Time-like proton form factors and heavy lepton production at PANDA

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Proton electromagnetic form factors are fundamental quantities for the understanding of nucleon structure. Time-like proton form factors can be measured at the future experiment PANDA through the proton antiproton annihilation into a lepton pair. In this work we extend previous calculations of polarization observables for the annihilation reaction $\bar{p} + p \rightarrow e^- + e^+$ to the case of heavy leptons, such as the $\mu$, $\tau$-lepton. We consider the case when the beam and/or the target are polarized, as well as the polarization of the outgoing leptons. A numerical application is done using a VDM parametrization of proton form factors. Our results show an enhancement of the polarization observables when the transverse polarization of these heavy unstable leptons is considered.
1. Introduction

Since long time electromagnetic probes are used to study the internal structure of the proton where the interaction is assumed to occur through the exchange of one virtual photon.

Elastic $ep$ scattering maps the spatial distributions of electric charge and magnetic current inside the nucleon by measuring the magnetic and electric form factors (FFs). Deep-Inelastic Scattering (DIS), in which a quark in a nucleon is knocked out by the virtual photon, allows to extract the quark and gluon distributions. Experimental studies with antiproton beams are also powerful tools to learn about hadronic structure and to test fundamental theories like quantum chromodynamics. Collisions of antiproton-proton beams accelerated by the Tevatron at Fermilab probed the structure of matter and antimatter at a very small scale. CERN’s Low Energy Antiproton Ring (LEAR) was also constructed to study the antimatter properties.

The PANDA experiment [1] is a particle physics experiment which is under construction at the Facility for Antiproton and Ion Research (FAIR) in Darmstadt. PANDA will use the antiproton beam produced at FAIR to study the properties of strong interaction. Among the many possible final channels in proton antiproton annihilation, we are interested here in the creation of heavy lepton ($\ell$) pairs $\bar{p} + p \rightarrow \ell^- + \ell^+$, $\ell = \mu$, $\tau$, through the annihilation into a virtual photon of four-momentum transfer squared $q^2$. The measurement of experimental observables related to these channels allows to access the electromagnetic structure of the proton, parametrized in terms of FFs.

2. Electromagnetic form factors

Proton FFs can be measured in space-like (SL) region by unpolarised elastic $ep$ scattering using Rosenbluth separation and by the polarization method suggested by Akhiezer and Rekalo [2]. Both methods assume the interaction between the electron and the proton via one photon exchange. The matrix element of the current between the proton states is expressed via the Dirac ($F_1$) and Pauli ($F_2$) FFs. Sachs electric $G_E$ and magnetic $G_M$ FFs are introduced as a combination of $F_1$ and $F_2$. Their physical interpretation in the non relativistic domain is related to three-dimension Fourier transform of charge and magnetic densities. In relativistic approaches, the proton recoil effects become important and one needs to consider the scattering in the Breit frame in which the initial and final state nucleons have momenta with the same magnitude, hence similar Lorentz contraction effects.

Crossing symmetry related reactions $e^+ + e^- \leftrightarrow \bar{p} + p$ carry the same information on the hadronic vertex. They allow one to measure FFs in the time-like (TL) region (positive 4-momentum transfer squared) where they are complex functions due to the initial or final hadron state interaction.

The reaction $\bar{p} + p \rightarrow e^+ + e^- + \pi^0$ probes the unphysical region below the threshold of proton antiproton annihilation [3]. From theoretical point of view, FFs are analytical functions which satisfy the analyticity and unitarity. Models which parametrize FFs in both regions exploit this fact to connect the SL and TL (including the unphysical) regions. The connection can be done using the dispersion relation approach. FFs from TL and SL region are constrained at very large momentum.
transfer by the Phragmén-Lindelhof theorem, and at the photon point, the Sachs FFs are normalised to the static values of electric charge and magnetic moment of the proton.

Contrary to the SL region, few data exist for proton FFs in TL region. Due to the low statistics in the experiments where the data were taken (BaBar, E835, Fenice, BES,...) they are extracted upon the assumption $G_E = 0$ or $G_E = G_M$. Data in the unphysical region are absent up to now. The future PANDA experiment is the first experiment where the TL proton FFs will be measured separately in a large range of momentum transfer squared, up to $14 \text{ GeV}^2$.

