

Thermal duality & non-singular superstring cosmology

Hervé Partouche

Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau cedex, France*

E-mail: herve.partouche@cpht.polytechnique.fr

We review the construction of superconformal field theories on the worldsheet, which describe superstring models where only a finite number of states are effectively thermalized. Compared to conventional superstring models at finite temperature, they are obtained by switching on suitable Wilson lines along the Euclidean time circle $S^1(R_0)$. This discrete deformation forbids the appearance of tachyons at any radius R_0 and restores a T-duality symmetry on the temporal cycle. This implies the existence of a maximal temperature T_c , which is equal to twice the standard Hagedorn temperature. The models obtained this way differ substantially from the usual thermal ones only in the regime where the temperature is of order of the string scale, when the canonical ensemble of the full superstring spectrum breaks down. In the tachyon free models, a transition occurs at the temperature T_c , which transforms a phase of pure Kaluza-Klein excitations along $S^1(R_0)$ into a T-dual phase of pure winding modes. Thanks to the consistency of these thermal backgrounds, cosmological evolutions induced by the free energy are found in various dimensions, with neither Hagedorn instabilities nor initial singularities. They describe bouncing universes, which can be described consistently in perturbation theory throughout the evolution.

Proceedings of the Corfu Summer Institute 2011 “School and Workshops on Elementary Particle Physics and Gravity”

September 4-18, 2011

Corfu, Greece

*Unité mixte du CNRS et de l’Ecole Polytechnique, UMR 7644.

1. Introduction

General relativity coupled to a finite set of local quantum fields drives the cosmological evolution of the universe to an initial curvature singularity. On the contrary, the consistency of string theory may lead to think that in this framework, the Big Bang may be resolved. To discuss this belief, we may start by asking what is usually meant by “consistency” in this context. A possible answer from an infrared (IR) point of view is that the spectrum should not contain tachyons, so that the one-loop vacuum amplitude Z is finite. From a string field theory point of view, when tachyons are present in a spectrum, they should acquire vacuum expectation values (VEV’s) in order to bring the system in a true, stable vacuum. Modular invariance on the worldsheet allows to rephrase the above IR picture in a more conventional ultra-violet (UV) point of view, by writing Z in the general form,

$$Z = \int_0^{+\infty} \frac{d\ell}{2\ell} V_d \int \frac{d^d k}{(2\pi)^d} \int_0^{+\infty} dM \left(\rho_B(M) - \rho_F(M) \right) e^{-(k^2 + M^2)\ell/2}, \quad (1.1)$$

where ρ_B and ρ_F are the densities of bosons and fermions at level mass M . In this expression, V_d is very large and regularizes the volume of the d -dimensional space-time. Strictly speaking, the integral over M should be replaced by a discrete sum over the mass spectrum. The dangerous region of integration over the proper time ℓ along the virtual loop is the UV region, $\ell \rightarrow 0$. In fact, for Z to be finite, a drastic cancellation between the bosonic and fermionic densities must occur. This is a very strong constraint on the consistency of the model since, as is generic in string theory, both ρ_B and ρ_F are exponentially growing for large masses, $\rho_{B,F} \sim e^{\beta_H M}$ as $M \rightarrow +\infty$. Actually, a careful analysis leads to the conclusion that the effective number of degrees of freedom $\rho_B - \rho_F$ grows like in a two-dimensional *quantum field theory* [1]. In other words, in a consistent string background, everything happens as if there were effectively only a finite number of particles running into the virtual loops. The supersymmetric models certainly fulfill this condition since bosons and fermions cancel exactly and Z vanishes. A less trivial class of consistent models includes the theories, where supersymmetry is spontaneously broken at a scale M_{susy} . In this case, the mass degeneracy is restored in the large M/M_{susy} limit, a fact that deserves the denomination of “asymptotic supersymmetry” [1]. In these models, the effective number of degrees of freedom is of the order of the number of states below the scale M_{susy} .

For cosmological purposes, the question of consistency should be reconsidered for conformal fields theories on the worldsheet, which describe string backgrounds at finite temperature T . In this case, the one-loop vacuum amplitude Z is evaluated in a target space, where time is Euclidean and compactified on a circle of perimeter $\beta = 2\pi R_0 = 1/T$. Bosonic and fermionic degrees of freedom are imposed periodic and antiperiodic boundary conditions along $S^1(R_0)$. Under these circumstances, one finds

$$\begin{aligned} Z &= V \int \frac{d^{d-1} \mathbf{k}}{(2\pi)^{d-1}} \int_0^{+\infty} dM \left[\rho_B(M) \ln \left(\frac{1}{1 - e^{-\beta \sqrt{\mathbf{k}^2 + M^2}}} \right) + \rho_F(M) \ln \left(1 + e^{-\beta \sqrt{\mathbf{k}^2 + M^2}} \right) \right] \\ &= \ln \text{Tr} e^{-\beta H}, \end{aligned} \quad (1.2)$$

