

## Bessel-weighted asymmetries and the Sivers effect

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We consider the cross section in Fourier space, conjugate to the outgoing hadron's transverse momentum, where convolutions of transverse momentum dependent parton distribution functions and fragmentation functions become simple products. Individual asymmetric terms in the cross section can be projected out by means of a generalized set of weights involving Bessel functions. Advantages of employing these Bessel weights are that they suppress (divergent) contributions from high transverse momentum and that soft factors cancel in (Bessel-) weighted asymmetries. Also, the resulting compact expressions immediately connect to previous work on evolution equations for transverse momentum dependent parton distribution and fragmentation functions and to quantities accessible in lattice QCD. Bessel-weighted asymmetries are thus model independent observables that augment the description and our understanding of correlations of spin and momentum in nucleon structure.

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## 1. Weighted asymmetries

In the factorized picture of semi-inclusive processes, where the transverse momentum of the detected hadron  $\mathbf{P}_{h\perp}$  is small compared to the photon virtuality  $Q^2$ , transverse momentum dependent (TMD) parton distribution functions (PDFs) characterize the spin and momentum structure of the proton [1, 2, 3, 4, 5, 6, 7]. At leading twist there are 8 TMD PDFs. They can be studied experimentally by analyzing angular modulations in the differential cross section, so called spin and azimuthal asymmetries. These modulations are a function of the azimuthal angles of the final state hadron momentum about the virtual photon direction, as well as that of the target polarization (see Fig. 1 and e.g., Ref. [8] for a review). TMD PDFs enter the SIDIS cross section in momentum space convoluted with transverse momentum dependent fragmentation functions (TMD FFs). However, after a two-dimensional Fourier transform of the cross section with respect to the transverse hadron momentum  $\mathbf{P}_{h\perp}$ , these convolutions become simple products of functions in Fourier  $\mathbf{b}_T$ -space. The usefulness of such Fourier-Bessel transforms in studying the factorization as well as the scale dependence of transverse momentum dependent cross section has been known for some time [9, 10, 11, 12, 13]. Here we exhibit the structure of the cross section in  $\mathbf{b}_T$ -space and demonstrate how this representation results in model independent observables [14] which are generalizations of the conventional weighted asymmetries [15, 6, 7]. Additionally, we explore the impact that these observables have in studying the scale dependence of the SIDIS cross section at small to moderate transverse momentum where the TMD framework is designed to give a good description of the cross section. In particular we study how the so called soft factor cancels from these observables. The soft factor [16, 13, 17, 11, 12, 18] is an essential element of the cross section that arises in TMD factorization expressions [9, 10, 11, 13]. It accounts for the collective effect of soft momentum gluons not associated with either the distribution or fragmentation part of the process and it is shown to be universal in hard processes [17].

The concept of transverse momentum weighted Single Spin Asymmetries (SSA) was proposed some time ago in Ref. [6, 7]. Using the technique of weighting enables one to disentangle in a model independent way the cross sections and asymmetries in terms of the transverse momentum moments of TMDs. A comprehensive list of such weights was derived in Ref. [7] for semi-inclusive deep inelastic scattering (SIDIS). In SIDIS and Drell-Yan scattering, proofs of TMD factorization contain an additional factor, the soft factor [16, 13, 17, 11, 12, 18]. At tree level (zeroth order in  $\alpha_S$ ) the soft factor is unity, which explains its absence in the factorization formalism considered for example in Ref. [8]. Consequently it is also absent in tree level phenomenological analyses of the experimental data (see for example Refs. [19, 20, 21, 22]). In principle, the results of tree level analyses at different energies cannot be compared. For a correct description of the energy scale dependence of the cross sections and asymmetries involving TMDs, the soft factor is essential to include [11, 12]. However, it is possible to consider observables where the soft factor is indeed absent or cancels out. These are precisely the weighted asymmetries [14].