3. Heavy lepton production

The reaction $\bar{p} + p \rightarrow e^+ + e^-$ has been first studied in Ref. [4] in connection with the possibility to extract proton FFs in the annihilation region. In Ref. [5, 6], polarization observables have been derived under the assumption $m_e = 0$. Following the formalism of Ref. [6], we extend the calculations of polarization observables for the annihilation reaction $\bar{p} + p \rightarrow \ell^- + \ell^+$, to the case of heavy leptons, such as $\mu$ or $\tau$ where the mass of the lepton can not be neglected. The interest of FFs measurement in $\bar{p} + p$ annihilation into heavier leptons is related to the following facts: - Polarization observables corresponding to the transverse polarization of the lepton contain the factor $m_\ell/E$ ($m_\ell$ is the lepton mass, $E$ is the incident energy) which corresponds to a huge suppression, in case of electron, whereas, in case of $\tau$-lepton it becomes an enhancement in the GeV range; - The polarization of unstable particles ($\mu$ and $\tau$) can be measured, in principle, through the angular distribution of their decay products; - Radiative corrections, which are a critical issue for the extraction of proton FFs [7], are essentially suppressed.

3.1 Unpolarized cross section

Let us consider the reaction:

$$\bar{p}(p_1) + p(p_2) \rightarrow \ell^- (k_1) + \ell^+(k_2),$$

where $\ell = e, \mu$ or $\tau$ and the four-momenta of the particles are written in parenthesis. We limit our study to the Born approximation where $q = k_1 + k_2 = p_1 + p_2$ is the four momentum of the exchanged virtual photon.

The following analysis of the cross section and polarization observables is done in the center of mass system (CMS). We define a coordinate frame, where the $z$-axis is directed along the antiproton momentum $\bar{p}$, the $y$-axis is directed along the vector $\bar{p} \times \vec{k}$ ($\vec{k}$ is the negative lepton momentum in CMS) and the $x$ axis in order to form a left handed coordinate system.

The unpolarized differential cross section can be written as:

$$\frac{d\sigma_0}{d\Omega} = \frac{\alpha^2}{4 q^2} \beta_{\ell} \beta_{p} \mathcal{D}, \quad \mathcal{D} = \frac{|G_E|^2}{\eta_p} (1 - \beta_\ell^2 \cos^2 \theta) + |G_M|^2 (2 - \beta_p^2 \sin^2 \theta),$$

where $\theta$ is the CMS angle of $\ell^-$ with respect to the antiproton beam, $\beta_{\ell}^2 = 1 - 4 m_\ell^2/q^2$ is the velocity squared of the lepton $\ell$, $\beta_{p}^2 = 1 - 4 M^2/q^2 = 1 - 1/\eta_p$ is the antiproton velocity squared and $M$ is the hadron mass.

As in the case of electron pair production, the unpolarised cross section depends on the moduli squared of FFs, and does not contain any interference term. One can see also that the terms due
to the lepton mass does not change the even nature of the differential cross section with respect to \( \cos \theta \), as expected from the one photon exchange mechanism, but changes the ratio of the cross section at \( \theta = 0^\circ \) or \( 180^\circ \) with respect to the cross section at \( \theta = 90^\circ \).

The ratio between the integrated cross section over the lepton solid angle for the production of an heavy lepton pair \( \ell^- \ell^+ \), \( \ell = \tau \) \((m_\tau=1776.82 \text{ MeV})\), or \( \ell = \mu \) \((m_\mu=105.66 \text{ MeV})\), with respect to the integrated cross section for the production of an electron pair \((m_e=0.511 \text{ MeV})\), is:

\[
R_\ell = \frac{\sigma(\ell^+\ell^-)}{\sigma(e^+e^-)} = \frac{1}{2} \beta_\ell (3 - \beta_\ell^2).
\]

The corrections to the ratio due to the mass are of the fourth order and proportional to \((m_\ell/\sqrt{q^2})^4\); therefore, over the kinematical threshold, the \( \mu \) cross section is similar to the electron one. But for \( \tau \) production, the variation is significant in the energy region over the \( \tau^+ \tau^- \) threshold.

3.2 Polarization observables

The unpolarized cross section contains only the moduli squared of the FFs. The investigation of reaction (3.1) with polarized antiproton beam and/or polarized proton target carries information about the phase difference of the nucleon FFs, \( \Phi = \Phi_M - \Phi_E \), where \( \Phi_M, E = \text{arg}G_{M,E} \). This phase difference contains important information on the nucleon FFs and its determination represents a stringent test of nucleon models. The same information is also carried by the measurement of the polarization of produced leptons.

In the following sections, we report on two types of polarization observables, the single spin and the double spin observables. The triple spin polarization coefficients can be found in Ref. [8].

