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Extraction of the Sivers functions with TMD evolution

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The QCD evolution of the unpolarized Transverse Momentum Dependent (TMD) distribution and fragmentation functions and of the Sivers function have been discussed in recent papers [1, 2, 3, 4]. Following these results we reconsider previous extractions of the Sivers function from SIDIS data and propose a simple strategy which allows to take into account the Q^2 dependence of the TMDs. Clear evidence of the TMD evolution can be seen in the available data, mostly in the newest COMPASS results for Semi Inclusive Deep Inelastic Scattering (SIDIS) off a transversely polarized proton target

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The exploration of the 3-dimensional structure of the nucleon, both in momentum and in configuration space, is one of the major issues in high energy hadron physics. Information on the partonic 3-dimensional momentum structure is embedded in the Transverse Momentum Dependent distributions (TMDs). Among them, the Sivers function, which describes the number density of unpolarized quarks inside a transversely polarized proton, is particularly interesting, as it is expected to provide information on the partonic orbital angular momentum [5].

The Sivers functions for u and d quarks have been extracted from SIDIS data by several groups, with consistent results [6, 7, 8, 9, 10, 5]. However, all these phenomenological fits have been performed so far either neglecting the QCD scale dependence of the TMDs – which was unknown – or limiting it to the DGLAP evolution of the collinear unpolarized parton distributions (which appear as factors in the parameterization of the Sivers functions).

We present here, following Ref. [11], a simple strategy which allows, in the extraction of the Sivers function from SIDIS data, to take into account the TMD evolution scheme proposed by Aybat, Collins, Qiu and Rogers [1, 2, 3]; we then show how this analysis compares with the previous extractions, with no TMD evolution.

In Ref. [11] we recasted the QCD evolution equation of the TMDs in the coordinate space proposed in Refs. [2] and [3] in a simplified way, taking the renormalization scale μ^2 and the regulating parameters ζ_F and ζ_D all equal to Q^2, as

$$\widetilde{F}(x,b_T;Q) = \widetilde{F}(x,b_T;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-g_K(b_T)\ln\frac{Q}{Q_0}\right\}\,, \tag{1}$$

where $\widetilde{F}$ can be either the unpolarized parton distribution, $\widetilde{F}(x,b_T;Q) = \widetilde{f}_{q/p}(x,b_T;Q)$, the unpolarized fragmentation function, $\widetilde{F}(x,b_T;Q) = \widetilde{D}_{h/q}(z,b_T;Q)$, or the first derivative, with respect to the parton impact parameter b_T, of the Sivers function, $\widetilde{F}(x,b_T;Q) = \widetilde{f}_{1T}^{\prime\perp f}(x,b_T;Q)$; $g_K(b_T)$ is an unknown, but universal and scale independent, function, while $\widetilde{R}(Q,Q_0,b_T)$ is the evolution kernel defined as

$$\widetilde{R}(Q,Q_0,b_T) \equiv \exp\left\{\ln\frac{Q}{Q_0}\int_{Q_0}^{\mu_b}\frac{d\mu'}{\mu'}\gamma_K(\mu') + \int_{Q_0}^{Q}\frac{d\mu}{\mu}\gamma_F\left(\mu,\frac{Q^2}{\mu^2}\right)\right\}\,. \tag{2}$$

The anomalous dimensions γ_F and γ_K appearing in Eq. (2), are given, at order α_s, by [2]

$$\gamma_F(\mu;\frac{Q^2}{\mu^2}) = \alpha_s(\mu)\,\frac{C_F}{\pi}\left(\frac{3}{2} - \ln\frac{Q^2}{\mu^2}\right) \qquad\qquad \gamma_K(\mu) = \alpha_s(\mu)\,\frac{2C_F}{\pi}\,. \tag{3}$$

The Q^2 evolution is therefore driven by the functions $g_K(b_T)$ and $\widetilde{R}(Q,Q_0,b_T)$. While the latter, Eq. (2), can be easily evaluated, numerically or even analytically, the former, is essentially unknown and will need to be taken from independent experimental inputs.

