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The study of $D_{s1}(2460)$ and $D_{s1}(2536)$ mixing

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In this proceeding, we report our recent study of the mixing mechanism between $D_{s1}(2460)$ and $D_{s1}(2536)$. On the basis of Godfrey-Isgur model, we consider the D^*K hadron loop effect on the $^3P_1$ and $^1P_1$ $c\bar{s}$ states. We construct the propagator matrix of these two-state system, from which we can extract the poles as well as the mixing angles. Through this method, we simultaneously determine the masses, widths and mixing angle of these two physical states and the results agree well with the experimental measurement. Besides the mass shift, we also find that the hadron loop effects can cause a significant shift for the mixing angles from the value determined in the heavy quark symmetry limit.

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1. Introduction

Last year, BaBar Collaboration updated their measurement of the mass and width of $D_{s1}(2536)$ meson [1]. Before this measurement, only an up limit of the width had been set [2]. This new measurement should be useful for the study of the internal structures of $D_{s1}(2536)$ and its axial-vector partner $D_{s1}(2460)$. Thus, it initiates a lot of theoretical interests in the mixing of these two states.

We recall first some observations concerning the P-wave D_s state. Theoretically, the Godfrey-Isgur (GI) model [3] has made a great success in providing an overall description of charmonium and charmed meson spectra. However, it is also realized that there exist apparent discrepancies between theory and experiment when a state is located close to an open threshold. In the D_s spectrum, the lowest S-wave states D_s and D_s^* can be well explained by the GI model. But for those four P-wave states, i.e. $D_{s0}(2317)$, $D_{s1}(2460)$, $D_{s1}(2536)$, and $D_{s2}(2573)$, there are significant discrepancies between the quark model prediction and experimental observation. For instance, the masses of $D_{s0}(2317)$ and $D_{s1}(2460)$ are about 100 MeV lower than the quark model prediction, and a dynamical reason for such large mass shifts may be due to the DK and D^*K thresholds, respectively [4, 5, 6, 7].

State mixing makes these two axial-vector states $D_{s1}(2460)$ and $D_{s1}(2536)$ more interesting. Such a heavy-light quark-antiquark system could be an ideal place to test the heavy quark symmetry. Namely, in the heavy quark symmetry limit, $D_{s1}(2460)$ is a pure $j = 1/2$ state and couples to D^*K channel through an S wave, while $D_{s1}(2536)$ is a pure $j = 3/2$ state and couples to D^*K channel through a D wave. The heavy quark symmetry is expected to be broken at the order of $1/m_c$. Explicit introduction of a broken mechanism for the heavy quark symmetry in these two axial-vector states is thus necessary.

Since both $D_{s1}(2460)$ and $D_{s1}(2536)$ are not charge conjugation eigenstates, and they both have strong S-wave couplings to D^*K, $D_s^*\eta$, and DK^*, we can then assume $D_{s1}(2460)$ and $D_{s1}(2536)$ to be mixed states of the pure ${}^3P_1$ and ${}^1P_1$ $c\bar{s}$ states through intermediate meson loops. We mention that the D^*K channel plays a dominant role in such a mechanism. By calculating the intermediate meson loop transitions, we can simultaneously determine the masses, widths and mixing angles of the two physical states. A similar mechanism has been applied to the $a_0(980) - f_0(980)$ mixing [8].

We mention that many other scenarios are also proposed to explain the experimental data for $D_{s1}(2460)$ and $D_{s1}(2536)$, such as D^*K molecule, tetra-quark state and dynamically generated state [9, 10, 11, 12]. Besides the mass shift, the hadron loop effect as a general mechanism can also affect the decay and production process. For instance, it has been used to evade the helicity selection rule in $\chi_{c1} \to VV$ and $\chi_{c2} \to VP$ [13], account for the large branch ratio of the non-$D\bar{D}$ decay of $\psi(3770)$ [14, 15], explain the large isospin violation process $\eta(1405/1475) \to 3\pi$ [16], and predict a direct measurement of open threshold effects in $e^+e^- \to J/\psi\pi^0$ [17].

2. Mixing through intermediate meson loops

In this part, we show our basic formulas for the mixing scheme through intermediate meson loops. If two states $|a\rangle$ and $|b\rangle$ can transit to each other through hadron loops, the propagator of such a two-state system can be written as a 2×2 matrix G_{ab} [18]. In general, the propagator matrix

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has two poles in the complex energy plane, which correspond to the physical states. From the pole position, we can determine the masses and widths of the physical states. The physical states $|A\rangle$ and $|B\rangle$ should be a mixture of $|a\rangle$ and $|b\rangle$,

$$\begin{pmatrix} |A\rangle \\ |B\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & \cos\theta \end{pmatrix} \begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} = R(\theta,\phi) \begin{pmatrix} |a\rangle \\ |b\rangle \end{pmatrix} \tag{2.1}$$

where θ is the mixing angle, and ϕ is a possible relative phase between $|a\rangle$ and $|b\rangle$. Then the propagator matrix of $|A\rangle$ and $|B\rangle$ becomes $G_{AB} = RG_{ab}R^{\dagger}$ and should be diagonal. So we can obtain the mixing parameters (θ,ϕ) by diagonalizing G_{ab}.