3.3 Single spin observables: the analyzing power

Unlike elastic \( e^- p \) scattering in one-photon exchange approximation, the hadronic tensor in the reaction (3.1) contains an antisymmetric part due to the fact that nucleon FFs are complex functions [6]. Therefore, in the present case, the polarization of the antiproton may lead to nonzero spin asymmetry. As a starting point, we consider the case when only the antiproton beam is polarized, which leads to the following expression of the differential cross section

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} (1 + A_y \chi_I), \quad A_y = \frac{\beta_y^2 \sin 2\theta}{\sqrt{\eta_p}} \text{Im}G_M G_E^*.
\]

where \( A_y \) is the only non vanishing single spin asymmetry due to the antiproton polarization, \( \chi_I \) is the polarization of the antiproton in its rest frame. Taking into account the mass of the lepton leads to a factor \( \beta_y^2 \) which decreases when the antiproton energy increases. The single spin asymmetry due to the polarization of the \( \tau \)-lepton vanishes in the Born approximation, therefore its measurement is a direct test of the presence of the two-photon exchange mechanism [6]. The contribution of two-photon exchange is suppressed by the presence of a factor \( m_\ell/M \) in this asymmetry, but in case of \( \tau \), this factor constitutes an enhancement.

3.4 Double spin polarization observables

The double spin observables describe the cases where two particles involved in the annihilation reaction are polarized: the polarization transfer coefficients \( T_{ij} \) from initial to final state, the
analyzing powers in polarized proton-antiproton collisions $A_{ij}$, and the correlation coefficients of polarized proton-antilepton pair in the final state $C_{ij}$.

We denote the direction of the lepton polarization vector (in its rest system) by the indices: $\ell$ (longitudinal) along its momentum, $t$ (transverse) which is orthogonal to the momentum in the reaction plane and $n$ (normal) which is perpendicular to the reaction plane.

The non-vanishing transfer polarization coefficients, where the antiproton beam is polarized and the polarization of the produced negative lepton is measured, are:

$$D_{T lx} = \frac{2 \sin \theta}{\sqrt{\eta_p}} \Re G_M G_E^*,$$  
$$D_{T tz} = 2 \cos \theta |G_M|^2,$$  
$$D_{T ny} = \frac{2}{M} \frac{m_\ell}{\eta_p} \Re G_E G_M^*,$$

(3.5)

The coefficients $T_{ny}, T_{lx}, T_{tz}$ are proportional to the mass of the produced lepton and they are suppressed by the factor $m_\ell/M$ for $\ell = e$, contrary to the case of $\mu$ or $\tau$.

The nonzero polarization correlation coefficients where the polarizations of produced leptons are measured in the annihilation of unpolarized antiproton-proton, have the following form

$$D_{C mn} = \sin^2 \theta \beta^{\ell}_{\eta_p} \beta^{\ell}_{\eta_p} + |G_E|^2 \left( \frac{1}{\eta_p^2} \right),$$  
$$D_{C lt} = \sin^2 \theta \left( 1 + \frac{1}{\eta_p^2} \right),$$  
$$D_{C ml} = \frac{2 \sin \theta}{\sqrt{\eta_p^2}} \beta^{\ell}_{\eta_p} \beta^{\ell}_{\eta_p},$$  
$$D_{C ll} = \frac{2 \sin \theta}{\sqrt{\eta_p^2}} \beta^{\ell}_{\eta_p} \beta^{\ell}_{\eta_p}.$$  

(3.6)

From these expressions one can see that, for the $\tau$-lepton, the large mass leads to an increase of the $|G_E|^2$ term in the angular regions $\theta \sim 0^\circ$ and $\theta \sim 180^\circ$. This effect essentially decreases for $e$ and $\mu$. For the analyzing powers in polarized proton-antiproton collisions, the expressions of the non vanishing coefficients are given in Ref. [8].

The non vanishing double spin observables are shown in Fig. 5 for the parametrization of Ref. [9] as a function of $\cos \theta$ in CMS for $q^2 = 14 \text{ GeV}^2$. Note that $A_{yy}$ coincides with $C_{mn}$ and it is not shown. For the $\tau$-meson, the effect of the mass is sizable in all the observables. The difference between $\mu$ and $e$ is tiny and it is best seen in the observables related to the transverse polarization, such as $T_{nz}$ and $C_{ll}$.

4. Conclusion

The calculation of polarization observables for the annihilation of proton-antiproton into a lepton pair was extended to the case of heavy leptons, such as $\tau$ or $\mu$ in the one-photon exchange approximation. The expressions of the observables are given in terms of nucleon electromagnetic FFs. We investigated the dependence of the unpolarized cross section, of the angular asymmetry and of various polarization observables on the mass of the lepton.

Our results show an enhancement of the polarization observables of these heavy unstable particles, in particular when the transverse polarization of the leptons is considered. The measurement of lepton polarization can be done in principle through the angular distribution of the decay products. The feasibility measurement of the TL FFs at PANDA using the production of heavy leptons is subject of actual studies.
Figure 1: Double polarization observables as a function of $\cos \theta$, for $q^2 = 14 \text{ GeV}^2$ in CMS, for $\mu$ (blue solid line), for $e$ (black dash-dotted line), and for $\tau$ (red dotted line).

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References