We focus on the Siverts asymmetry [2]. With a general  $|\mathbf{P}_{h\perp}|$ -weight, this asymmetry can be written as [7]

$$A_{UT}^{w_1 \sin(\phi_h - \phi_S)}(x, z, y) \equiv \frac{2 \int d|\mathbf{P}_{h\perp}| d\phi_h d\phi_S w_1(|\mathbf{P}_{h\perp}|) \sin(\phi_h - \phi_S) (d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi))}{\int d|\mathbf{P}_{h\perp}| d\phi_h d\phi_S w_0(|\mathbf{P}_{h\perp}|) (d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi))}. \quad (1.1)$$

In the numerator, the angular weight  $\sin(\phi_h - \phi_S)$  projects out the structure function  $F_{UT,T}^{\sin(\phi_h - \phi_S)}$  [8] from the cross section, while the weight- $w_1(|\mathbf{P}_{h\perp}|)$  leads to a “deconvolution” of the structure function into a product of the first  $\mathbf{p}_T$ -moment of the Siverts function  $f_{1T}^{\perp(1)}$  and the lowest moment of the unpolarized fragmentation function  $D_1^{(0)}$  [7]. In a similar manner using the weight  $w_0(|\mathbf{P}_{h\perp}|)$  the unpolarized structure function  $F_{UU,T}$  is written zeroth  $\mathbf{p}_T$ -moments,  $f_1^{(0)}(x)$  and  $D_1^{(0)}$ .

In [14], we show that in TMD factorization the soft factor cancels in the asymmetry due to the “deconvolution” achieved by appropriate  $|\mathbf{P}_{h\perp}|$ -weighting. We demonstrate that it is natural to employ Bessel functions as weights,  $w_n \propto J_n(|\mathbf{P}_{h\perp}| \mathcal{B}_T)$ , where  $\mathcal{B}_T$  is the Fourier conjugate variable of  $|\mathbf{P}_{h\perp}|$ . This generalized weighting procedure addresses a problem related to the perturbative tail of TMDs when we weight with conventional weights [6, 7],  $w_n \propto |\mathbf{P}_{h\perp}|^n$ . Using Bessel functions as weights, the respective integrals become convergent while the “deconvolution” property and soft factor cancellation are preserved. However, it is important to stress that while the integrals are convergent, the scale dependence of the resulting functions, and consequently also the  $Q^2$  dependence of the Bessel-weighted asymmetries remains to be studied.

### 1.1 Bessel-weighted asymmetries

To deconvolute or convert the convolutions of TMD PDFs and TMD FFs in the SIDIS cross section into products, one can perform a multipole expansion and a subsequent Fourier transform of the cross section with respect to the transverse components  $\mathbf{P}_{h\perp}$  of the hadron momentum. In general, one can write a transverse momentum dependent cross section  $\sigma(|\mathbf{P}_{h\perp}|, \phi_h)$  as a two-dimensional multipole expansion of the cross section in Fourier space [14]. The  $n^{\text{th}}$  harmonic in  $\phi_h$  is given by the  $n^{\text{th}}$  Bessel function of the first kind  $J_n$ . With such definitions, the relevant terms of the SIDIS cross section can be written as

$$\frac{d\sigma}{dx dy d\phi_S dz d\phi_h |\mathbf{P}_{h\perp}| d|\mathbf{P}_{h\perp}|} = \frac{\alpha^2}{xyQ^2} \frac{y^2}{(1-\varepsilon)} \left(1 + \frac{\gamma^2}{2x}\right) \int_0^\infty \frac{d|\mathbf{b}_T|}{(2\pi)} |\mathbf{b}_T| \times \left\{ J_0(|\mathbf{b}_T| |\mathbf{P}_{h\perp}|) \tilde{F}_{UU,T} + |\mathbf{S}_\perp| \sin(\phi_h - \phi_S) J_1(|\mathbf{b}_T| |\mathbf{P}_{h\perp}|) \tilde{F}_{UT,T}^{\sin(\phi_h - \phi_S)} + \dots \right\}, \quad (1.2)$$

where the ellipsis represent 16 more terms. Note that there is only a finite number of multipoles in the SIDIS cross section; the Bessel function of highest order is  $J_3$ . Here we show only two terms in the cross section and omit for now regularization parameters needed beyond tree level, see Ref. [14] and references therein for more details. Introducing the Fourier-transformed TMDs and fragmentation functions,

$$\tilde{f}(x, \mathbf{b}_T^2) \equiv \int d^2 \mathbf{p}_T e^{i\mathbf{p}_T \cdot \mathbf{b}_T} f(x, \mathbf{p}_T^2) = 2\pi \int_0^\infty d|\mathbf{p}_T| |\mathbf{p}_T| J_0(|\mathbf{b}_T| |\mathbf{p}_T|) f(x, \mathbf{p}_T^2),$$

