

## Baryon resonances at large $N_c$ , or Quark Nuclear Physics

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We suggest a new point of view according to which baryon resonances can be understood as collective excitations about intrinsic one-quark excitations in a mean field of definite symmetry. This approach is justified in the limit of large number of colours  $N_c$ , and is similar to the physics of large- $A$  nuclei, hence “quark nuclear physics”. Although in the real world  $N_c$  is only three, we obtain a good agreement with the observed resonance spectrum of light baryons up to 2 GeV, and of lowest charmed baryon multiplets. A by-product of the scheme is the prediction of a new exotic charmed (and bottom) baryons that may be stable against strong decays.

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## 1. Introduction

Baryons from the large- $N_c$  point of view have been studied by quite a number of people who derived many algebraic relations between baryon observables, following from the requirement that physical quantities should have “natural” behaviour in  $N_c$ , see *e.g.* [1, 2, 3] and references therein. In this report, we suggest a simple physical picture that results in those relations, and derive some new ones. A longer paper has been recently published [4], and a detailed report is currently in preparation.

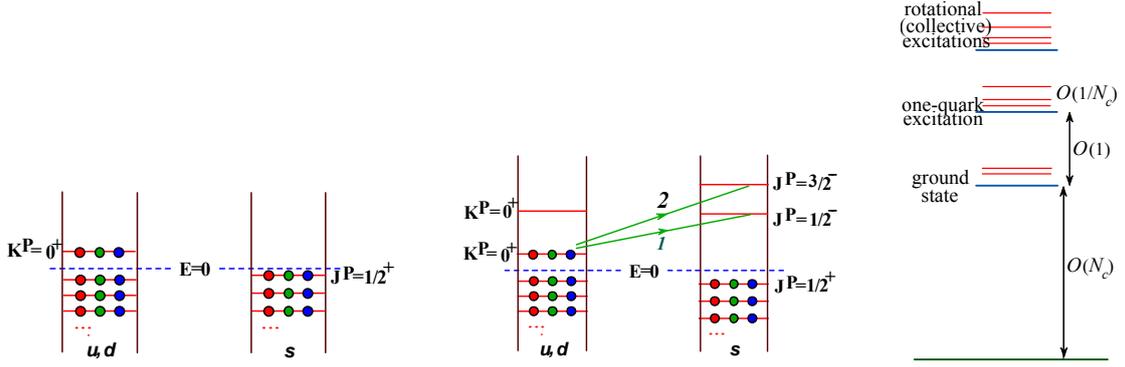
If the number of colours  $N_c$  is large the  $N_c$  quarks constituting a baryon can be considered in a mean (non-fluctuating) field that is stable as  $N_c \rightarrow \infty$  [5]. At the microscopic level quarks experience colour interactions, however gluon field fluctuations are not suppressed if  $N_c$  is large; the mean field can be only ‘colourless’. An example how originally colour interactions transform mathematically into interactions of dynamically massive quarks with mesonic fields is provided by the instanton liquid model, see *e.g.* [6]. A non-fluctuating confining chiral bag model gives another example of a ‘colourless’ mean field. A modern example is the 5- or 6-dimensional ‘gravitational’, plus mesonic, background field in the holographic QCD models. They also need large  $N_c$  for a justification.

The advantage of the large- $N_c$  approach is that at large  $N_c$  baryon physics simplifies considerably. It becomes possible to take into full account the important relativistic and field-theoretic effects that are often ignored. Baryons are not just three (or  $N_c$ ) quarks but contain additional quark-antiquark pairs, as it is well known experimentally. The number of antiquarks in baryons is, theoretically, also proportional to  $N_c$  [7], which means that antiquarks cannot be obtained from adding one or two mesons to a baryon: one needs  $\mathcal{O}(N_c)$  mesons to explain  $\mathcal{O}(N_c)$  antiquarks, implying in fact a classical mesonic field.

If the mean field at large  $N_c$  is a reality, one can consider quarks in that mean field – similarly to nucleons in the mean field of large- $A$  nuclei. Generally, quarks obey the Dirac equation in the background field, that may be in fact non-local. All intrinsic quark Dirac levels in the mean field are stable in  $N_c$ . All negative-energy levels should be filled in by  $N_c$  quarks in the antisymmetric state in colour, corresponding to the zero baryon number state. Filling in the lowest positive-energy level by  $N_c$  ‘valence’ quarks makes a baryon, see Fig. 1, left. Exciting higher quark levels or making particle-hole excitations produces baryon resonances [8, 9], see Fig. 1, middle. The baryon mass is  $\mathcal{O}(N_c)$ , and the excitation energy is  $\mathcal{O}(1)$ . When one excites one quark the change of the mean field is  $\mathcal{O}(1/N_c)$  that can be neglected to the first approximation.