where V is the volume of space, and the second equality makes the link with quantum statistical physics. The exponential of Z equals the canonical partition function of bosonic and fermionic modes, whose Hamiltonian is free *i.e.* in the perfect gas approximation. Since the logarithmic

terms in Eq. (1.2) behave as $e^{-\beta M}$ in the large M limit, the thermal amplitude Z diverges when $\beta_H > \beta$. Thus, T larger than the so-called Hagedorn temperature $T_H = 1/\beta_H$ is not allowed. It is believed that this singular behavior in the UV signals a transition at the maximal temperature T_H , which brings the thermal system into a distinct phase. One way to argue this is to write the amplitude (1.2) as an integral over the fundamental domain of $SL(2, \mathbb{Z})$ and observe that in a Hamiltonian formulation of the integrand, winding modes around the Euclidean temporal circle become tachyonic when $T > T_H$. Thus, they should acquire VEV's when they become massless at the Hagedorn temperature, and bring the thermal system into a stable configuration [2]. However, from a cosmological point of view, the dynamical description of this process is not understood yet.

In this note, our aim is to review the existence and cosmological consequences of conformal field theories on the worldsheet, which are describing models where only a finite number of string modes are effectively thermalized. The amplitude Z is well defined for arbitrary radius R_0 and the models actually admit a T-duality symmetry on the temporal circle, $R_0/R_c \rightarrow R_c/R_0$, which implies the existence of a maximal temperature, $T_c = 1/2\pi R_c$. Since the masses of the string excitations which are not thermalized are larger than T_c , the system can never differ substantially from a conventional thermal one. At the critical temperature T_c , a phase transition occurs, transforming pure Kaluza-Klein (KK) excitations along $S^1(R_0)$ into pure winding modes. Thanks to the consistency of these thermal backgrounds, cosmological evolutions free of Hagedorn and initial singularities are found. They describe bouncing universes, which can be described consistently in perturbation theory throughout the evolution [3, 4].

2. Superstring models with no Hagedorn instabilities

In this section, we make the comparison between conventional models at finite temperature and those, which are free of Hagedorn divergences [5]. We consider the type IIB superstring compactified on the Euclidean background $S^1(R_0) \times T^{d-1} \times T^{9-d} \times S^1(R_9)$. The torus T^{d-1} has volume V , which regularizes the infinite size of flat space, while $T^{9-d} \times S^1(R_9)$ is the internal manifold. All space-time supersymmetries generated by the right-moving sector are spontaneously broken by coupling the lattice $\Gamma_{(1,1)}$ of zero modes associated to $S^1(R_9)$ to the right-moving Ramond charge \bar{a} (equal to 0, 1 modulo 2).¹ At finite temperature, the partition function takes the following form

$$Z = \frac{V}{(2\pi)^{d-1}} \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^{\frac{d+1}{2}}} \frac{\Gamma_{(9-d,9-d)}}{\eta^8 \bar{\eta}^8} \frac{1}{2} \sum_{a,b} (-1)^{a+b+ab} \frac{\theta_{[b]}^{[a]^4}}{\eta^4} \frac{1}{2} \sum_{\bar{a}, \bar{b}} (-1)^{\bar{a}+\bar{b}+\bar{a}\bar{b}} \frac{\bar{\theta}_{[\bar{b}]}^{[\bar{a}]^4}}{\bar{\eta}^4} \frac{R_0}{\sqrt{\tau_2}} \sum_{n_0, \bar{m}_0} e^{-\frac{\pi R_0^2}{\tau_2} |n_0 \tau + \bar{m}_0|^2} (-1)^{(a+\bar{a})\bar{m}_0 + (b+\bar{b})n_0} \frac{R_9}{\sqrt{\tau_2}} \sum_{n_9, \bar{m}_9} e^{-\frac{\pi R_9^2}{\tau_2} |n_9 \tau + \bar{m}_9|^2} (-1)^{\bar{a}\bar{m}_9 + \bar{b}n_9 + \bar{m}_9 n_9}, \quad (2.1)$$

where the lattice of zero modes $\Gamma_{(1,1)}$ of the Euclidean time circle is coupled to the fermion number $F = a + \bar{a}$, the left plus right Ramond charges. At the fermionic point $R_9 = R_c := 1/\sqrt{2}$, the $U(1)_R$ gauge symmetry associated to the direction 9 is extended to $SU(2)_R$. This fact implies the thermal effective potential $-Z$ admits a minimum at this point, so that we can suppose from now on that R_9 is stabilized at R_c [7, 8]. As a result, the scale $1/(2\pi R_9)$ of right-moving supersymmetry breaking is of order the string scale. In Eq. (2.1), the remaining left-moving supersymmetries are

¹More general models where the spontaneous breaking involves more internal directions can be considered [6].

spontaneously broken by the presence of finite temperature, at the scale $T = 1/(2\pi R_0)$. Clearly, they could have been broken at the same scale by coupling the zero modes of $S^1(R_0)$ to the left-moving Ramond charge a only, (equal to 0, 1 modulo 2). Technically, this amounts to changing the phase

$$(-1)^{(a+\bar{a})\bar{m}_0+(b+\bar{b})n_0} \longrightarrow (-1)^{a\bar{m}_0+bn_0+\bar{m}_0n_0} \quad (2.2)$$

in Eq. (2.1). To show that the model obtained this way admits an interesting thermal interpretation is the aim of the remaining part of this section [5].