Here, we show the propagator matrix. For the spin-0 states, i.e. scalar or pseudo-scalar, the propagator matrix is

$$G_{ab} = \frac{1}{D_a D_b - D_{ab}^2} \begin{pmatrix} D_b & D_{ab} \\ D_{ba} & D_a \end{pmatrix} , \tag{2.2}$$

where D_a and D_b are the denominators of the single propagators of $|a\rangle$ and $|b\rangle$, respectively, and D_{ab} is the mixing term. For the spin-1 states, i.e. vector and axial-vector, the propagator matrix is

$$G_{ab}^{\mu\nu} = iP^{\mu\nu} \frac{\bar{G}_{ab}(s)}{\det \bar{G}_{ab}(s)} + iQ^{\mu\nu} \frac{G_{ab}^L}{\det G_{ab}^L} , \tag{2.3}$$

where $P^{\mu\nu} \equiv g^{\mu\nu} - p^\mu p^\nu/p^2$ and $Q^{\mu\nu} \equiv p^\mu p^\nu/p^2$ are the transverse and longitudinal projector, respectively. The poles and mixing angles are only related to the transverse part and

$$\bar{G}_{ab}(s) \equiv M_{ab}^2 - \delta_{ab} s = \begin{pmatrix} m_b^2 + \Pi_b(s) - s & -\Pi_{ab}(s) \\ -\Pi_{ab}(s) & m_a^2 + \Pi_a(s) - s \end{pmatrix} , \tag{2.4}$$

where Π_a, Π_b and Π_{ab} are the transverse coefficients of the self-energy functions and the mixing term, respectively.

For the case of $D_{s1}(2460)$ and $D_{s1}(2536)$, we apply the following mixing scheme

$$\begin{aligned} |D_{s1}(2460)\rangle &= \cos\theta|^3P_1\rangle - \sin\theta e^{i\phi}|^1P_1\rangle \\ |D_{s1}(2536)\rangle &= \sin\theta e^{-i\phi}|^3P_1\rangle + \cos\theta|^1P_1\rangle . \end{aligned} \tag{2.5}$$

In the heavy quark limit, the ideal mixing angle is $\theta_0 = 35.26^\circ$ in our convention. Considering parity conservation and the OZI rule, the important intermediate states that can couple to $D_{s1}(2460)$ and $D_{s1}(2536)$ are D^*K, $D_s^*\eta$ and DK^*. To compute these diagrams, we need to know the couplings of the vertices as well as form factors to remove the ultraviolet (UV) divergences in the loop integrals.

3. Result and discussion

At hadronic level, the Axial-vector-Vector-Pseudoscalar vertex can be written as $i(g_S g^{\mu\nu} + g_D p^\mu p^\nu)$, where g_S and g_D are the S and D-wave couplings. Near threshold, the D-wave couplings are suppressed due to the small momentum. So we only consider the S-wave couplings, and use the chiral quark model to evaluate g_S [19, 20, 21, 22]. In the chiral quark model, the light pseudoscalars and light vectors are treated as chiral fields.

Our result for the S-wave couplings are shown in Fig. 1. We can see that they are insensitive to the initial meson mass. The couplings to D^*K are strong, the couplings to $D^*\eta$ are rather weak, and the couplings to DK^* are vanishing as a leading approximation. To remove the UV divergences in the loop integrals, we modify the non-relativistic (NR) exponential form factors in the quark model to a covariant form, i.e. $\exp(-q_1^2/4\alpha^2) \to \exp(q^2 - m^2/\Lambda^2)$, where Λ is the cutoff energy and can be fixed by the chiral quark model.

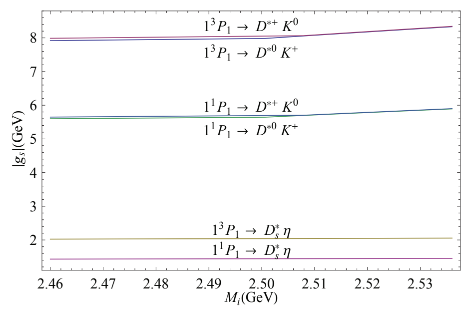

Figure 1: The absolute values of coupling g_S as a function of the initial meson mass M_i [18].

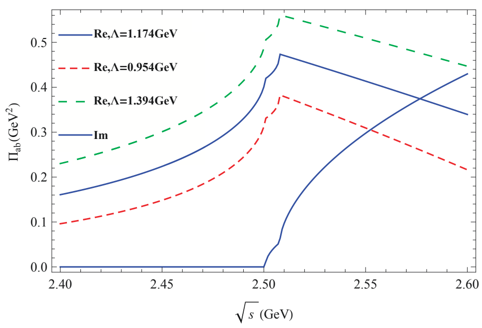

Figure 2: The mixing term Π_{ab} with different cutoff values for the exponential form factor.

