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Enhanced non-quark-antiquark and non-glueball N_c behavior of light scalar mesons

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We report our results on the nature of the lightest scalar resonances, where we show that a $\bar{q}q$ or glueball interpretation of the scalars $f_0(600)$ and $K_0^*(800)$ requires a very unnatural fine tuning to satisfy $1/N_c$–expansion predictions for $\bar{q}q$ or glueball states, which is not needed in the case of the lightest vector mesons $\rho(770)$ and $K^*(892)$. For this we consider scattering observables whose value is fixed to 1 for $\bar{q}q$ and glueball states up to corrections suppressed by more than one power of $1/N_c$, thus enhancing contributions of other nature. This allows us to evaluate these observables and check the $1/N_c$ predictions at $N_c = 3$ without the need to extrapolate to unphysical N_c values. This is done using recent and very precise dispersive $\pi\pi$ and πK scattering data analyses.

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1. Introduction

Light scalar mesons are an object of great interest in hadron and nuclear physics. They are largely responsible for the attractive part [1] of the nucleon–nucleon interaction; some have the quantum numbers of the lightest glueball, which is interesting for the non–abelian nature of Quantum Chromodynamics (QCD); also, some have the quantum numbers of the vacuum, so they should play a relevant role in the spontaneous Chiral Symmetry breaking of QCD. However, the precise properties of the light scalar mesons, as their nature, spectroscopic classification, and even their existence —as in the case of the $K_0^*(800)$— are still the subject of an intense debate. Regarding their spectroscopic nature, several models [4] suggest that they might not be of ordinary $\bar{q}q$ nature, but of other kind of spectroscopic classification, such as tetraquarks, meson–meson molecules, glueballs, or a complicated mixture of all these.

A powerful tool to study the spectroscopic nature of mesons is the QCD $1/N_c$ expansion [5]. It is valid in the entire energy range and gives a clear definition of different spectroscopic components in terms of their mass and width $1/N_c$ scaling, which is well known for $\bar{q}q$ and glueball states. By combining the $1/N_c$ expansion with Chiral Perturbation Theory (ChPT) [3] unitarized with the Inverse Amplitude Method [14], some of us studied [6, 7] the $1/N_c$ behavior of light resonances. It was found that whereas the $\rho(770)$ and $K^*(892)$ vectors behave predominantly as expected for $\bar{q}q$ states, the scalars $f_0(600)$ and $K_0^*(800)$ do not [6]. However, the two–loop analysis [7] showed that a possible subdominant $\bar{q}q$ component for the $f_0(600)$ may exist, but with a mass around 1 GeV or more.

In [6, 7] unitarized ChPT was used to change N_c and study the $1/N_c$ scaling of the mass and width of the light resonances generated. However, the $1/N_c$ leading $\bar{q}q$ scaling, $M = O(1)$, $\Gamma = O(1/N_c)$ receives subleading corrections suppressed by $1/N_c$, and for physical $N_c = 3$ this may not seem a large suppression. Thus, we report here our results [8] using adimensional observables with corrections suppressed further than $1/N_c$, that allow us to obtain conclusions directly from real data at $N_c = 3$, without the need to extrapolate to larger N_c using unitarized ChPT.

The observables mentioned above are related to the three different criteria commonly used to identify resonances in elastic two–body scattering, which are equivalent for large N_c. One of these criteria is the position of the pole associated to the resonance in the unphysical sheet, s_R, which gives a definition of the resonance mass and width, $s_R = m_R^2 - im_R\Gamma_R$. A second possibility is to define the mass as the energy at which the phase shift reaches $\pi/2$, which for both $\pi\pi$ and πK scalar scattering phase shifts occur relatively far from the pole position. Third, the resonance mass can also be identified with the point where the phase derivative is maximum. The relation between the first two criteria, which are equivalent up to $O(1/N_c^2)$ corrections for $\bar{q}q$ states [9], was studied in [9] for the $f_0(600)$ with a relatively inconclusive result about its assumed $\bar{q}q$ nature. A more reliable parametrization and better data were called for and we will use it here with more conclusive results.

