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Semileptonic decays of spin-1/2 doubly charmed baryons

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We evaluate exclusive semileptonic decays of ground-state spin-1/2 doubly heavy charmed baryons. The decays are driven by a $c \to s, d$ transition at the quark level. Our form factors are consistent with Heavy Quark Symmetry constraints. The latter are valid in the limit of infinitely heavy quark mass at zero recoil.

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1. Decay width and form factor decomposition of the hadronic current

The total decay width for semileptonic $c \to l$ transitions, with $l = s, d$, is given by

$$\Gamma = |V_{cl}|^2 \frac{G_F^2}{8\pi^4} \frac{M'^2}{M} \int \sqrt{w^2-1}\, \mathscr{L}^{\alpha\beta}(q) \mathscr{H}_{\alpha\beta}(P,P')\, dw \tag{1.1}$$

where $|V_{cl}|$ is the modulus of the corresponding Cabibbo–Kobayashi–Maskawa (CKM) matrix element for a $c \to l$ quark transition, for which we shall use $|V_{cs}| = 0.97345$ and $|V_{cd}| = 0.2252$ taken from Ref. [1]. $G_F = 1.16637(1) \times 10^{-11}\,\mathrm{MeV}^{-2}$ [1] is the Fermi decay constant, P, M (P', M') are the four-momentum and mass of the initial (final) baryon, $q = P - P'$ and w is the product of the baryons four-velocities $w = v \cdot v' = \frac{P}{M} \cdot \frac{P'}{M'} = \frac{M^2+M'^2-q^2}{2MM'}$. In the decay, w ranges from $w = 1$, corresponding to zero recoil of the final baryon, to a maximum value given, neglecting the neutrino mass, by $w = w_{\max} = \frac{M^2+M'^2-m^2}{2MM'}$, which depends on the transition and where m is the final charged lepton mass. Finally $\mathscr{L}^{\alpha\beta}(q)$ is the leptonic tensor after integrating in the lepton momenta and $\mathscr{H}_{\alpha\beta}(P,P')$ is the hadronic tensor.

The leptonic tensor is given by

$$\mathscr{L}^{\alpha\beta}(q) = A(q^2)\, g^{\alpha\beta} + B(q^2)\, \frac{q^\alpha q^\beta}{q^2} \tag{1.2}$$

where

$$A(q^2) = -\frac{I(q^2)}{6}\left(2q^2 - m^2 - \frac{m^4}{q^2}\right)\ ,\ \ B(q^2) = \frac{I(q^2)}{3}\left(q^2 + m^2 - 2\frac{m^4}{q^2}\right) \tag{1.3}$$

with $I(q^2) = \frac{\pi}{2q^2}(q^2 - m^2)$.

The hadronic tensor reads

$$\mathscr{H}^{\alpha\beta}(P,P') = \frac{1}{2J+1} \sum_{r,r'} \left\langle B', r'\ \vec{P}' \middle| J_{cl}^\alpha(0) \middle| B, r\ \vec{P} \right\rangle \left\langle B', r'\ \vec{P}' \middle| J_{cl}^\beta(0) \middle| B, r\ \vec{P} \right\rangle^* \tag{1.4}$$

with J the initial baryon spin, $\left|B, r\ \vec{P}\right\rangle$ $\left(\left|B', r'\ \vec{P}'\right\rangle\right)$ the initial (final) baryon state with three-momentum $\vec{P}$ $(\vec{P}')$ and spin third component r (r') in its center of mass frame. $J_{cl}^\mu(0)$ is the charged weak current for a $c \to l$ quark transition

$$J_{cl}^\mu(0) = \overline{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0) \tag{1.5}$$

Baryonic states are normalized as $\left\langle B, r'\ \vec{P}' \,|\, B, r\ \vec{P}\right\rangle = 2E\,(2\pi)^3\,\delta_{rr'}\,\delta^3(\vec{P} - \vec{P}')$, with E the baryon energy for three-momentum $\vec{P}$.