In terms of $SO(8)$ affine characters, the partition function Z of the model obtained by the substitution (2.2) involves in the left-moving sector the combination

$$\sum_{m_0, l_0} \left[\left(\Gamma_{m_0, 2l_0} V_8 - \Gamma_{m_0 + \frac{1}{2}, 2l_0} S_8 \right) + \left(\Gamma_{m_0, 2l_0 + 1} O_8 - \Gamma_{m_0 + \frac{1}{2}, 2l_0 + 1} C_8 \right) \right] \Big|_{R_0}, \quad (2.3)$$

where $\Gamma_{\alpha, \delta}(R) := q^{[(\alpha/R)^2 + (\delta R)^2]/4}$. This shows that the GSO projection is reversed in the odd winding sector along the Euclidean time circle, $n_0 = 2l_0 + 1$. Similarly, the right-moving GSO projection is reversed in the odd winding sector along $S^1(R_0)$. Thus, there exists a potentially dangerous $O_8 \bar{O}_8$ sector that may yield tachyonic modes. However, the lowest squared mass in this sector is non-negative,

$$M_{O\bar{O}}^2 = \left(\frac{1}{2R_0} + R_0 \right), \quad (2.4)$$

a fact which is in contrast with the conventional thermal model, where $M_{O\bar{O}}^2 = R_0^2 - 2$. In the latter case, a Hagedorn radius $R_H = \sqrt{2}$ exists, below which the partition function is ill-defined. On the contrary, even better than being defined for all values of R_0 , the tachyon free model admits a T-duality symmetry

$$\left(\frac{R_0}{R_c}, S_8, C_8 \right) \longrightarrow \left(\frac{R_c}{R_0}, C_8, S_8 \right), \quad (2.5)$$

which implies type IIB and type IIA models are identified. Finally, an important fact to which we return later, is that at the fermionic point $R_0 = R_c$ fixed by the T-duality, the $U(1)_L$ Euclidean gauge symmetry is extended to $SU(2)_L$, coupled to matter in the adjoint representation.

To make contact between the thermal partition function (2.1) and statistical physics, we “unfold the fundamental domain \mathcal{F} ”. In practice, this means the integral of \mathcal{F} can be extended to the whole upper half strip, while restricting the winding number n_0 to zero. Formally, one has

$$Z = \int_{\mathcal{F}} d^2\tau \sum_{n_0} (\dots) |_{n_0} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\tau_1 \int_0^{+\infty} d\tau_2 (\dots) |_{n_0=0}. \quad (2.6)$$

Since $n_0 = 0$ is even, the left-moving characters involved in the integrand are V_8 and S_8 only. It is then straightforward to implement level matching by integrating over τ_1 and write Z as the logarithm of a canonical partition function of point-like particles, when it is expressed in first quantized formalism as an integral over the proper time τ_2 . As reviewed in Eq. (1.2), one obtains

$$e^Z = \text{Tr} e^{-\beta H} \quad \text{where} \quad (2.7)$$

$$\beta = 2\pi R_0 \quad \text{and} \quad \text{the left-moving Ramond character is } S_8.$$

Noticing that $(-1)^a = (-1)^{a+\bar{a}}(-1)^{\bar{a}}$, the above manipulations can be applied to the tachyon free model to yield a similar result, up to a remnant phase $(-1)^{\bar{a}}$ in the final expression. Actually, the unfolding procedure is valid when the integrand converges absolutely, which is guaranteed as long as $R_0 > R_c$. The analysis in the regime $R_0 < R_c$ can be derived by T-duality and amounts to setting \tilde{m}_0 instead of n_0 to zero (see Eqs (2.1) and (2.2)). Altogether, the amplitude Z in the tachyon free model is related to a *deformed canonical partition function* defined as [5]

$$e^Z = \text{Tr} \left[(-1)^{\bar{a}} e^{-\beta H} \right] \quad \text{where} \quad (2.8)$$

$$\begin{cases} \beta = 2\pi R_0 & \text{and the left-moving Ramond character is } S_8, \text{ when } R_0 > R_c \\ \beta = 2\pi/(2R_0) & \text{and the left-moving Ramond character is } C_8, \text{ when } R_0 < R_c \end{cases}$$

In this expression, \bar{a} stands for the total right-moving Ramond charge of the multiparticle states contributing to the trace. In the phase $R_0 > R_c$, the excitations along $S^1(R_0)$ are pure KK states, while in the T-dual phase $R_0 < R_c$, they are pure winding modes. Moreover, the spinorial representations of the massless space-time fermions differ, as follows from the exchange $S_8 \leftrightarrow C_8$. An important consequence of the definition of the parameter β in Eq. (2.8) is that $T := 1/\beta$ is bounded, $T \leq T_c := 1/(2\pi R_c)$. However, the physical interpretation of the parameter T is at this stage rather obscure.