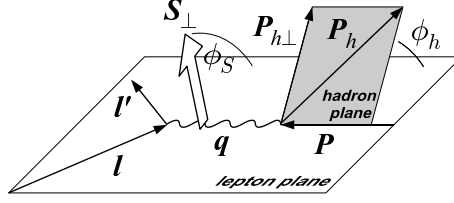
and the derivative (or  $\mathbf{b}_T$  moment)

$$\tilde{f}^{(1)}(x, \mathbf{b}_T^2) \equiv -\frac{2}{M^2} \partial_{\mathbf{b}_T^2} \tilde{f}(x, \mathbf{b}_T^2) = \frac{2\pi}{M^2} \int_0^\infty d|\mathbf{p}_T| \frac{|\mathbf{p}_T^2|}{|\mathbf{b}_T|} J_1(|\mathbf{b}_T| |\mathbf{p}_T|) f(x, \mathbf{p}_T^2), \quad (1.3)$$

the structure functions in Fourier space in Eq. (1.2) are given by

$$\tilde{F}_{UU,T} = x \sum_a e_a^2 \tilde{f}_1^{(0)a}(x, z^2 \mathbf{b}_T^2) \tilde{D}_1^{(0)a}(z, \mathbf{b}_T^2) \tilde{S}(\mathbf{b}_T^2) H_{UU,T}(Q^2), \quad (1.4)$$

$$\tilde{F}_{UT,T}^{\sin(\phi_h - \phi_S)} = -x \sum_a e_a^2 |\mathbf{b}_T| z M \tilde{f}_{1T}^{\perp(1)a}(x, z^2 \mathbf{b}_T^2) \tilde{D}_1^{(0)a}(z, \mathbf{b}_T^2) \tilde{S}(\mathbf{b}_T^2) H_{UT,T}(Q^2), \quad (1.5)$$



**Figure 1:** Kinematics of the SIDIS process. The in- and out-going lepton momenta are  $l$  and  $l'$ , respectively. The momentum transfer is  $q$ . The target nucleon carries momentum  $P$  and its transverse spin components are labelled  $\mathbf{S}_\perp$ . The momentum of the measured hadron  $P_h$  has transverse components  $\mathbf{P}_{h\perp}$ , which define an angle  $\phi_h$  with the lepton plane.

where  $\tilde{f}_1^{(0)a}$  and  $\tilde{f}_{1T}^{\perp(1)a}$  are the Fourier transformed unpolarized and Siverts TMD PDFs respectively, and  $\tilde{D}_1^{(0)a}$  is a Fourier transformed fragmentation function. We have used the kinematic variables  $Q^2 \equiv -q^2$ ,  $M^2 = P^2$ ,  $x \approx x_B \equiv Q^2/P \cdot q$ ,  $y = P \cdot q/P \cdot l$ , and  $z \approx z_h \equiv P \cdot P_h/P \cdot q$  and assume  $M \ll Q$ ,  $|\mathbf{P}_{h\perp}| \ll zQ$ . The sum  $\sum_a$  runs over quark flavors  $a$  and  $e_a$  is the corresponding electric charge of the quark. In contrast to the tree-level equation [8, 14], we include here an explicit soft factor  $\tilde{S}(\mathbf{b}_T^2)$  (in Fourier space) and a scale dependent hard part  $H(Q^2)$ , as in reference [11, 12]. For brevity we suppress the dependencies on a renormalization scale  $\mu$  and on rapidity cutoff parameters (e.g.,  $\zeta$ ,  $\hat{\zeta}$ ,  $\rho$  in [11, 12, 14]). It is now clear that the cross section Eq. (1.2) is a multipole series, and that projection on Fourier modes in polar coordinates  $\phi_h$ ,  $|\mathbf{P}_{h\perp}|$  will give access to the right hand sides of Eqs. (1.4) and (1.5). Calculating the weighted asymmetry Eq. (1.1) with weights  $w_1(|\mathbf{P}_{h\perp}|) = 2J_1(|\mathbf{P}_{h\perp}|/\mathcal{B}_T)/zM\mathcal{B}_T$  and  $w_0(|\mathbf{P}_{h\perp}|) = J_0(|\mathbf{P}_{h\perp}|/\mathcal{B}_T)$  thus yields