Moreover, if one replaces one light ( $u, d$  or  $s$ ) quark in light baryons by a heavy ( $c, b$ ) one, as in charmed or bottom baryons, the change in the mean field is also  $\mathcal{O}(1/N_c)$ . Therefore, the spectrum of heavy baryons is directly related to that of light baryons [9, 10]. In light baryons made of  $u, d, s$  quarks, we first take the chiral limit of zero quark masses. It implies that all resonances appear as degenerate  $SU(3)_{\text{flav}}$  multiplets that are later split by nonzero  $m_s$ .

Next, we argue that the mean field in baryons has a definite symmetry, namely it breaks spontaneously the symmetry under separate  $SU(3)_{\text{flav}}$  and  $SO(3)_{\text{space}}$  rotations but does not change under simultaneous  $SU(2)_{\text{iso+space}}$  rotations in ordinary space and a compensating rotation in isospace [8, 9]. Similarly, in nuclear physics the  $SO(3)_{\text{space}}$  symmetry is broken spontaneously in the ground state since most of the heavy nuclei are elliptical-, not spherical-symmetric.



**Figure 1:** *Left:* Filling in  $u, d$ - and, separately,  $s$ -quark levels for the ground-state light baryons. *Middle:* Examples of one-quark excitations. *Right:* Hierarchy of scales in large- $N_c$  baryons.

Spontaneous breaking of  $SU(3)_{\text{flav}}$  means, in particular, that one-quark levels for  $u, d$  quarks on the one hand, and for  $s$  quarks on the other, are totally, 100% different even in the chiral limit  $m_s \rightarrow 0$ . The latter levels are characterized by total angular momentum  $\mathbf{J} = \mathbf{L} + \mathbf{S}$ , and parity, whereas the former are characterized by the grand spin  $\mathbf{K} = \mathbf{J} + \mathbf{T}$  where  $\mathbf{T}$  is the isospin, and parity.

The original  $SU(3)_{\text{flav}} \times SO(3)_{\text{space}}$  symmetry is restored by the (quantized) rotation of the mean field in flavour and ordinary spaces. It implies that each intrinsic quark state, be it the ground state or a one-quark excitation in the Dirac spectrum, generates a band of  $SU(3)$  multiplets appearing as collective rotational excitations of a given intrinsic state. The quantum numbers of those multiplets, their total number and the  $\mathcal{O}(1/N_c)$  splittings between them are unequivocally dictated by the symmetry of the mean field, see Fig. 1, right. Assuming the  $SU(2)_{\text{iso+space}}$  symmetry of the mean field, we obtain exactly the spin and flavour multiplets that are observed in Nature.

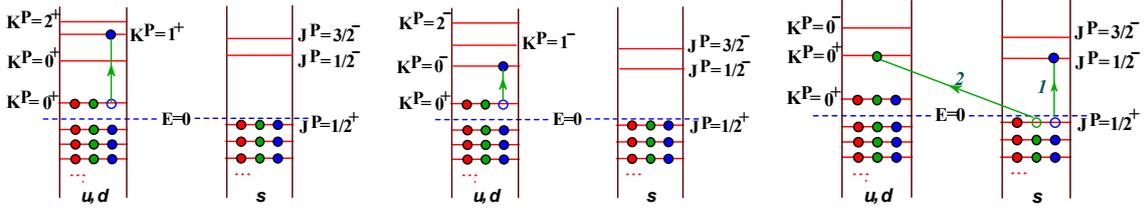
All properties of baryon resonances belonging to one band associated with a given one-quark excitation are related by symmetry. For example, the splittings inside several  $SU(3)$  multiplets as due to the nonzero  $m_s$ , are related to each other: these relations are satisfied with high accuracy, in some cases better than the Gell-Mann–Okubo relations for separate  $SU(3)$  multiplets.

At this time we do not consider any specific dynamical model but concentrate mainly on symmetry. A concrete dynamical model would say what is the intrinsic relativistic quark spectrum in baryons. Instead of calculating the intrinsic Dirac spectrum of quarks from a model, we extract it from the experimentally known baryon spectrum by interpreting baryon resonances as collective excitations about the ground state and about the one-quark transitions. However, we show in Ref. [4] that the needed intrinsic quark spectrum can be obtained from a natural choice of the mean field satisfying the  $SU(2)_{\text{iso+space}}$  symmetry.

In summary, we show that it is possible to obtain a realistic spectrum of baryon resonances up to 2 GeV, starting from the large- $N_c$  limit. The lowest  $SU(3)$  multiplets of charmed baryons also fit nicely into this universal picture [9, 10].