Hadronic matrix elements can be parameterized in terms of form factors. For $1/2 \to 1/2$ transitions the commonly used form factor decomposition reads

$$\begin{aligned} &\left\langle B'(1/2), r'\ \vec{P}' \middle| \overline{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0) \middle| B(1/2), r\ \vec{P} \right\rangle = \\ &\bar{u}_{r'}^{B'}(\vec{P}')\Big\{\gamma^\mu\left[F_1(w) - \gamma_5 G_1(w)\right] + v^\mu\left[F_2(w) - \gamma_5 G_2(w)\right] + v'^\mu\left[F_3(w) - \gamma_5 G_3(w)\right]\Big\} u_r^B(\vec{P}) \end{aligned} \tag{1.6}$$

The u_r are Dirac spinors normalized as $(u_{r'})^\dagger u_r = 2E\,\delta_{rr'}$. v^μ, v'^μ are the four velocities of the initial and final baryons. The three vector F_1, F_2, F_3 and three axial G_1, G_2, G_3 form factors are functions of w or equivalently of q^2.

For $1/2 \to 3/2$ transitions we follow Llewellyn Smith [2] to write

$$\begin{aligned}\left\langle B'(3/2), r'\vec{P}' \,|\, \overline{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0)\,|\, B(1/2), r\vec{P}\right\rangle &= \bar{u}^{B'}_{\lambda\, r'}(\vec{P}')\,\Gamma^{\lambda\mu}(P,P')\,u^B_r(\vec{P})\\ \Gamma^{\lambda\mu}(P,P') &= \Big[\frac{C^V_3(w)}{M}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^V_4(w)}{M^2}(g^{\lambda\mu}qP' - q^\lambda P'^\mu)\\ &\quad + \frac{C^V_5(w)}{M^2}(g^{\lambda\mu}qP - q^\lambda P^\mu) + C^V_6(w)g^{\lambda\mu}\Big]\gamma_5\\ &+\left[\frac{C^A_3(w)}{M}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C^A_4(w)}{M^2}(g^{\lambda\mu}qP' - q^\lambda P'^\mu) + C^A_5(w)g^{\lambda\mu} + \frac{C^A_6(w)}{M^2}q^\lambda q^\mu\right]\end{aligned} \tag{1.7}$$

Here $u^{B'}_{\lambda\, r'}$ is the Rarita-Schwinger spinor of the final spin 3/2 baryon normalized such that $(u^{B'}_{\lambda\, r'})^\dagger u^{B'\lambda}_r = -2E'\,\delta_{rr'}$, and we have four vector ($C^V_{3,4,5,6}(w)$) and four axial ($C^A_{3,4,5,6}(w)$) form factors.

2. Heavy quark spin symmetry

In hadrons with a single heavy quark the dynamics of the light degrees of freedom becomes independent of the heavy quark flavor and spin when the mass of the heavy quark is much larger than Λ_{QCD} and the masses and momenta of the light quarks. This is the essence of heavy quark symmetry (HQS) [3, 4, 5, 6]. However, HQS can not be directly applied to hadrons containing two heavy quarks. The static theory for a system with two heavy quarks has infra-red divergences which can be regulated by the kinetic energy term $\overline{h}_Q(D^2/2m_Q)h_Q$. This term breaks the heavy quark flavor symmetry, but not the spin symmetry for each heavy quark flavor [7]. This is known as heavy quark spin symmetry (HQSS). HQSS implies that all baryons with the same flavor wave-function are degenerate. The invariance of the effective Lagrangian under arbitrary spin rotations of the c quark leads to relations, near the zero recoil point ($w = 1 \leftrightarrow q^2 = (M-M')^2 \leftrightarrow |\vec{q}\,| = 0$), between the form factors for vector and axial-vector currents between the Ξ_{cc} and Ω_{cc} baryons and the single charmed baryons. These decays are induced by the semileptonic weak decay of the c quark to a d or a s quark. The consequences of spin symmetry for weak matrix elements can be derived using the “trace formalism” [8].