The appropriate Fourier transform allows us to obtain the distribution and fragmentation functions in the momentum space:

$$\widehat{f}_{q/p}(x,k_\perp;Q) = \frac{1}{2\pi}\int_0^\infty db_T\,b_T\,J_0(k_\perp b_T)\,\widetilde{f}_{q/p}(x,b_T;Q) \tag{4}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q) = \frac{1}{2\pi}\int_0^\infty db_T\,b_T\,J_0(\mathrm{k}_T b_T)\,\widetilde{D}_{h/q}(z,b_T;Q) \tag{5}$$

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$$\widehat{f}_{1T}^{\perp f}(x,k_\perp;Q) = \frac{-1}{2\pi k_\perp}\int_0^\infty \mathrm{d}b_T\, b_T\, J_1(k_\perp b_T)\, \widetilde{f}_{1T}^{\prime\perp q}(x,b_T;Q)\,, \tag{6}$$

where J_0 and J_1 are Bessel functions, while $\widehat{f}_{q/p}$ is the unpolarized TMD distribution function for a parton of flavor q inside a proton, $\widehat{D}_{h/q}$ is the unpolarized TMD fragmentation function for hadron h inside a parton q and $\widehat{f}_{1T}^{\perp q}$ is the Sivers distribution defined, for unpolarized partons inside a transversely polarized proton, as:

$$\widehat{f}_{q/p^\uparrow}(x,k_\perp,S;Q) \;=\; \widehat{f}_{q/p}(x,k_\perp;Q) - \widehat{f}_{1T}^{\perp q}(x,k_\perp;Q)\frac{\varepsilon_{ij}\,k_\perp^i\,S^j}{M_p} \tag{7}$$

$$= \widehat{f}_{q/p}(x,k_\perp;Q) + \frac{1}{2}\Delta^N\widehat{f}_{q/p^\uparrow}(x,k_\perp;Q)\frac{\varepsilon_{ij}\,k_\perp^i\,S^j}{k_\perp}\,. \tag{8}$$

The unknown input functions $g_K(b_T)$ and $\widetilde{F}(x,b_T;Q_0)$ inside Eq. (1) have to be appropriately parameterized. As already anticipated, $g_K(b_T)$ is a non-perturbative, but universal function, which in the literature is usually parameterized in a quadratic form: $g_K(b_T) = \frac{1}{2}g_2\,b_T^2$. As in Ref. [3], we will adopt the results provided by a recent fit of Drell-Yan data [12], and assume $g_2 = 0.68$ GeV^2.

The input functions $\widetilde{F}(x,b_T;Q_0)$ are parameterized by requiring that their Fourier-transforms, which give the corresponding TMD functions in the transverse momentum space, coincide with the previously adopted $k_\perp$-Gaussian forms, with the x dependence factorized out. As shown in Ref. [11], one finds

$$\widetilde{f}_{q/p}(x,b_T;Q) = f_{q/p}(x,Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-b_T^2\left(\alpha^2+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\} \tag{9}$$

$$\widetilde{D}_{h/q}(z,b_T;Q) = \frac{1}{z^2}D_{h/q}(z,Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,\exp\left\{-b_T^2\left(\beta^2+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\}\,, \tag{10}$$

$$\widetilde{f}_{1T}^{\prime\perp}(x,b_T;Q) = -2\,\gamma^2\,f_{1T}^\perp(x;Q_0)\,\widetilde{R}(Q,Q_0,b_T)\,b_T\,\exp\left\{-b_T^2\left(\gamma^2+\frac{g_2}{2}\ln\frac{Q}{Q_0}\right)\right\} \tag{11}$$

with

$$\alpha^2 = \langle k_\perp^2\rangle/4 \qquad \beta^2 = \langle p_\perp^2\rangle/(4z^2) \qquad 4\,\gamma^2 \equiv \langle k_\perp^2\rangle_S = \frac{M_1^2\,\langle k_\perp^2\rangle}{M_1^2+\langle k_\perp^2\rangle} \tag{12}$$

and $\widetilde{R}(Q,Q_0,b_T)$ as in Eq. (2). Q_0 is taken to be 1 GeV.

Eqs. (9)-(11) show that the Q^2 evolution is controlled by the logarithmic Q dependence of the b_T Gaussian width, together with the factor $\widetilde{R}(Q,Q_0,b_T)$: for increasing values of Q^2, they are responsible for the typical broadening effect already observed in Refs. [2] and [3].