## 2. Baryons made of $u, d, s$ quarks

The ground-state baryons are obtained from the level-filling scheme shown in Fig. 1, left. The



**Figure 2:** *Left:* The origin of parity-plus light resonances. *Middle:* The origin of parity-minus light baryons. *Right:* Pentaquarks are particle-hole excitations of a negative-energy  $s$  quark. The exotic parity-plus  $\Theta^+$  (recently seen again at JLab [11]) corresponds to a non-diagonal or ‘Gamov–Teller’ transition 2.

quantization of the rotations (restoring the original flavour and space symmetries) of the  $K^P = 0^+$  valence level gives rise to the lowest baryons multiplets,  $(\mathbf{8}, 1/2^+)$  and  $(\mathbf{10}, 3/2^+)$ .

The singlets,  $(\mathbf{1}, 1/2^-)$  and  $(\mathbf{1}, 3/2^-)$  (the  $\Lambda$  hyperons), are obtained from ‘‘Gamov–Teller’’ excitations of the  $s$ -quark levels, see Fig. 1, middle. In principle, these two transitions entail rotational bands, however the rotational splitting of higher rotational excitations are not  $\mathcal{O}(1/N_c)$  but  $\mathcal{O}(1)$  [10], therefore all states except the lowest-mass singlets must be ignored in this approach.

The parity-plus resonances are obtained from the  $u, d$  one-quark excitations  $0^+ \rightarrow 0^+$ ,  $0^+ \rightarrow 1^+$  and  $0^+ \rightarrow 2^+$ , whereas parity-minus resonances can be obtained from the  $u, d$  transitions  $0^+ \rightarrow 0^-$ ,  $0^+ \rightarrow 1^-$  and  $0^+ \rightarrow 2^-$ , see [4]. All known baryon resonances below 2 GeV can be nicely fit into the rotational states stemming from those one-quark excitations. There are no resonances from the PDG tables that are left unaccounted for, and there are no extra states from the theory side, except the  $\Delta(3/2^+)$  resonance coming from the rotational excitation on top of the  $0^+ \rightarrow 1^+$  transition. It is therefore our prediction.

### 3. Splittings inside $SU(3)$ multiplets

Properties of baryon resonances, such as the couplings, partial decay widths, mass splittings, *etc.*, arising from one rotational band about a given one-quark excitation are related to each other by symmetry. Typically, the observables for resonances from *different* multiplets are expressed through a small number of constants, which leads to numerical relations between the observables, that can be tested against the data.

For example, in the linear order in  $m_s$  the splittings inside an octet are determined by two constants  $\mu_{1,2}^{(8)}$  while those in a decuplet are determined by only one constant  $\mu^{(10)}$ . We write the masses of individual members of the octets,

$$\mathcal{M}_N = M_8 - \frac{7}{4}\mu_1^{(8)} - \mu_2^{(8)}, \quad \mathcal{M}_\Lambda = M_8 - \mu_1^{(8)}, \quad \mathcal{M}_\Sigma = M_8 + \mu_1^{(8)} \quad \mathcal{M}_\Xi = M_8 + \frac{3}{4}\mu_1^{(8)} + \mu_2^{(8)},$$

and the masses of the members of the decuplets,

$$\mathcal{M}_\Delta = M_{10} - \mu^{(10)}, \quad \mathcal{M}_\Sigma = M_{10}, \quad \mathcal{M}_\Xi = M_{10} + \mu^{(10)} \quad \mathcal{M}_\Omega = M_{10} + 2\mu^{(10)}.$$

From this parametrization one gets automatically the Gell-Mann–Okubo relations for the octets and decuplets separately. However, the symmetry breaking pattern  $SU(3)_{\text{flav}} \times SO(3)_{\text{space}} \rightarrow SU(2)_{\text{iso+space}}$  that we advocate enables one to relate the splitting parameters related to *different* multiplets with

various spins. In particular, we obtain the following new relations for the different multiplets stemming from the  $0 \rightarrow 1$  and  $0 \rightarrow 2$  one-quark transitions, respectively,

$$7\mu^{(10)} \left( \frac{1}{2} \right) + 3\mu_2^{(8)} \left( \frac{3}{2} \right) = 10\mu^{(10)} \left( \frac{3}{2} \right),$$

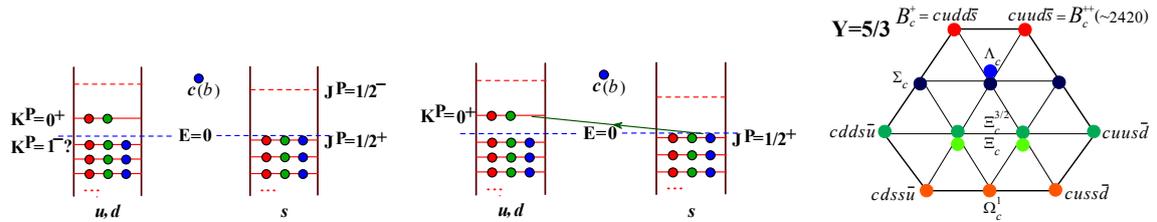
$$5\mu_2^{(8)} \left( \frac{5}{2} \right) + 11\mu^{(10)} \left( \frac{3}{2} \right) = 16\mu^{(10)} \left( \frac{5}{2} \right)$$