Near the zero recoil point, where the spin symmetry should work best, HQSS considerably reduces the number of independent form factors, and it relates those that correspond to transitions where the spin of the two light quarks in the final baryon is $S = 1$. Indeed, we find at $w = 1$ [9]

- $1/2 \to 1/2$ transitions ($\Xi_{cc} \to \Lambda_c, \Xi_c$ and $\Omega_{cc} \to \Xi_c$), where the total spin of the two light quarks in the final baryon is $S = 0$:

$$F_1 + F_2 + F_3 = 3G_1 \equiv \eta_0 \tag{2.1}$$

In the equal mass transition case one would find that η_0 is normalized as $\eta_0(w=1) = \sqrt{\frac{3}{2}}$.

- Total spin of the two light quarks in the final baryon is $S = 1$.

* $1/2 \to 1/2$ transitions ($\Xi_{cc} \to \Sigma_c, \Xi'_c$ and $\Omega_{cc} \to \Xi'_c, \Omega_c$) .

$$F_1 + F_2 + F_3 = \frac{3}{5} G_1 \equiv \eta_1 \tag{2.2}$$

* $1/2 \to 3/2$ transitions ($\Xi_{cc} \to \Sigma_c^*, \Xi_c^*$ and $\Omega_{cc} \to \Xi_c^*, \Omega_c^*$).

$$\frac{\sqrt{3}}{2}\left(C_3^A \frac{M-M'}{M} + C_4^A \frac{M'(M-M')}{M^2} + C_5^A\right) = \eta_1 \tag{2.3}$$

In the equal mass transition case one would have that $\eta_1(w=1) = \frac{1}{\sqrt{2}}$ when the two light quarks in the final state are different and $\eta_1(w=1) = 1$ when they are equal (Ω_c and Ω_c^*).

Relations (2.1), (2.2) and (2.3) are exactly satisfied in the quark model when the heavy quark mass is made arbitrarily large, and thus the calculation is consistent with HQSS constraints.

3. Results

We start by checking that our calculation respects the constraints on the form factors deduced from HQSS. In Figs. 1 and 2, we show to what extent the relations of (2.1), (2.2) and (2.3) summarized above are satisfied for the actual m_c value. In all cases we see moderate deviations, that stem from $1/m_c$ corrections, at the level of about 10% near zero recoil, though larger than those found in [10] for the $b \to c$ transitions of the Ξ_{bc} and Ξ_{bb} baryons. These discrepancies tend to disappear when the mass of the heavy quark is made arbitrarily large [9].

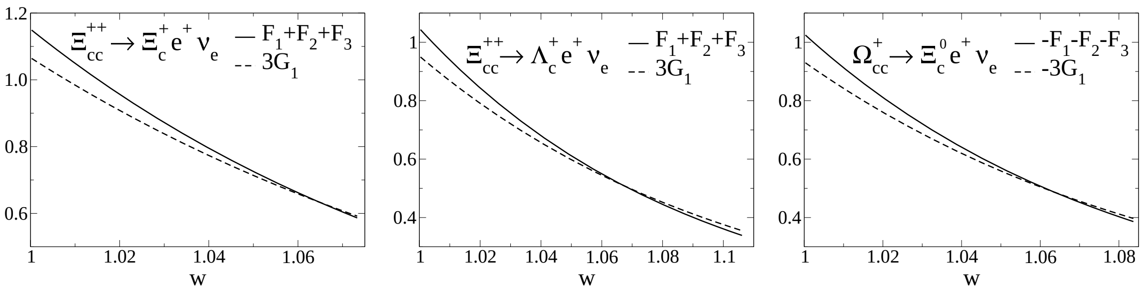

Figure 1: Comparison of $F_1 + F_2 + F_3$ (solid) and $3G_1$ (dashed) for the specified transitions. The two light quarks in the final baryon have total spin $S = 0$.