$$A_{UT}^{\frac{2J_1(|\mathbf{P}_{h\perp}|/\mathcal{B}_T)}{zM\mathcal{B}_T} \sin(\phi_h - \phi_s)}(x, y, z; \mathcal{B}_T) = \frac{-2 \sum_a e_a^2 H_{UT,T}(Q^2) \tilde{f}_{1T}^{\perp(1)a}(x, z^2 \mathcal{B}_T^2) \tilde{S}(\mathcal{B}_T^2) \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}{\sum_a e_a^2 H_{UU,T}(Q^2) \tilde{f}_1^{(0)a}(x, z^2 \mathcal{B}_T^2) \tilde{S}(\mathcal{B}_T^2) \tilde{D}_1^{(0)a}(z, \mathcal{B}_T^2)}. \quad (1.6)$$

Due to the ‘‘deconvolution’’ of the structure functions in the weighted asymmetries, and universality of the soft factor,  $\tilde{S}$  cancels in the numerator and the denominator.

Further, note that  $\mathcal{B}_T$  enters the weights  $w_0$  and  $w_1$  as a free parameter that we can scan over a whole range in order to compare the transverse momentum dependence of the distributions in the numerator and denominator relative to each other (in Fourier space). At the operator level,  $\mathcal{B}_T (= |\mathbf{b}_T|)$  controls the space-like transverse distance between quark fields in the correlation functions we measure. The Bessel-weighted asymmetries are a natural extension of conventional weighted asymmetries [6, 7] with weights  $w_1$  proportional to powers of  $|\mathbf{P}_{h\perp}|$ . Indeed, in the limit  $\mathcal{B}_T \rightarrow 0$ , Eq. (1.6) results in the often encountered special case for the SIDIS cross section

$$A_{UT}^{\frac{|\mathbf{P}_{h\perp}|}{zM} \sin(\phi_h - \phi_s)}(x, z, y) = -2 \frac{\sum_a e_a^2 H_{UT,T}(Q^2) \tilde{f}_{1T}^{\perp(1)a}(x) D_1^{(0)a}(z)}{\sum_a e_a^2 H_{UU,T}(Q^2) \tilde{f}_1^{(0)a}(x) D_1^{(0)a}(z)}, \quad (1.7)$$

where formally  $\tilde{f}_1^{(0)}(x, 0) = f_1^{(0)}(x)$ ,  $\tilde{f}_{1T}^{\perp(1)}(x, 0) = f_{1T}^{\perp(1)}(x)$ , and  $\tilde{D}_1^{(0)}(x, 0) = D_1^{(0)}(x)$  are

$$\tilde{f}^{(n)}(x, 0) = f^{(n)}(x) \equiv \int d^2 \mathbf{p}_T \left( \frac{\mathbf{p}_T^2}{2M^2} \right)^n f(x, \mathbf{p}_T^2). \quad (1.8)$$

However we caution that these moments are not well-defined without some regularization. It is therefore safer to study Bessel-weighted asymmetries at finite  $\mathcal{B}_T$ , where the Bessel functions

suppress contributions from large transverse momenta. Without further regularization, the integrals defining the moments  $f_{1T}^{\perp(1)}(x)$  and  $f^{(0)}(x)$  are ill-defined due to the asymptotic behavior  $f_{1T}^{\perp}(x, \mathbf{p}_T^2) \propto 1/\mathbf{p}_T^4$ ,  $f_1(x, \mathbf{p}_T^2) \propto 1/\mathbf{p}_T^2$  at large  $\mathbf{p}_T$  (see [23]).

## 1.2 Cancellation of soft factor at the level of matrix elements

In a similar manner to the discussion above, we now consider the soft factor cancellation in the average transverse momentum shift of unpolarized quarks in a transversely polarized nucleon for a given longitudinal momentum fraction  $x$  [24] defined by a ratio of the  $\mathbf{p}_T$ -weighted correlator [5]

$$\langle p_y(x) \rangle_{TU} = \frac{\int d^2 \mathbf{p}_T \mathbf{p}_y \Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)}{\int d^2 \mathbf{p}_T \Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)} \Big|_{S^\pm=0, \mathbf{S}_T=(1,0)} = M \frac{f_{1T}^{\perp(1)}(x; \mu^2, \zeta, \rho)}{f_1^{(0)}(x; \mu^2, \zeta, \rho)}, \quad (1.9)$$