To clarify this point, we note that the mass M of any state with right-moving Ramond charge $\bar{a} = 1$ satisfies

$$M \geq \frac{1}{2R_9} \text{ or } R_9 \implies M \geq \pi T_c > T, \quad (2.9)$$

as follows from the stabilization of R_9 at the fermionic point R_c . As a result, the contributions in the trace (2.8) of the multiparticle states with total Ramond charges $\bar{a} = 1$ are weighted with exponentially small Boltzmann factors, so that

$$e^Z = \text{Tr} \left[(-1)^{\bar{a}} e^{-\beta H} \right] \simeq \text{Tr}_{M < T_c} e^{-\beta H}, \quad (2.10)$$

where the trace in the r.h.s. is restricted to the multiparticle states built out of modes with masses below T_c . As a result, T deserves the denomination of temperature for this restricted canonical ensemble. Actually, the conventional thermal model and the tachyon free one differ substantially only in the regime where the temperature is of order of the string scale, when the canonical ensemble of the full superstring spectrum breaks down and the partition function (2.7) is blowing up.

In fact, the maximal temperature T_c is larger than the Hagedorn temperature T_H of the conventional thermal case, $T_c = 2T_H$. The convergence of the amplitude Z in the tachyon free model for any temperature below the maximal value T_c follows from the alternative signs in the deformed canonical partition function (2.8). To precise this remark, let us focus for a moment on the deformed trace associated to a gas made of a single species, with fermion number F , mass M and right-moving Ramond charge \bar{a} . A direct evaluation based on the Bose-Einstein (or Fermi-Dirac) statistics leads [5]

$$\ln \left(\text{Tr} \left[(-1)^{\bar{a}} e^{-\beta H} \right] \right) = -(-1)^F \sum_{\mathbf{k}} \ln \left(1 - (-1)^{F+\bar{a}} e^{-\beta \sqrt{\mathbf{k}^2 + M^2}} \right). \quad (2.11)$$

This shows that the contributions of a bosonic particle with charge \bar{a} and a fermionic one with charge $1 - \bar{a}$ are opposite, when the masses are degenerate. This fact implies the effective number of degrees of freedom contributing to the total tachyon free amplitude Z is reduced.

In extreme cases, the pairing of a boson of charge \bar{a} with a fermion of charge $1 - \bar{a}$ and equal masses can be an exact symmetry of the entire massive spectrum of the theory. Under these circumstances, a perfect cancellation of the massive contributions occurs and e^Z takes the form of a conventional canonical partition function for thermal radiation associated to a finite number of massless bosonic and fermionic degrees of freedom. Actually, such a symmetry can only exist in two dimensions and is realized when the right-moving sector admits a so-called MSDS structure, for ‘‘Massive Spectrum Degeneracy Symmetry’’ [9]. The ‘‘hybrid model’’ realizes these ideas in type II superstring [6, 3]. Its Euclidean one-loop amplitude is defined as

$$Z = \frac{V}{2\pi} \int_{\mathcal{F}} \frac{d^2\tau}{2\tau_2^{3/2}} \frac{\Gamma_{(8,0)}}{\eta^8} \frac{1}{2} \sum_{a,b} (-1)^{a+b+ab} \frac{\theta_{[b]}^{[a]^4}}{\eta^4} (\bar{V}_{24} - \bar{S}_{24}) \frac{R_0}{\sqrt{\tau_2}} \sum_{n_0, \bar{m}_0} e^{-\frac{\pi R_0^2}{\tau_2} |n_0\tau + \bar{m}_0|^2} (-1)^{a\bar{m}_0 + b n_0 \bar{m}_0 n_0}, \quad (2.12)$$

where V is the regularized ‘‘volume’’ of the one-dimensional space and $\Gamma_{(8,0)}$ is the E_8 root lattice. The zero modes of the Euclidean time circle are coupled to the left-moving Ramond charge a and a deformed canonical partition function is obtained, as defined in Eq. (2.8). The MSDS structure follows from the choice of right-moving conformal blocks,

$$\bar{V}_{24} - \bar{S}_{24} = 24 \quad \text{where} \quad \bar{S}_{24} = 2^{11} \bar{q} + \mathcal{O}(\bar{q}^2). \quad (2.13)$$