Thus in section 2 we define and obtain the $1/N_c$ scaling of the observables used to test the $1/N_c$ predictions suppressed by more than one power of $1/N_c$. These are related to the phase shift and its derivative evaluated at the resonance "pole" mass $m_R^2 = \mathrm{Re}\,(s_R)$. In section 3 we discuss the results obtained, where we see that the coefficients needed for considering the $f_0(600)$ and $K_0^*(800)$ as $\bar{q}q$ or glueball states are unnaturally large by two orders of magnitude.

2. Highly suppressed $1/N_c$ observables

Consider the elastic scattering of two mesons with a resonance associated to a pair of conjugate poles on the unphysical sheet of the scattering amplitude, located at $s_R = m_R^2 \pm i m_R \Gamma_R$, where m_R and Γ_R are the resonance mass and width. It was found in [9] that if the resonance behaves as a $\bar{q}q$ state, i. e., $m_R = O(1)$, $\Gamma_R = O(1/N_c)$, then the phase shift satisfies

$$\delta(m_R^2) = \frac{\pi}{2} - \underbrace{\left.\frac{\mathrm{Re}\,t^{-1}}{\sigma}\right|_{m_R^2}}_{O(N_c^{-1})} + O(N_c^{-3}), \qquad \delta'(m_R^2) = -\underbrace{\left.\frac{(\mathrm{Re}\,t^{-1})'}{\sigma}\right|_{m_R^2}}_{O(N_c)} + O(N_c^{-2}), \tag{2.1}$$

where $t(s)$ is the scattering partial wave, $\sigma = 2k/\sqrt{s}$, k is the center of mass momentum of one of the mesons and s is the usual Mandelstam variable. The prime denotes derivatives with respect to s. Note that the subleading $1/N_c$ corrections are suppressed by two powers of $1/N_c$. This particular $1/N_c$ counting, as shown in [9], comes from the expansion of the real and imaginary parts of the pole equation, as we detail next.

The inverse of the partial wave, which generically scales as N_c, can be written as $t^{-1} = R + iI$, where R and I are analytic functions that coincide with the real and imaginary parts of t^{-1} over the right cut, i. e., $R(s) = \mathrm{Re}\,t^{-1}(s)$ and $I(s) = \mathrm{Im}\,t^{-1}(s) = -\sigma(s)$ for $s > s_{th}$. Then, the inverse partial wave on the second sheet is given by $t_{II}^{-1} = R - iI$, and the equation for the resonance pole position, $t_{II}^{-1}(s_R) = 0$, can be written as $R(s_R) = iI(s_R)$. If the resonance is a $\bar{q}q$ state, $m_R = O(1)$ and $\Gamma_R = O(N_c^{-1})$, and we take the real and imaginary parts of the expansion of the pole equation around m_R^2, we arrive at

$$\begin{aligned} \mathrm{Re}\,t^{-1}(m_R^2) &= \underbrace{m_R\Gamma_R\left[\frac{m_R\Gamma_R}{2}(\mathrm{Re}\,t^{-1})''_{s=m_R^2} - \sigma'(m_R^2)\right]}_{O(N_c^{-1})} + O(N_c^{-3}), \\ (\mathrm{Re}\,t^{-1})'_{s=m_R^2} &= \underbrace{\frac{\sigma(m_R^2)}{m_R\Gamma_R}}_{O(N_c)} + O(N_c^{-1}). \end{aligned} \tag{2.2}$$

Since the expansion parameter $im_R\Gamma_R \sim 1/N_c$ is purely imaginary, the different orders in the expansion, which are suppressed by the corresponding $1/N_c$ factors, are real or purely imaginary alternatively. Then, when taking the real and imaginary parts of the equation, the different orders are suppressed by two powers of $1/N_c$, as shown in Eqs. (2.2), from where we also see that the inverse amplitude scales as $1/N_c$ instead of as the generic N_c when evaluated at m_R^2. Then, Eqs. (2.1) are obtained noting that the phase shift $\delta(s)$ satisfies $\delta - \pi/2 = -\arctan(\mathrm{Re}\,t^{-1}/\sigma)$, and using Eqs. (2.2) to expand the arctan function in $1/N_c$ powers.