where  $f_{1T}^{\perp(1)}$  and  $f_1^{(0)}$  are the moments defined in Eq. (1.8). Obviously, the average transverse momentum shift is very similar in structure to the weighted asymmetry Eq. (1.6). While the weighted asymmetries are accessible directly from the  $\mathbf{p}_{h\perp}$ -weighted cross section, the average transverse momentum shifts are obtained from the  $\mathbf{p}_T$ -weighted correlator and could in principle be accessible from weighted jet SIDIS asymmetries. Therefore we generalize the above quantity, weighting with Bessel functions of  $|\mathbf{p}_T|$ . In particular, we replace

$$\mathbf{p}_y = |\mathbf{p}_T| \sin(\phi_p) \quad \longrightarrow \quad \frac{2J_1(|\mathbf{p}_T| \mathcal{B}_T)}{\mathcal{B}_T} \sin(\phi_p - \phi_S), \quad (1.10)$$

where  $\phi_S = 0$  for the choice  $\mathbf{S}_T = (1, 0)$  in Eq. (1.9). The Bessel-weighted analog of Eq. (1.9) is thus

$$\begin{aligned} \langle p_y(x) \rangle_{TU}^{\mathcal{B}_T} &\equiv \frac{\int d|\mathbf{p}_T| |\mathbf{p}_T| \int d\phi_p \frac{2J_1(|\mathbf{p}_T| \mathcal{B}_T)}{\mathcal{B}_T} \sin(\phi_p - \phi_S) \Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)}{\int d|\mathbf{p}_T| |\mathbf{p}_T| \int d\phi_p J_0(|\mathbf{p}_T| \mathcal{B}_T) \Phi^{(+)[\gamma^+]}(x, \mathbf{p}_T, P, S, \mu^2, \zeta, \rho)} \Big|_{|\mathbf{S}_T|=1} \\ &= M \frac{\tilde{f}_{1T}^{\perp(1)}(x, \mathcal{B}_T^2; \mu^2, \zeta, \rho)}{\tilde{f}_1^{(0)}(x, \mathcal{B}_T^2; \mu^2, \zeta, \rho)}. \end{aligned} \quad (1.11)$$

where the correlator  $\Phi^{(+)[\gamma^+]}$  is given in [25]. Again, the soft factors cancel where the independence of the soft factor on  $v \cdot b / \sqrt{v^2}$  is crucial [14]. Further, weighting with Bessel functions at various lengths  $\mathcal{B}_T$  thus allows us to map out, e.g., ratios of Fourier-transformed TMDs [14]. In the limit  $\mathcal{B}_T \rightarrow 0$ , we recover Eq. (1.9),  $\langle p_y(x) \rangle_{TU}^0 = \langle p_y(x) \rangle_{TU}$ , which we have thus shown to be formally free of any soft factor contribution. Again we caution that the expressions at  $\mathcal{B}_T = 0$  are ill-defined without an additional regularization. Finally we note that the study of the ratio Eq. (1.11) in lattice QCD gives direct numerical evidence that non-zero  $T$ -odd TMDs are a consequence of the first principles of QCD [25].

## 1.3 Conclusions

Rewriting the SIDIS cross-section in coordinate space displays the important feature that structure functions become simple products of Fourier transformed TMD PDFs and FFs, or derivatives thereof. The angular structure of the cross section naturally suggests weighting with Bessel functions in order to project out these Fourier-Bessel transformed distributions, which serve as well-defined replacements of the transverse moments entering conventional weighted asymmetries. In

addition, Bessel-weighted asymmetries provide a unique opportunity to study nucleon structure in a model independent way due to the absence of the soft factor which cancels from these observables. This cancellation is based on the fact that the soft factor is flavor blind in hard processes, and it depends only on  $\mathbf{b}_T^2, \mu^2$  and the rapidity cutoff parameter  $\rho$  [11, 12]. Moreover, evolution equations for the distributions are typically calculated in terms of the (derivatives of) Fourier transformed TMD PDFs and FFs. As a result, the study of the scale dependence of Bessel-weighted asymmetries should prove more straightforward [26, 27, 28]. Thus, we propose Bessel-weighted asymmetries as clean observables to study the scale dependence of TMD PDFs and FFs.

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