These relations satisfied by the $SO(24)$ affine characters show the right-moving sector is almost supersymmetric, since only 24 states at level 0 in the NS sector $\bar{a} = 0$ are not paired with degenerate modes in the Ramond sector $\bar{a} = 1$. Due to this property, level matching projects out all massive contributions in Z , which takes the explicitly T-duality invariant simple form

$$\frac{Z}{V} = 24 \begin{cases} 1/R_0 & \text{for } R_0 > R_c \\ 2R_0 & \text{for } R_0 < R_c \end{cases} = 24 \left(\frac{1}{2R_0} + R_0 \right) - 24 \left| \frac{1}{2R_0} - R_0 \right|. \quad (2.14)$$

Given the definition of the duality invariant temperature $T = 1/\beta$ (see Eq. (2.8)), the free energy density is exactly that of thermal radiation in two dimensions, in either of the winding phase ($R_0 < R_c$) or momentum phase ($R_0 > R_c$),

$$\frac{F}{V} \equiv -\frac{Z}{\beta V} = -T^2 n \Sigma_2, \quad (2.15)$$

where n counts the number of massless states and Σ_d is Stefan’s constant for radiation in d dimensions.

Coming back to the more general class of tachyon free models that yield Eq. (2.8) in arbitrary dimensions, the MSDS structure of the Hybrid model is replaced by asymptotic supersymmetry in the right-moving sector, where supersymmetry is spontaneously broken at the scale $T_c = 1/(2\pi R_9)$ [4]. As a result, the perfect cancellation of the massive modes in the Hybrid case is only approximate in the general case, as indicated in Eq. (2.10). The free energy density then satisfies

$$\frac{F}{V} \equiv -\frac{Z}{\beta V} \simeq -T^d n \Sigma_d, \quad (2.16)$$

which ensures the positivity of the energy and specific heat,

$$U = -\frac{\partial Z}{\partial \beta} > 0, \quad C_V = \beta^2 \frac{\partial^2 Z}{\partial \beta^2} > 0. \quad (2.17)$$

In particular, redefining the duality-invariant temperature as $T = T_c e^{-|\sigma|}$, where $R_0 = R_c e^\sigma$, the amplitude Z depends on $|\sigma|$ only and shows a conical singularity at the fermionic point $\sigma = 0$. In other words, $Z(\sigma)$ is not derivable at $\sigma = 0$, a fact that would imply the existence of a temperature range, where the specific heat would be negative.

After having described the system in the winding ($R_0 < R_c$) and momentum ($R_0 > R_c$) phases, we would like to understand its behavior at the fermionic point $R_0 = R_c$, where a $U(1)_L \rightarrow SU(2)_L$ extension of the Kac-Moody algebra occurs at level 2. Euclidean gauge bosons and adjoint matter with momentum and winding numbers $m_0 = n_0 = \pm 1$ become massless and are actually at the origin of the above mentioned non-analyticity of the amplitude Z . For instance, this is explicitly seen in the Hybrid model, where the term in absolute value in Eq. (2.14) is the vanishing mass of 8 complex scalars in the adjoint representation of $SU(2)_L$ (in two dimensions, the gauge bosons have no degrees of freedom). At the fermionic point, they have $p_L^0 = \pm 1$, $p_R^0 = 0$, which is precisely what is needed to map the spectrum of the momentum phase into that of the winding phase, and vis versa. For instance, acting with the holomorphic current $\psi^0 e^{-iX_L^0}$ on a pure KK state in the spinorial representation S_{10} of $SO(10)$, one obtains [3]

$$\psi^0 e^{-iX_L^0}(z) e^{-\phi/2} S_{10,\alpha} e^{\frac{i}{2}X_L^0 + \frac{i}{2}X_R^0} \bar{V}_{24}(w) = \frac{1}{z-w} \gamma_{\alpha\beta}^0 e^{-\phi/2} C_{10,\beta} e^{-\frac{i}{2}X_L^0 + \frac{i}{2}X_R^0} \bar{V}_{24}(w) + \text{regular}, \quad (2.18)$$

where the r.h.s. involves a pure winding mode of flipped spinorial representation. In fact, the additional marginal operators at the fermionic point trigger a pure winding to pure momentum phase transition, at the level of the conformal field theory on the worldsheet.