As $R(Q,Q_0,b_T)$ shows only a weak dependence on (small) b_T (i.e. large $k_\perp$) through the upper integration limit μ_b [11], we can assume $R(Q,Q_0,b_T)$ to be constant in b_T and Fourier-transform the evolution equations (9), (10) and (11) analytically within this approximation, to find

$$\widehat{f}_{q/p}(x,k_\perp;Q) \;=\; f_{q/p}(x,Q_0)\,R(Q,Q_0)\,\frac{e^{-k_\perp^2/w^2}}{\pi\,w^2} \tag{13}$$

$$\widehat{D}_{h/q}(z,p_\perp;Q) \;=\; D_{h/q}(z,Q_0)\,R(Q,Q_0)\,\frac{e^{-p_\perp^2/w_F^2}}{\pi w_F^2} \tag{14}$$

$$\Delta^N\widehat{f}_{q/p^\uparrow}(x,k_\perp;Q) \;=\; \frac{k_\perp}{M_1}\sqrt{2e}\,\frac{\langle k_\perp^2\rangle_S^2}{\langle k_\perp^2\rangle}\,\Delta^N f_{q/p^\uparrow}(x,Q_0)\,R(Q,Q_0)\,\frac{e^{-k_\perp^2/w_S^2}}{\pi w_S^4} \tag{15}$$

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Table 1: The total χ^2 corresponding to the TMD-fit, the TMD-analytical-fit and the DGLAP-fit. The most significant partial contributions, enhancing the difference between TMD and non-TMD evolutions are shown.

	TMD Evolution (exact)	TMD Evolution (analytical)	DGLAP Evolution
	$\chi^2_{tot} = 255.8$ $\chi^2_{d.o.f} = 1.02$	$\chi^2_{tot} = 275.7$ $\chi^2_{d.o.f} = 1.10$	$\chi^2_{tot} = 315.6$ $\chi^2_{d.o.f} = 1.26$
HERMES π^+	$\chi^2_x = 10.7$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 9.1$	$\chi^2_x = 12.9$ $\chi^2_z = 4.3$ $\chi^2_{P_T} = 10.5$	$\chi^2_x = 27.5$ $\chi^2_z = 8.6$ $\chi^2_{P_T} = 22.5$
COMPASS h^+	$\chi^2_x = 6.7$ $\chi^2_z = 17.8$ $\chi^2_{P_T} = 12.4$	$\chi^2_x = 11.2$ $\chi^2_z = 18.5$ $\chi^2_{P_T} = 24.2$	$\chi^2_x = 29.2$ $\chi^2_z = 16.6$ $\chi^2_{P_T} = 11.8$

where $f_{q/p}(x,Q_0)$ and $D_{h/q}(z,Q_0)$ are the usual integrated PDF evaluated at the initial scale Q_0, and $\Delta^N f_{q/p^\uparrow}(x,Q_0)$ gives the x dependence of the Sivers function [11]. Most importantly, w^2, w_F^2 and w_S^2 are the "evolving" Gaussian widths, defined as:

$$w^2 = \langle k_\perp^2 \rangle + 2\, g_2 \ln \frac{Q}{Q_0} \,, \qquad w_F^2 = \langle p_\perp^2 \rangle + 2 z^2 g_2 \ln \frac{Q}{Q_0} \,, \qquad w_S^2 = \langle k_\perp^2 \rangle_S + 2 g_2 \ln \frac{Q}{Q_0} \,\cdot \tag{16}$$

Notice that the Q^2 evolution of the TMD PDFs is now determined by the overall factor $R(Q,Q_0)$ and, most crucially, by the Q^2 dependent Gaussian width $w(Q,Q_0)$.

It is interesting to point that the evolution factor $R(Q,Q_0)$, controlling the TMD evolution, is the same for all functions (TMD PDFs, TMD FFs and Sivers) and is flavor independent: consequently it will appear, squared, in both numerator and denominator of the Sivers azimuthal asymmetry and, approximately, cancel out. Therefore, we can safely conclude that most of the TMD evolution of the azimuthal asymmetries is controlled by the logarithmic Q dependence of the $k_\perp$ Gaussian widths $w^2(Q,Q_0)$, Eq. (16).

The aim of our paper is to analyze the available polarized SIDIS data from the HERMES and COMPASS collaborations in order to understand whether or not they show signs of the TMD evolution proposed in Ref. [3].

In particular we perform three different data fits of the SIDIS Sivers single spin asymmetry $A_{UT}^{\sin(\phi_h-\phi_S)}$ measured by HERMES and COMPASS: a fit (TMD-fit) in which we adopt the TMD evolution equations shown in Eqs. (9)-(11); a second fit (TMD-analytical-fit) in which we apply the same TMD evolution, but using the analytical approximation of Eqs. (13)–(15); a fit (DGLAP-fit) in which we follow our previous work, as done so far in Ref. [10, 13], using the DGLAP evolution equation only in the collinear part of the TMDs.