We can now define from Eqs. (2.1) the following adimensional observables,

$$\left.\frac{\frac{\pi}{2} - \mathrm{Re}\,t^{-1}/\sigma}{\delta}\right|_{m_R^2} \equiv \Delta_1 = 1 + \frac{a}{N_c^3}, \qquad -\left.\frac{[\mathrm{Re}\,t^{-1}]'}{\delta'\sigma}\right|_{m_R^2} \equiv \Delta_2 = 1 + \frac{b}{N_c^2}, \tag{2.3}$$

whose value should be one for predominantly $\bar{q}q$ resonances up to $O(1/N_c^3)$ and $O(1/N_c^2)$ corrections, respectively. We have written explicitly the corresponding $1/N_c$ powers in the subleading

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terms, so the coefficients a and b should naturally be $O(1)$ or less. Note that it is relatively simple to make a and b much smaller than one by taking into account higher order contributions of natural size, but very unnatural to make them much larger. In the case of a glueball nature of the resonance, whose mass and width scale as $m_R = O(1)$ and $\Gamma_R = O(1/N_c^2)$, the above derivations can be repeated, but now the subleading corrections are even more suppressed since the width scales as $1/N_c^2$ instead of only $1/N_c$. Then, for a glueball resonance, the observables Δ_1 and Δ_2 satisfy

$$\Delta_1 = 1 + \frac{a'}{N_c^6}, \qquad \Delta_2 = 1 + \frac{b'}{N_c^4}, \tag{2.4}$$

where a' and b' should of natural $O(1)$ size.

In the following section we will calculate these observables to see how well the above predictions for Δ_1 and Δ_2 are fulfilled assuming a $\bar{q}q$ nature (or also glueball for the $f_0(600)$) for the resonances found in elastic of $\pi\pi$ and πK scattering.

3. Results

In Table 1 we show the values of the a and b parameters for the lightest resonances found in $\pi\pi$ and πK scattering, which have been calculated from the data analyses that we detail below. Let us first note that *for the $\rho(770)$ and $K^*(892)$ vector resonances all parameters are of order one or less, as expected for $\bar{q}q$ states*. In contrast, *for the $f_0(600)$ and $K_0^*(800)$ scalar resonances we find that all parameters are larger, by two orders of magnitude, than expected for $\bar{q}q$ states*. This is one of our main results and make the $\bar{q}q$ interpretation of both scalars extremely unnatural.

The data analyses that we have used in each case are the following. For the $\pi\pi$ scattering phase shifts we use the very precise and reliable output of the data analysis in [10] constrained to satisfy Roy equations, once subtracted Roy–like equations (GKPY equations) and forward dispersion relations, which is therefore model independent and specially suited to obtain the $f_0(600)$ pole [11]. This analysis is also in good agreement with previous dispersive result based on Roy equations [12]. For the case of isospin $1/2$ scalar channel of πK scattering, where we find the $K_0^*(800)$, we have also used the rigorous dispersive calculation in [13] that uses Roy-Steiner equations, although in this case we can only provide a central value. For the isospin $1/2$ vector channel of πK scattering, where we find the $K^*(892)$, there are no very precise purely dispersive descriptions of data, so we use unitarized ChPT in the form of the elastic IAM [14]. We have checked that using the IAM for the $\rho(770)$ we obtain results within 50% of the results using the GKPY dispersive representation. Since the $K^*(892)$ is narrower than the $\rho(770)$, the IAM is likely to provide a better approximation than in the $\rho(770)$ case, but even with that 50% uncertainty we can check that the a and b parameters are smaller than one.

	$\rho(770)$	$K^*(892)$	$f_0(600)$	$K_0^*(800)$
a	-0.06 ± 0.01	0.02	-252^{+119}_{-156}	-2527
b	$0.37^{+0.04}_{-0.05}$	0.16	77^{+28}_{-24}	162

Table 1: Normalized coefficients of the $1/N_c$ expansion for different resonances. For $\bar{q}q$ resonances, all them are expected to be of order one or less.