Now we discuss the results for the decay widths. Those are shown in Table 1 for the dominant ($c \to s$) and sub-dominant ($c \to d$) exclusive semileptonic decays of the Ξ_{cc} and Ω_{cc} to ground state, $1/2^+$ or $3/2^+$, single charmed baryons and with a positron in the final state[1]. For the Ω_{cc}^+ baryon, semileptonic decays driven by a $s \to u$ transition at the quark level are also possible. However, in this latter case phase space is very limited and we find the decay widths are orders of magnitude smaller than the ones shown. To our knowledge there are just a few previous theoretical evaluations of the Ξ_{cc} semileptonic decays. In Ref. [11] the authors use the relativistic three-quark model

[1] Similar results are obtained for $\mu^+ \nu_\mu$ leptons in the final state.

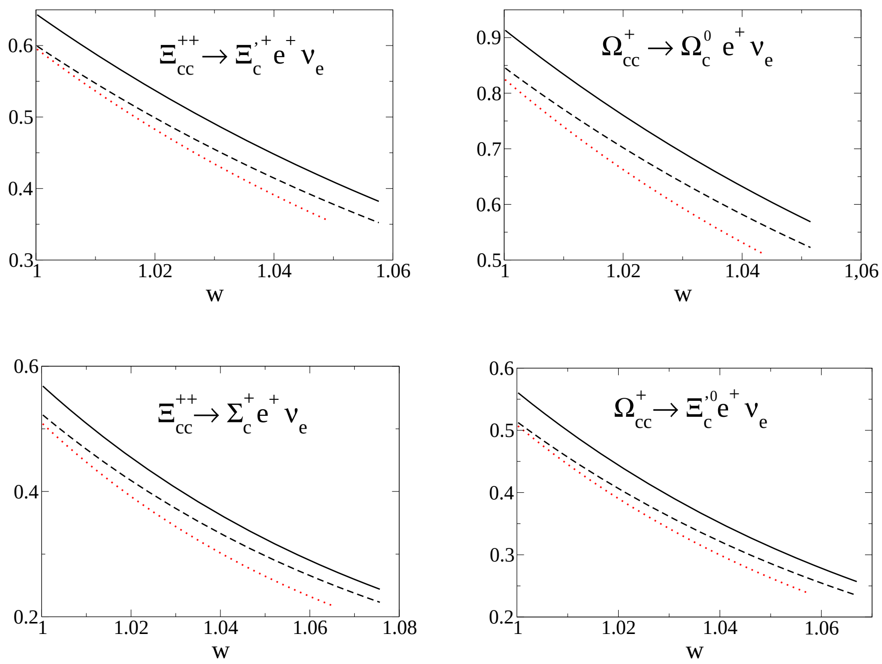

Figure 2: Solid (dashed): $F_1 + F_2 + F_3$ $(3G_1/5)$ for the specified transitions. Dotted: the combination $\frac{\sqrt{3}}{2}\left(C_3^A \frac{M-M'}{M} + C_4^A \frac{M'(M-M')}{M^2} + C_5^A\right)$ for the transition with the corresponding 3/2 baryon (Σ_c^*, Ξ_c^* or Ω_c^*) in the final state. In all cases the two light quarks in the final baryon have total spin $S = 1$.

to evaluate the $\Xi_{cc} \to \Xi_c' e^+ \nu_e$ decay, while in Ref. [12], using heavy quark effective theory and non-relativistic QCD sum rules, they give both the lifetime of the Ξ_{cc} baryon and the branching ratio for the combined decay $\Xi_{cc} \to \Xi_c e^+ \nu_e + \Xi_c' e^+ \nu_e + \Xi_c^* e^+ \nu_e$ from which we have evaluated the semileptonic decay widths shown in the table. We find a fair agreement of our predictions with both calculations. In Ref. [13], using the optical theorem and the operator product expansion, the authors evaluated the total semileptonic decay rate finding it to be $0.151\,\mathrm{ps}^{-1}$ for Ξ_{cc}^{++} and $0.166\,\mathrm{ps}^{-1}$ for Ξ_{cc}^{+}. These values are roughly a factor of two smaller than the sum of our partial decay widths or the results in Ref. [12]. For the Ω_{cc}^+ a total semileptonic decay width of $0.454\,\mathrm{ps}^{-1}$ is given in Ref. [13]. In this case this is in better agreement with the sum of our partial semileptonic decay widths which add up to $0.353\,\mathrm{ps}^{-1}$.

