcal{T}_{JI}\,\Theta^J F^I\Big)\,. \tag{4.3}$$

These variations are very similar to the anomalous variation of the effective action and can thus be used to arrive and more general anomaly conditions [53, 54]:

$$\text{Pure anomalies:} \qquad \mathcal{T}_{II} = \frac{1}{2}\Big(Q_I\cdot Q_I - q_I\cdot q_I\Big)\,, \qquad I = J\,, \tag{4 a}$$

$$\text{Mixed anomalies:} \qquad \mathcal{T}_{IJ} = (1-c_{IJ})\,(Q_I\cdot Q_J - q_I\cdot q_J)\,, \qquad I < J\,, \tag{4 b}$$

$$\mathcal{T}_{JI} = c_{IJ}\,(Q_I\cdot Q_J - q_I\cdot q_J)\,, \qquad I > J\,. \tag{4 c}$$

Here we have included the freedom to shift mixed anomalies around by making specific choices for the coefficients c_{IJ}. As observed above, the GS–coefficients $\mathcal{T}_{IJ}$ are often not symmetric, hence we need the c_{IJ} freedom in order to increase the chance to cancel the anomalies.

Let us give one particular example of a $W_{\rm FI}$: Given that the chiral multiplets Ψ^a generically transform with chiral superfield phases, we obtain ρ_J that transform as shifts under gauge transformations. By taking

$$W_{\text{log FI}} = \frac{1}{2\pi}\,\rho_J(\Psi)F^J\,, \qquad \rho_J(\Psi) = \rho_J^0 + T_{XJ}\ln R^X(\Psi)\,, \tag{4.5}$$

we obtain the GS–coefficients

$$\mathcal{T}_{IJ} = r_I^X\,T_{XJ}\,. \tag{4.6}$$

Here ρ_J^0 are constants and $R^X(\Psi)$ are homogeneous polynomials with $U(1)$ charges r_I^X. For simplicity we take below $R^X(\Psi) = \Psi^a$ so that $r_I^X = (q_I)^a$.

Looking at the conditions (4 a)–(4 c), one might get the impression that one is able to cancel any kind of worldsheet gauge anomaly in this way be choosing the GS–coefficients $\mathcal{T}_{IJ}$. This is not the case, because the GS–coefficients $\mathcal{T}_{IJ}$ are subject to stringent quantization conditions. They are often incompatible with the anomaly conditions (4 a)–(4 c). To see how the quantization conditions on the GS–coefficients $\mathcal{T}_{IJ}$ arise, we first recall some basic facts concerning gauge instantons in two dimensions [55, 56]: For an Euclidean one–instanton solution the scalar z^a vanishes at a single point on the worldsheet, say $\sigma = 0$, and the phase of z^b winds non–trivially around this zero. The worldsheet gauge flux is then quantized as

$$\sum_J (q_J)^b \int \frac{f^J_{E\,2}}{2\pi} = 1\,, \tag{4.7}$$

where $f^J_{E\,2} = f^J_{E\,21}\,d\sigma^2 d\sigma^1$. Now the logarithmic FI terms (4.5) have to be well–defined under trivial phases transformations $z^a \rightarrow e^{2\pi i}z^a$. Since,

$$\int d^2\sigma d\theta^+\,W_{\text{log FI}} + \text{h.c.} \supset i\,T_{aJ}\int \text{Im}(\ln z^a)\,\tfrac{f^J_2}{2\pi}\,, \tag{4.8}$$

the anomaly coefficients are quantized as

$$T_{aJ} \int \frac{f_2^J}{2\pi} \in \mathbb{Z} \,, \tag{4.9}$$

under the assumption that $R^X(\Psi) = \Psi^a$.

Possible interpretation: Torsion and NS5–branes

The next question we would like to address is what is the interpretation of such logarithmic FI–terms on the worldsheet. The Kalb–Ramond two–form B_2 can be expanded as $B_2(z) = \beta_I(z) F_2^I$ in harmonic two–forms F_2^I. The coefficients $\beta_I(z)$ can be interpreted as axions with a non–trivial background over the target space geometry. Given that the axions $\beta_I(z) = \text{Im}(\rho_I(z))$ transform with shifts under the worldsheet gauge transformations (4.2), the three–form field strength H_3 of B_2 has to be modified to [52]

$$H_3 = \big(d\beta_J + r_I^X T_{XJ} A_1^I\big) F_2^J \,, \tag{4.10}$$

in order to be globally well–defined. This is the GLSM realization of the effect discussed in [57, 58]: The anomalies in transformations of the worldsheet fermions induce the target space GS–mechanism. The target space is no longer Kähler, i.e. there is torsion, since $H_3 = i(\bar{\partial} - \partial) J_2 \neq 0$ implies that the fundamental two–form J_2 is no longer closed.