Table I shows the main results of our fitting procedure. The best total χ^2_{tot}, which amounts to 256, is obtained by using the TMD evolution, followed by a slightly higher χ^2_{tot} of the analytical approximation, and a definitely larger $\chi^2_{tot} \simeq 316$ corresponding to the DGLAP fit. To examine the origin of this difference between TMD and DGLAP evolution, in Ref. [11] we have shown

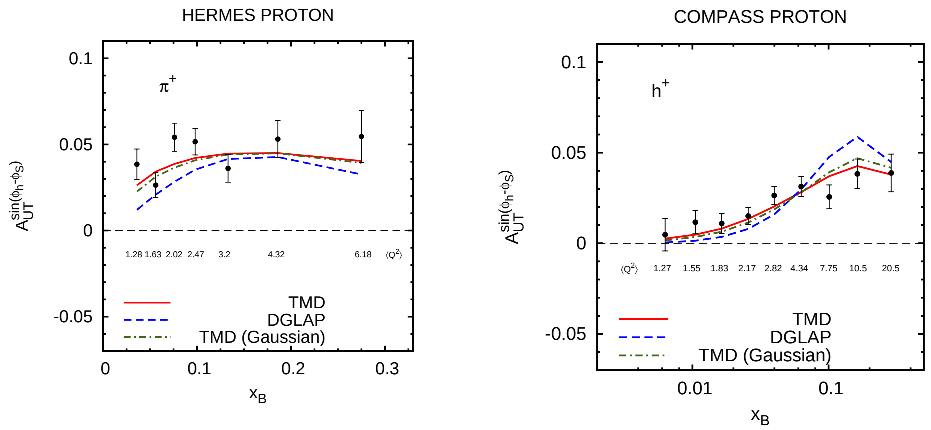

Figure 1: The results obtained from our fit of the SIDIS $A_{UT}^{\sin(\phi_h-\phi_S)}$ Sivers asymmetries applying TMD evolution (red, solid lines) are compared with the analogous results found by using DGLAP evolution equations (blue, dashed lines). The green, dash-dotted lines correspond to the results obtained by using the approximated analytical TMD evolution (see text for further details). The experimental data are from HERMES [14] (left panel) and COMPASS [15] (right panel) Collaborations.

the individual contributions to χ^2_{tot} of each experiment (HERMES, COMPASS on NH_3 and on LiD targets), for all types of detected hadrons and for all variables observed (x, z and P_T). It turns out that the difference of about 60 χ^2-points between the TMD and the DGLAP fits is not equally distributed among all χ^2s per data point; rather, it is mainly concentrated in the asymmetry for π^+ production at HERMES and for h^+ production at COMPASS, especially when this asymmetry is observed as a function of the x-variable. These differences are explicitly shown in Table I.

It is important to stress that, as x is directly proportional to Q^2 through the kinematical relation $Q^2 = xys$, the x behavior of the asymmetries is intimately connected to their Q^2 evolution. While the HERMES experimental bins cover a very modest range of Q^2 values, from 1.3 GeV^2 to 6.2 GeV^2, COMPASS data raise to a maximum Q^2 of 20.5 GeV^2, enabling to test more severely the TMD Q^2 evolution in SIDIS.

These aspects are illustrated in Fig. 1, where the SIDIS Sivers asymmetries $A_{UT}^{\sin(\phi_h-\phi_S)}$ obtained in the three fits are shown in the same plot. It is evident that the DGLAP evolution seems to be unable to describe the correct x trend, *i.e.* the right Q^2 behavior, while the TMD evolution (red solid line) follows much better the large Q^2 data points, corresponding to the last x-bins measured by COMPASS.

In conclusions, we have reconsidered the Sivers effect in SIDIS experiments, by upgrading old fits with the addition of the most recent HERMES and COMPASS data, and by implementing, for the first time, the newly introduced TMD evolution equations in our analysis. We have compared the results obtained using the TMD evolution equations with the results found by considering only the DGLAP evolution of the collinear part of the TMDs. We find some clear evidence that the available SIDIS data, in particular those at the largest Q^2 values, support the TMD evolution scheme. Further experimental data, covering a yet wider range of Q^2, are necessary to confirm this.

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