One might argue that, since the first of Eqs. (2.1) comes from the expansion of $\arctan(x) = x - x^3/3 + \ldots$, the correction a/N_c^3 to $\Delta_1 = 1$ is really the cube of a $O(N_c^{-1})$ quantity, $(\tilde{a}/N_c)^3$, where now the coefficient that should be natural is $\tilde{a}$ instead of a. That explains the very small values obtained for the $\rho(770)$ and the $K^*(892)$, that come from $a = \tilde{a}^3/3$, with $\tilde{a} = 0.56 \pm 0.03$ and $\tilde{a} = -0.4$ for the $\rho(770)$ and $K^*(892)$ respectively, which are quite natural values. For the $f_0(600)$ and the $K_0^*(800)$ we obtain $\tilde{a} = 9.1^{+1.3}_{-2.5}$ and -19.6, still rather unnatural values. In the case of Δ_2, where the corrections are only suppressed by $1/N_c^2$ instead of $1/N_c^3$, we do not find this issue, because the b/N_c^2 term is not the square of a natural $1/N_c$ quantity,

$$\frac{b}{N_c^2} = \frac{\mathrm{Re}\,t^{-1}}{\sigma}\left[\frac{\sigma'}{(\mathrm{Re}\,t^{-1})'} - \frac{\mathrm{Re}\,t^{-1}}{\sigma}\right] + O(N_c^{-4}). \tag{3.1}$$

Despite containing a cancellation between two $1/N_c$ terms, its value for the $\rho(770)$ and $K^*(892)$ is rather natural. However, the value for the scalars is almost two orders of magnitude larger than expected for predominantly $\bar{q}q$ states.

In the case of a glueball interpretation of the $f_0(600)$, the coefficients a' and b' from Eqs. (2.4) are even more unnatural, this time too large by three or four orders of magnitude, $a' = -6800^{+3200}_{-4200}$ and $b' = 2080^{+760}_{-650}$. In other words, a very dominant or pure glueball nature for the $f_0(600)$ is very disfavored by the $1/N_c$ expansion. Of course, as in the $\bar{q}q$ case we could worry about the fact that, due to the $\arctan(x) = x - x^3/3 + \ldots$ expansion, the a' should be interpreted as $a' = \tilde{a}'/3$. However, even with that interpretation we would still find $\tilde{a}' = 27^{+5}_{-7}$, again rather unnatural. Once more, in the case of b', its value is genuinely unnatural, disfavoring the glueball interpretation.

Finally, in [8] we also showed that what really happens for the scalars is that they do not even follow the $1/N_c$ expansion of $\bar{q}q$ or glueball states given in Eqs. (2.3) and (2.4). This was done by calculating the $1/N_c$ scaling of the quantities $\Delta_i - 1$ for the different resonances using the Inverse Amplitude Method, where the $1/N_c$ expansion can be implemented through the ChPT low energy constants. We refer however the reader to our original work [8] for further details.

4. Summary

We have reviewed our results in [8] where we study the $1/N_c$ expansion of elastic meson–meson scattering phase shifts around the pole mass of a $\bar{q}q$ or glueball resonance. In particular, we have defined the observables (2.3) and (2.4), whose value is fixed to one up to corrections suppressed by more than one power of $1/N_c$ for $\bar{q}q$ or glueball states. Using very precise dispersive analyses of $\pi\pi$ and πK scattering data we have shown that a $\bar{q}q$ or glueball interpretation for the $f_0(600)$ or $K_0^*(800)$ needs unnaturally large coefficients in the expansion. Thus, a predominant $\bar{q}q$ or glueball nature for these resonances is heavily disfavored by the $1/N_c$ expansion, and this has been shown without extrapolating beyond $N_c = 3$. However, when extrapolating to larger N_c using the IAM, we checked in [8] that the scalars do not follow the pattern of the $1/N_c$ expansion expected for $\bar{q}q$ or glueball states.

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