The logarithmic worldsheet FI–terms lead to a more drastic modification of the target space geometry. The Kalb–Ramond Bianchi identity reads

$$dH_3 = X_4 + \text{tr}\mathscr{R}_2^2 - \text{tr}\mathscr{F}_2^2 \,, \tag{4.11}$$

where $\mathscr{R}_2$ and $\mathscr{F}_2$ are the curvature and the gauge field strength, respectively. The additional contribution $X_4 = d(d\beta_J) F_2^J$ arises when β_I can become singular. As it measures the failure in the exactness of $\text{tr}\mathscr{R}_2^2 - \text{tr}\mathscr{F}_2^2$, it signals the presence of NS5 branes [59, 60] (also sometimes referred to as H5 branes [61, 62]). Apparently, even though the perturbative heterotic worldsheet theory is incapable of describing the NS5 brane dynamics, it definitely feels their effects.

Consequences for the geometry

The logarithmic FI–term (4.5) the worldsheet D–term potential become

$$V_{\text{D}} = \frac{e_I^2}{2} \Big((q_I)^a \, |z^a|^2 + (q_I)^m \, |z^m|^2 - \frac{1}{2\pi} (b_I^0 + T_{XI} \ln |R^X(z)|) \Big)^2 \,. \tag{4.12}$$

In particular, when $R^X(\Psi) = \Psi^a$, this implies that it is not possible anymore to set $z^a = 0$. This means that the corresponding divisor $D_a := \{z^a = 0\}$ no longer exists: The infinitely thin NS5 brane is replaced by a non–trivial modification of the target space geometry near the cycle that this brane used to wrapped.

The logarithm in potential (4.12) can have further consequences: Depending on the relative signs of the charges and the FI–parameter T_{XI} the geometry can even decompactify [53, 54]. In this case we interpret the singular FI–terms to describe the presence of anti–NS5 branes.

5. Conclusions

In this proceedings we have reported on recent progress on heterotic compactification using gauged linear sigma models. There are two main approaches to obtained interesting models for string phenomenology fro the heterotic string: Exact orbifold constructions or large volume Calabi–Yau compactifications. Even though it is expected that these two approaches are closely related, their precise relation has so far not been fully understood.

In this work we propose to use (2,0) GLSMs to smoothly interpolate between non–compact heterotic orbifold models and large volume Calabi–Yau compactifications. The charges of these GLSMs are determined from the shifted momenta that characterize the twisted states that take VEVs that generate the blow–up.

To allow for more flexibility to obtain solutions to the worldsheet anomaly cancellation conditions we can consider a worldsheet version of the Green–Schwarz mechanism involving field dependent Fayet–Iliopoulos terms. We show that such worldsheet modifications describe torsion target spaces in general. By investigating consequences for the target space B–field Bianchi identity we establish that singular field dependent FI–terms describe NS5 branes, which can have drastic effects on the target space geometry.

References

[1] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, *Heterotic string theory. 1. The free heterotic string*, *Nucl. Phys.* **B256** (1985) 253.

[2] D. J. Gross, J. A. Harvey, E. J. Martinec, and R. Rohm, *Heterotic string theory. 2. The interacting heterotic string*, *Nucl. Phys.* **B267** (1986) 75.

[3] P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, *Vacuum configurations for superstrings*, *Nucl. Phys.* **B258** (1985) 46–74.

[4] S. Donalson, *Anti-self-dual Yang–Mills connections over complex algebraic surfaces and stable vector bundles*, *Proc. Londan Math. Soc.* **50** (1985) 1–26.

[5] K. Uhlenbeck and S. Yau, *On the existence of Hermitian-Yang-Mills connections in stable bector Bundles*, *Comm. Pure and Appl. Math.* **19** (1986) 257–293.

[6] T. L. Gomez, S. Lukic, and I. Sols, *Constraining the Kahler moduli in the heterotic standard model*, *Commun.Math.Phys.* **276** (2007) 1–21 [arXiv:hep-th/0512205].

[7] R. Donagi, B. A. Ovrut, T. Pantev, and D. Waldram, *Standard models from heterotic M-theory*, *Adv. Theor. Math. Phys.* **5** (2002) 93–137 [arXiv:hep-th/9912208].

[8] R. Donagi, B. A. Ovrut, T. Pantev, and D. Waldram, *Standard-model bundles*, *Adv. Theor. Math. Phys.* **5** (2002) 563–615 [arXiv:math/0008010].

[9] R. Donagi, B. A. Ovrut, T. Pantev, and D. Waldram, *Spectral involutions on rational elliptic surfaces*, *Adv. Theor. Math. Phys.* **5** (2002) 499–561 [arXiv:math/0008011].

[10] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, *A heterotic standard model*, *Phys. Lett.* **B618** (2005) 252–258 [arXiv:hep-th/0501070].