Before presenting the cosmological evolutions arising from the tachyon free models, we would like to show the discrete deformation of the canonical partition function introduced in Eq. (2.8) can be interpreted in terms of Wilson lines [5]. We saw in the second line of Eq. (2.1) and Eq. (2.2) that tachyon free models can be constructed by coupling the $\Gamma_{(1,1)}$ lattices of the factorized circles $S^1(R_0) \times S^1(R_9)$ with $(-1)^a$ and $(-1)^{\bar{a}}$, respectively. However, this lattice contribution to Z can be written as a non-factorized lattice sum, thermally coupled to $(-1)^{a+\bar{a}}$,

$$\frac{\sqrt{G}}{\tau_2} \sum_{n_0, \bar{m}_0} \sum_{n_9, \bar{m}_9} e^{-\frac{\pi}{2}(n\tau + \bar{m})_i (G+B)_{ij} (n\bar{\tau} + \bar{m})_j} (-1)^{(a+\bar{a})\bar{m}_0 + (b+\bar{b})n_0} (-1)^{\bar{a}\bar{m}_9 + \bar{b}n_9 + \bar{m}_9 n_9}. \quad (2.19)$$

In this expression, the 2×2 metric G_{ij} and antisymmetric tensor B_{ij} are

$$G_{ij} = \begin{pmatrix} R_0^2 + A_0^2 R_9^2 & A_0 R_9^2 \\ A_0 R_9^2 & R_9^2 \end{pmatrix}, \quad B_{ij} = \begin{pmatrix} 0 & A_0' R_9^2 \\ -A_0' R_9^2 & 0 \end{pmatrix}, \quad (2.20)$$

$G = \det G$ and $A_0 = 1$, $A_0' = 1/2$. In these notations, the model with Hagedorn transition at $R_0 = R_H$ in Eq. (2.1) is recovered by choosing $A_0 = A_0' = 0$ instead. Therefore, the tachyon free model amounts to switching on constant gauge potentials $A_\mu = (A_0, \vec{0})$ and $A'_\mu = (A_0', \vec{0})$ along the temporal cycle, for the $U(1)$'s obtained by dimensional reduction of G_{ij} and B_{ij} on the internal $S^1(R_9)$.

From this point of view, the conventional trace in Eq (2.7) found by computing a path integral in compact Euclidean time with periodic and antiperiodic boundary conditions for bosons and fermions can be deformed by arbitrary Wilson lines A_0 and A'_0 . The latter cannot be gauged away and are therefore true vacuum parameters. The result takes the form

$$\text{Tr} e^{-\beta H - 2i\pi(A_0 Q + A'_0 Q')}, \quad (2.21)$$

where Q and Q' are the total $U(1)$'s charges of the multiparticle states. In the models defined in Eqs (2.1) and (2.2), one has

$$Q = \tilde{m}_9 - \frac{\bar{a} + n_9}{2}, \quad Q' = n_9, \quad (2.22)$$

so that the deformation in Eq. (2.8) is recovered,

$$e^{-2i\pi(A_0 Q + A'_0 Q')} = (-1)^{\bar{a}} \quad \text{when} \quad A_0 = 2A'_0 = 1. \quad (2.23)$$

3. Induced bouncing cosmologies

Our aim in this section is to describe the cosmological evolution induced by the thermal/quantum corrections of the tachyon free models on the tree level static Minkowski space-time in d dimensions [4, 3, 10]. For this purpose, we first consider the low energy effective action of the massless modes in compact Euclidean time and integrate out all massive states. In either the winding or momentum phase, focussing for simplicity on homogeneous and isotropic evolutions of the metric and dilaton, the action at the one-loop level is

$$\int dx^0 d^{d-1} \mathbf{x} \beta a^{d-1} \left[e^{-2\phi} \left(\frac{R}{2} + 2(\partial\phi)^2 \right) + \frac{Z}{\beta V} \right], \quad (3.1)$$

where we have chosen natural definitions of Euclidean times corresponding to laps functions equal to the perimeters β of the temporal circle $S^1(R_0)$ in the momentum phase and T-dual circle in the winding phase.

At some instant x_c^0 such that $R_0 = R_c$, the set of massless states is enhanced and contains in particular complex scalar fields φ^I , with both winding and momentum quantum numbers $m_0 = n_0 = \pm 1$. These extra massless scalars parametrize a moduli space, which is a coset

$$\mathcal{M} = \frac{SO(2, 12-d)}{SO(2) \times SO(12-d)}. \quad (3.2)$$

Thus, their tree level action is a $(d-1)$ -dimensional non-linear σ -model,

$$\int d^{d-1} \mathbf{x} a(x_c^0)^{d-1} e^{-2\phi(x_c^0)} \left(-h_{I\bar{J}} \partial_{\hat{\mu}} \varphi^I \partial^{\hat{\mu}} \bar{\varphi}^{\bar{J}} \right), \quad (3.3)$$

where $\hat{\mu} = 1, \dots, d-1$ and $h_{I\bar{J}}$ is the φ -dependent metric of \mathcal{M} . The classical backgrounds allowed by the above action are embeddings of space into the moduli space, $\mathbb{R}^{d-1} \rightarrow \mathcal{M}$. Simple solutions exist for $d=2, 3$ and less trivial ones are found for $2 \leq d \leq 7$ at the boundary of \mathcal{M} . In any case, one obtains

$$h_{I\bar{J}} \partial_{\hat{\mu}} \varphi^I \partial^{\hat{\mu}} \bar{\varphi}^{\bar{J}} = \kappa^2, \quad (3.4)$$