(in the parentheses we indicate the spin of the multiplet). Fitting the masses of individual multiplet members by the parameters  $\mu$  one can find them for various multiplets and then plug into the above relations. It turns out that these very nontrivial relations are satisfied to an accuracy of 1-2%! It gives a strong support to the scheme suggested. Another support comes from the fact that the  $\mathcal{O}(1/N_c)$  splittings between the *centers* of the  $SU(3)$  multiplets are expressed through two moments of inertia, and phenomenologically many splittings can be described, to a fair accuracy, with the same moments of inertia. Surprisingly, approximately the same moments of inertia are needed to explain the splittings in heavy baryons.

#### 4. Heavy baryons

If one of the light quarks in a light baryon is replaced by a heavy  $b$  or  $c$  quark, there are still  $N_c - 1$  light quarks left. At large  $N_c$ , they form *the same* mean field as in light baryons, with the same sequence of Dirac levels, up to  $1/N_c$  corrections. The heavy quark contributes to the mean  $SU(3)_{\text{flav}}$  symmetric field but it is a  $1/N_c$  correction, too. It means that at large  $N_c$  one can *predict* the spectrum of the  $Qq \dots q$  (and  $Qq \dots qq\bar{q}$ ) baryons from the spectrum of light baryons.

The filling of Dirac levels for the ground-state  $c$  (or  $b$ ) baryon is shown in Fig. 3, left: there is a hole in the  $0^+$  shell for  $u, d$  quarks as there are only  $N_c - 1$  quarks there, in an antisymmetric state in colour. Adding the heavy quark makes the full state ‘colourless’.



**Figure 3:** *Left:* Ground-state charmed (or bottom) baryon with a hole in the valence  $u, d$  level. *Middle:* A particle-hole excitation of the  $s$ -quark from the negative-energy level. *Right:* The corresponding  $\bar{\mathbf{15}}$ -plet of heavy pentaquarks. The *Beta* baryons in the upper line may be stable against strong decays.

As in the case of light baryons, the filling scheme by itself does not tell us what are the quantum numbers of the state: they arise from quantizing the  $SU(3)_{\text{flav}}$  and  $SO(3)_{\text{space}}$  rotations of the given filling scheme. The quantization of the rotations about the ground state gives three  $SU(3)$  multiplets:  $(\bar{\mathbf{3}}, 1/2^+)$ ,  $(\mathbf{6}, 1/2^+)$  and  $(\mathbf{6}, 3/2^+)$ . These are exactly the lightest charmed multiplets observed in Nature. The centers of the last two multiplets are split by  $1/N_c^2$  and  $1/m_c$  corrections, and indeed the splitting is small (67 MeV), whereas the splitting between the centers of the  $\mathbf{6}$ 's and

the  $\bar{\mathbf{3}}$  is  $\mathcal{O}(1/N_c)$  and is in good accordance with the splitting between *light* rotational states, thus supporting the main idea [9, 10].

Finally, the hole in the ground-state  $0^+$  shell can be filled in either by exciting a  $u, d$  quark from lower lying shells, or by non-diagonal ‘Gamov-Teller’ transition from the highest filled  $s$  quark shell, see Fig. 3, middle. In the second case the corresponding charmed baryon resonance will be a pentaquark exotic, actually belonging to a  $\bar{\mathbf{15}}$ -plet from the  $SU(3)$  point of view, Fig. 3, right.

The masses of the exotic “Beta baryons”,  $\mathcal{B}_c^{++} = cuud\bar{s}$ ,  $\mathcal{B}_c^+ = cudd\bar{s}$ , and  $\mathcal{B}_b^+ = buud\bar{s}$ ,  $\mathcal{B}_b^0 = budd\bar{s}$  have been estimated in Ref. [9, 10] to be quite light,  $m_{\mathcal{B}} \approx 2420 \text{ MeV}$ , and hence they may be well stable against strong decays! See Ref. [10] for the discussion of the  $\bar{\mathbf{15}}$ -plet of the exotic charmed (or bottom) pentaquarks and of the possibilities of their observation at LHC and b-factories.

## 5. Conclusions

We have presented a new unified picture of baryon resonances, light and heavy, that is in fact quite similar to that of nuclei at large  $A$  whose rôle is played by large  $N_c$ . While  $N_c$  is only three in the real world, we see that the imprint of the large- $N_c$  picture is clearly visible. A by-product of this study is the prediction of a relatively light exotic charmed pentaquark, the *Beta*.

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