[11] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, *The exact MSSM spectrum from string theory*, *JHEP* **05** (2006) 043 [arXiv:hep-th/0512177].

[12] V. Bouchard and R. Donagi, *An SU(5) heterotic standard model*, *Phys. Lett.* **B633** (2006) 783–791 [arXiv:hep-th/0512149].

[13] V. Bouchard and R. Donagi, *On heterotic model constraints*, *JHEP* **08** (2008) 060 [arXiv:0804.2096].

[14] L. B. Anderson, J. Gray, Y.-H. He, and A. Lukas, *Exploring Positive Monad Bundles And A New Heterotic Standard Model*, *JHEP* **02** (2010) 054 [arXiv:0911.1569].

[15] L. B. Anderson, J. Gray, A. Lukas, and E. Palti, *Two Hundred Heterotic Standard Models on Smooth Calabi-Yau Threefolds*, [arXiv:1106.4804].

[16] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, *Strings on Orbifolds*, *Nucl. Phys.* **B261** (1985) 678–686.

[17] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, *Strings on Orbifolds. 2*, *Nucl. Phys.* **B274** (1986) 285–314.

[18] L. E. Ibáñez, J. Mas, H.-P. Nilles, and F. Quevedo, *Heterotic strings in symmetric and asymmetric orbifold backgrounds*, *Nucl. Phys.* **B301** (1988) 157.

[19] O. Lebedev *et al.*, *Low Energy Supersymmetry from the Heterotic Landscape*, *Phys. Rev. Lett.* **98** (2007) 181602 [arXiv:hep-th/0611203].

[20] W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz, *Supersymmetric Standard Model from the heterotic string. II*, *Nucl. Phys.* **B785** (2007) 149–209 [arXiv:hep-th/0606187].

[21] W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz, *Supersymmetric Standard Model from the heterotic string*, *Phys. Rev. Lett.* **96** (2006) 121602 [arXiv:hep-ph/0511035].

[22] O. Lebedev *et al.*, *A mini-landscape of exact MSSM spectra in heterotic orbifolds*, *Phys. Lett.* **B645** (2007) 88–94 [arXiv:hep-th/0611095].

[23] O. Lebedev *et al.*, *The Heterotic Road to the MSSM with R parity*, *Phys. Rev.* **D77** (2008) 046013 [arXiv:0708.2691].

[24] M. R. Douglas, B. R. Greene, and D. R. Morrison, *Orbifold resolution by D-branes*, *Nucl. Phys.* **B506** (1997) 84–106 [arXiv:hep-th/9704151].

[25] F. Denef, M. R. Douglas, and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [arXiv:hep-th/0404257].

[26] D. Lüst, S. Reffert, E. Scheidegger, and S. Stieberger, *Resolved toroidal orbifolds and their orientifolds*, *Adv. Theor. Math. Phys.* **12** (2008) 67–183 [arXiv:hep-th/0609014].

[27] S. Reffert, *The Geometer's Toolkit to String Compactifications*, [arXiv:0706.1310].

[28] W. Fulton, *Introduction to Toric Varieties*. Princeton University Press 1993.

[29] K. Hori *et al.*, *Mirror symmetry*, . Providence, USA: AMS (2003) 929 p.

[30] G. Honecker and M. Trapletti, *Merging heterotic orbifolds and K3 compactifications with line bundles*, *JHEP* **01** (2007) 051 [arXiv:hep-th/0612030].

[31] S. Groot Nibbelink, T.-W. Ha, and M. Trapletti, *Toric Resolutions of heterotic orbifolds*, *Phys. Rev.* **D77** (2008) 026002 [arXiv:0707.1597].

[32] S. Groot Nibbelink, D. Klevers, F. Plöger, M. Trapletti, and P. K. S. Vaudrevange, *Compact heterotic orbifolds in blow-up*, *JHEP* **04** (2008) 060 [arXiv:0802.2809].

[33] S. Groot Nibbelink, J. Held, F. Ruehle, M. Trapletti, and P. K. S. Vaudrevange, *Heterotic Z6-II MSSM orbifolds in blowup*, *JHEP* **03** (2009) 005 [arXiv:0901.3059].

[34] S. Groot Nibbelink, M. Trapletti, and M. Walter, *Resolutions of C^n/Z_n orbifolds, their U(1) bundles, and applications to string model building*, *JHEP* **03** (2007) 035 [arXiv:hep-th/0701227].

[35] S. Groot Nibbelink, H. P. Nilles, and M. Trapletti, *Multiple anomalous U(1)s in heterotic blow-ups*, *Phys. Lett.* **B652** (2007) 124–127 [arXiv:hep-th/0703211].