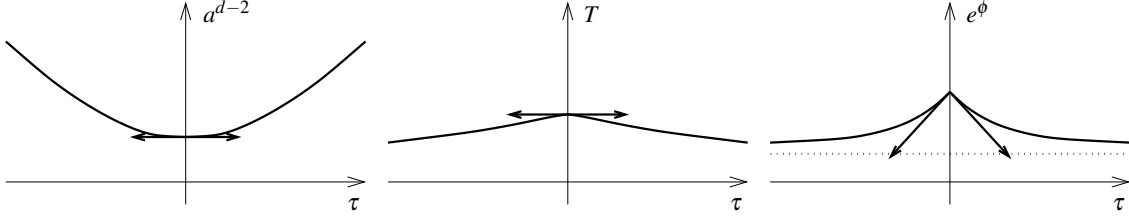


Figure 1: Bouncing cosmology arising from the tachyon free thermal models.

where κ^2 is an integration constant.

In total, the full effective action valid in the winding regime, momentum regime, as well as at the phase transition is the sum of the bulk part in Eq. (3.1) and brane-like contribution Eqs (3.3), (3.4). Switching to Lorentzian time by analytic continuation, $x^0 \rightarrow ix^0$, leads formally to an identical result,

$$\int dx^0 d^{d-1} \mathbf{x} \beta a^{d-1} \left[e^{-2\phi} \left(\frac{R}{2} + 2(\partial\phi)^2 \right) + \frac{Z}{\beta V} \right] - \kappa^2 \int d^{d-1} \mathbf{x} a^{d-1} e^{-2\phi} \delta(x^0 - x_c^0). \quad (3.5)$$

Thus, the brane tension κ^2 yields a negative contribution to the pressure localized in time, and vanishing energy. This corresponds to a very unusual state equation, which arises from the phase transition and not from any kind of exotic stringy matter.

Varying the action with respect to the laps function, scale factor and dilaton field, the resulting equations of motion can be solved explicitly using the approximation of Eq. (2.16). For $d > 2$, the cosmological evolution in string frame and conformal time τ takes the form [4],

$$\begin{aligned} \ln \frac{a}{a_c} = \ln \frac{T_c}{T} &= \frac{1}{d-2} \left[\eta_+ \ln \left(1 + \frac{\omega|\tau|}{\eta_+} \right) - \eta_- \ln \left(1 + \frac{\omega|\tau|}{\eta_-} \right) \right], \\ \phi &= \phi_c + \frac{\sqrt{d-1}}{2} \left[\ln \left(1 + \frac{\omega|\tau|}{\eta_+} \right) - \ln \left(1 + \frac{\omega|\tau|}{\eta_-} \right) \right], \end{aligned} \quad (3.6)$$

where a_c and ϕ_c are arbitrary integration constants in terms of which we have defined

$$\omega = \kappa^2 a_c \frac{d-2}{4\sqrt{d-1}}, \quad \kappa^2 = 2\sqrt{2(d-1)} \sqrt{n\Sigma_d} T_c^{d/2} e^{\phi_c}, \quad \eta_{\pm} = \sqrt{d-1} \pm 1. \quad (3.7)$$

A solution also exists for $d = 2$ and is consistently recovered by taking the limit $d \rightarrow 0$ in the above expressions [3, 4]. As shown in Figure 1, the evolution describes a universe, which bounces at the maximal temperature T_c . For $d > 2$, it is radiation dominated at late and early times, when the dilaton approaches its asymptote. The weak coupling approximation is valid throughout the evolution, provided the maximal value e^{ϕ_c} reached by the string coupling is chosen low enough. Moreover, since the time-dependencies of the fields arise from loop corrections, all time-derivatives are at most of order $\mathcal{O}(e^{\phi_c})$. Thus, the Ricci scalar is small and higher order derivative terms are consistently neglected.

In the string frame, the time derivatives of a and T vanish at the transition, while the dilaton field is non-derivable. However, the scale factor and temperature are conical in the Einstein frame (defined for $d > 2$), due to the dilaton dressing they acquire. When measured in this frame, the critical temperature is lower than the string scale, $T_c^{(E)} = e^{\frac{2\phi_c}{d-2}} T_c$.

As a conclusion, the tachyon free models obtained by switching on suitable Wilson lines along the Euclidean $S^1(R_0)$ lead to bouncing cosmologies, which are free of Hagedorn instabilities and initial singularities. In Euclidean time, the pre- and post-Big Bang eras differ in their realizations, in terms of pure winding or pure momentum states, respectively. In Lorentzian time, they are characterized by fermions in different spinorial representations and correspond to distinct phases connected by a transition at the maximal temperature $T_c = 1/(\sqrt{2}\pi)$. The cosmological evolutions are induced by the back-reaction of the one-loop quantum/thermal corrections, while higher genus and higher derivative terms can be neglected consistently. This is in contrast with previous attempts to construct bouncing cosmologies [11]. Finally, we mention that other cosmological solutions for the pre-Big Bang era ($\tau < 0$) have been found recently within the framework reviewed in this note, with identical post-Big Bang evolutions ($\tau > 0$). In the string frame, they are characterized by a static scale factor and a constant temperature equal to its critical value T_c , for all $\tau < 0$. In the Einstein frame, they describe a universe, whose time-evolution for $\tau < 0$ is identical to that of a contracting radiation era [12].