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		Γ [ps^{-1}]	
$B_{cc} \to B_c e^+ \nu_e$	Quark transition	This work	[11],[12]
$\Xi_{cc}^{++} \to \Xi_c^+ e^+ \nu_e$	$(c \to s)$	8.75×10^{-2}	
$\Xi_{cc}^{+} \to \Xi_c^0 e^+ \nu_e$	$(c \to s)$	8.68×10^{-2}	
$\Xi_{cc}^{++} \to \Xi_c'^+ e^+ \nu_e$	$(c \to s)$	0.146	$0.208 \div 0.258$[11]
$\Xi_{cc}^{+} \to \Xi_c'^0 e^+ \nu_e$	$(c \to s)$	0.145	$0.208 \div 0.258$[11]
$\Xi_{cc}^{++} \to \Xi_c^{*+} e^+ \nu_e$	$(c \to s)$	3.20×10^{-2}	
$\Xi_{cc}^{+} \to \Xi_c^{*0} e^+ \nu_e$	$(c \to s)$	3.20×10^{-2}	
$\Xi_{cc}^{++} \to \Xi_c'^+ e^+ \nu_e + \Xi_c^+ e^+ \nu_e + \Xi_c^{*+} e^+ \nu_e$	$(c \to s)$	0.266	$0.37 \pm 0.04^{(*)}$[12]
$\Xi_{cc}^{+} \to \Xi_c'^0 e^+ \nu_e + \Xi_c^0 e^+ \nu_e + \Xi_c^{*0} e^+ \nu_e$	$(c \to s)$	0.264	$0.47 \pm 0.15^{(*)}$[12]
$\Xi_{cc}^{++} \to \Lambda_c^+ e^+ \nu_e$	$(c \to d)$	4.86×10^{-3}	
$\Xi_{cc}^{++} \to \Sigma_c^+ e^+ \nu_e$	$(c \to d)$	7.94×10^{-3}	
$\Xi_{cc}^{+} \to \Sigma_c^0 e^+ \nu_e$	$(c \to d)$	1.58×10^{-2}	
$\Xi_{cc}^{++} \to \Sigma_c^{*+} e^+ \nu_e$	$(c \to d)$	1.77×10^{-3}	
$\Xi_{cc}^{+} \to \Sigma_c^{*0} e^+ \nu_e$	$(c \to d)$	3.54×10^{-3}	
$\Omega_{cc}^{+} \to \Omega_c^0 e^+ \nu_e$	$(c \to s)$	0.282	
$\Omega_{cc}^{+} \to \Omega_c^{*0} e^+ \nu_e$	$(c \to s)$	5.77×10^{-2}	
$\Omega_{cc}^{+} \to \Xi_c^0 e^+ \nu_e$	$(c \to d)$	4.11×10^{-3}	
$\Omega_{cc}^{+} \to \Xi_c'^0 e^+ \nu_e$	$(c \to d)$	7.44×10^{-3}	
$\Omega_{cc}^{+} \to \Xi_c^{*0} e^+ \nu_e$	$(c \to d)$	1.72×10^{-3}	

Table 1: Decay widths in units of ps^{-1}. We use $|V_{cs}| = 0.97345$ and $|V_{cd}| = 0.2252$ taken from Ref. [1]. Results with an $(*)$, our estimates from the total decay widths and branching ratios in [12]. Similar results are obtained for $\mu^+ \nu_\mu$ leptons in the final state.

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