[36] S. Groot Nibbelink, F. Paccetti Correia, and M. Trapletti, *Non-Abelian bundles on heterotic non-compact K3 orbifold blowups*, *JHEP* **11** (2008) 044 [arXiv:0809.4430].

[37] R. Donagi and K. Wendland, *On orbifolds and free fermion constructions*, *J.Geom.Phys.* **59** (2009) 942–968 [arXiv:0809.0330].

[38] M. Blaszczyk *et al.*, *A Z2xZ2 standard model*, *Phys. Lett.* **B683** (2010) 340–348 [arXiv:0911.4905].

[39] M. Blaszczyk, S. Groot Nibbelink, F. Ruehle, M. Trapletti, and P. K. S. Vaudrevange, *Heterotic MSSM on a resolved orbifold*, *JHEP* **09** (2010) 065 [arXiv:1007.0203].

[40] E. Witten, *Phases of N = 2 theories in two dimensions*, *Nucl. Phys.* **B403** (1993) 159–222 [arXiv:hep-th/9301042].

[41] S. Kachru and E. Witten, *Computing the complete massless spectrum of a Landau- Ginzburg orbifold*, *Nucl. Phys.* **B407** (1993) 637–666 [arXiv:hep-th/9307038].

[42] J. Distler and S. Kachru, *(0,2) Landau-Ginzburg theory*, *Nucl. Phys.* **B413** (1994) 213–243 [arXiv:hep-th/9309110].

[43] J. Distler, *Notes on (0,2) superconformal field theories*, [arXiv:hep-th/9502012].

[44] J. Distler, B. R. Greene, and D. R. Morrison, *Resolving singularities in (0,2) models*, *Nucl. Phys.* **B481** (1996) 289–312 [arXiv:hep-th/9605222].

[45] T.-M. Chiang, J. Distler, and B. R. Greene, *Some features of (0,2) moduli space*, *Nucl. Phys.* **B496** (1997) 590–616 [arXiv:hep-th/9702030].

[46] I. V. Melnikov and M. R. Plesser, *The Coulomb branch in gauged linear sigma models*, *JHEP* **06** (2005) 013 [arXiv:hep-th/0501238].

[47] P. S. Aspinwall, I. V. Melnikov, and M. R. Plesser, *(0,2) elephants*, [arXiv:1008.2156].

[48] M. Kreuzer, J. McOrist, I. V. Melnikov, and M. R. Plesser, *(0,2) deformations of linear sigma models*, [arXiv:1001.2104].

[49] J. McOrist, *The revival of (0,2) linear sigma models*, [arXiv:1010.4667].

[50] P. K. S. Vaudrevange, *Grand Unification in the Heterotic Brane World*, [arXiv:0812.3503]. PhD thesis.

[51] S. Groot Nibbelink, *Heterotic orbifold resolutions as (2,0) gauged linear sigma models*, *Fortsch.Phys.* **59** (2011) 454–493 [arXiv:1012.3350].

[52] A. Adams, M. Ernebjerg, and J. M. Lapan, *Linear models for flux vacua*, *Adv. Theor. Math. Phys.* **12** (2008) 817–851 [arXiv:hep-th/0611084].

[53] M. Blaszczyk, S. Groot Nibbelink, and F. Ruehle, *Green-Schwarz Mechanism in Heterotic (2,0) Gauged Linear Sigma Models: Torsion and NS5 Branes*, *JHEP* **1108** (2011) 083 [arXiv:1107.0320].

[54] C. Quigley and S. Sethi, *Linear Sigma Models with Torsion*, *JHEP* **1111** (2011) 034 [arXiv:1107.0714].

[55] A. Adams, A. Basu, and S. Sethi, *(0,2) duality*, *Adv. Theor. Math. Phys.* **7** (2004) 865–950 [arXiv:hep-th/0309226].

[56] A. Adams, *Orbifold phases of heterotic flux vacua*, [arXiv:0908.2994].

[57] C. M. Hull and E. Witten, *Supersymmetric sigma models and the heterotic string*, *Phys. Lett.* **B160** (1985) 398–402.

[58] C. Hull and P. Townsend, *World sheet supersymmetry and anomaly cancellation in the heterotic string*, *Phys.Lett.* **B178** (1986) 187.

[59] E. Witten, *Small instantons in string theory*, *Nucl.Phys.* **B460** (1996) 541–559 [arXiv:hep-th/9511030].

[60] D. Tong, *NS5-branes, T duality and world sheet instantons*, *JHEP* **0207** (2002) 013 [arXiv:hep-th/0204186].

[61] A. Strominger, *Heterotic solitons*, *Nucl.Phys.* **B343** (1990) 167–184.

[62] G. Honecker, *Massive U(1)s and heterotic five-branes on K3*, *Nucl.Phys.* **B748** (2006) 126–148 [arXiv:hep-th/0602101].