Acknowledgments

The works reviewed in this note have been done in collaboration with C. Angelantonj, I. Florakis, C. Kounnas and N. Toumbas I would like to thank warmly. I am also grateful to the organizers of the Corfu Summer Institute for the invitation to present these results. This work is partially supported by the EU contracts PITN GA-2009-237920, ERC-AG-226371 and IRSES-UNIFY, a French ANR 05-BLAN-NT09-573739 contract, the CEFIPRA/IFCPAR 4104-2 project, and PICS contracts France/Cyprus, France/Greece and France/USA.

References

- [1] D. Kutasov, N. Seiberg, “Number of degrees of freedom, density of states and tachyons in string theory and CFT,” Nucl. Phys. **B358** (1991) 600-618.
- [2] J. J. Atick and E. Witten, “The Hagedorn transition and the number of degrees of freedom of string theory,” Nucl. Phys. B **310** (1988) 291.
I. Antoniadis and C. Kounnas, “Superstring phase transition at high temperature,” Phys. Lett. B **261** (1991) 369.
I. Antoniadis, J. P. Derendinger and C. Kounnas, “Nonperturbative temperature instabilities in $\mathcal{N} = 4$ strings,” Nucl. Phys. B **551** (1999) 41 [arXiv:hep-th/9902032].
- [3] I. Florakis, C. Kounnas, H. Partouche, N. Toumbas, “Non-singular string cosmology in a 2d hybrid model,” Nucl. Phys. **B844** (2011) 89-114. [arXiv:1008.5129 [hep-th]].
- [4] C. Kounnas, H. Partouche, N. Toumbas, “Thermal duality and non-singular cosmology in d -dimensional superstrings,” Nucl. Phys. **B855** (2012) 280-307. [arXiv:1106.0946 [hep-th]].
- [5] C. Angelantonj, C. Kounnas, H. Partouche, N. Toumbas, “Resolution of Hagedorn singularity in superstrings with gravito-magnetic fluxes,” Nucl. Phys. **B809** (2009) 291-307. [arXiv:0808.1357 [hep-th]].
- [6] I. Florakis, C. Kounnas, N. Toumbas, “Marginal deformations of vacua with massive boson-fermion degeneracy symmetry,” Nucl. Phys. **B834** (2010) 273-315. [arXiv:1002.2427 [hep-th]].

- [7] P. H. Ginsparg and C. Vafa, “Toroidal compactification of nonsupersymmetric heterotic strings,” Nucl. Phys. B **289** (1987) 414.
V. P. Nair, A. D. Shapere, A. Strominger and F. Wilczek, “Compactification of the twisted heterotic string,” Nucl. Phys. B **287** (1987) 402.
S. P. Patil and R. Brandenberger, “Radion stabilization by stringy effects in general relativity,” Phys. Rev. D **71** (2005) 103522. [arXiv:hep-th/0401037].
- [8] F. Bourliot, J. Estes, C. Kounnas, H. Partouche, “Cosmological phases of the string thermal effective potential,” Nucl. Phys. **B830** (2010) 330-373. [arXiv:0908.1881 [hep-th]].
J. Estes, C. Kounnas and H. Partouche, “Superstring cosmology for $\mathcal{N}_4 = 1 \rightarrow 0$ superstring vacua,” Fortsch. Phys. **59** (2011) 861-895. [arXiv:1003.0471 [hep-th]].
J. Estes, L. Liu, H. Partouche, “Massless D-strings and moduli stabilization in type I cosmology,” JHEP **1106** (2011) 060. [arXiv:1102.5001 [hep-th]].
L. Liu and H. Partouche, “Moduli stabilization in type II Calabi-Yau compactifications at finite temperature,” arXiv:1111.7307 [hep-th].
- [9] C. Kounnas, “Massive boson-fermion degeneracy and the early structure of the universe,” Fortsch. Phys. **56** (2008) 1143-1156. [arXiv:0808.1340 [hep-th]].
- [10] H. Partouche, “Non-singular superstring cosmology in two dimensions,” [arXiv:1106.1309 [hep-th]].
- [11] M. Gasperini, G. Veneziano, “The pre-big bang scenario in string cosmology,” Phys. Rept. **373** (2003) 1-212. [hep-th/0207130].
- [12] C. Kounnas, H. Partouche and N. Toumbas, “S-brane to thermal non-singular string cosmology,” arXiv:1111.5816 [hep-th].