In the propagator matrix Eqs. (2.3-2.4), what we need to know are the self-energy functions Π_a and Π_b, and the mixing term Π_{ab}. They are found to be proportional to each other [18]. In Fig. 2 we show the energy dependence of Π_{ab}. The blue lines are the results with cutoff $\Lambda = 1.174$ GeV fixed by the chiral quark model. We can see that both the real and the imaginary part have two kinks due to the charged and neutral D^*K thresholds. Below the thresholds the imaginary part is zero, while above the thresholds it increases quickly. At the energy near 2460 MeV, only the real part contributes. At the energy near 2536 MeV, both the real and imaginary parts have contributions. We also vary Λ to see the cutoff dependence and find that the real part is sensitive to Λ, while the imaginary part is unchanged.

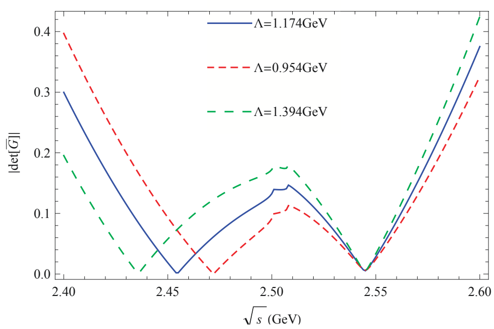

Figure 3: Pole structures highlighted by the zero values of $\det[\bar{G}]$ in the propagator matrix.

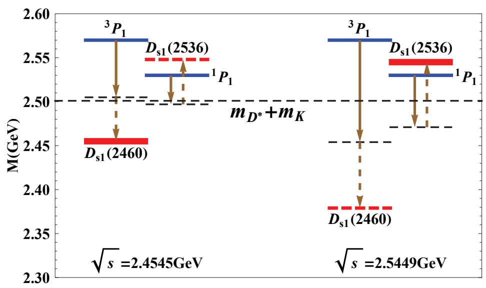

Figure 4: Schematic plot for the mass-shift procedure [18].

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The poles in the propagator matrix $G_{ab}^{\mu\nu}$ are equal to the zero points in $\bar{G}_{ab}$ as shown in Fig. 3. For the bare $c\bar{s}$ masses, we adopt the values from the GI model, $m[^3P_1] = 2.57$ GeV and $m[^1P_1] =$

2.53 GeV [3]. From Fig. 3 we can see two poles: one is near 2460 MeV and the other is near 2536 MeV. The higher pole is insensitive to the cutoff while the lower pole turns out to be rather sensitive.

We list the masses and widths in Table 1 with $\Lambda = 1.174$ GeV. The results agree with experiment, which indicate that the exponential form factor from the quark model can give a good estimate of the real part. The detailed mass shift procedures of these two physical states are demonstrated in Fig. 4.

Table 1: Masses and widths obtained from the pole analysis [18].

$[m - i\frac{\Gamma}{2}]$ (MeV)	$D_{s1}(2460)$	$D_{s1}(2536)$
Π^1_{ab}	2454.5	$2544.9 - 1.0i$
$\Pi^1_{ab} + \Pi^2_{ab}$	2455.8	$2544.9 - 1.1i$
Experiment	2459.5	$2535.08 - 0.46i$

Besides the masses and widths, we can also extract the mixing parameters and the results are shown in Table 2. We can see that the mixing angles are different for these two physical states at their own mass shells. It is also interesting to see the deviation of the mixing angles from the heavy quark limit. For $D_{s1}(2460)$ the deviation is large $\Delta\theta = 12.3^\circ$, and for $D_{s1}(2536)$ the deviation is relatively small $\Delta\theta = 4.4^\circ$. The new experimental data from BaBar put a strong constraint on the mixing angle of $D_{s1}(2536)$, but with two solutions, i.e. $\theta_1 = 32.1^\circ$ or $\theta_2 = 38.4^\circ$ [18], which are symmetric to the ideal mixing angle. It shows that our theoretical analysis favors the bigger one.

Table 2: The mixing angle θ and relative phase ϕ extracted at the two poles in those three diagonalization schemes [18].

$\{\theta,\phi\}[^\circ]$	$D_{s1}(2460)$			$D_{s1}(2536)$		
	I	II	III	I	II	III
Π^1_{ab}	{47.5, 0}	{47.5, 0}	{47.5, 0}	{39.7, −6.4}	{39.7, 6.4}	{39.7, 0}
$\Pi^1_{ab} + \Pi^2_{ab}$	{47.6, 0}	{47.6, 0}	{47.6, 0}	{39.8, −6.5}	{39.8, 6.5}	{39.7, 0}

4. Summary

When taking into account the D^*K loop corrections in the Godfrey-Isgur model, we can explain the masses and widths of the physical states $D_{s1}(2460)$ and $D_{s1}(2536)$, and extract their mixing angles. We also find that when there are strong S-wave coupling channels, the hadron loop corrections can cause both large mass shifts from the quark model prediction and large mixing angle shifts from the heavy quark limit. Also, from our calculation we can see that the exponential form factor from the quark model can give a good estimate of the real part of the meson loops. Our study may provide further insights into the role of hadron loops as an important unquenched correction to the potential quark model.

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References

[1] J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **83**, 072003 (2011) [arXiv:1103.2675 [hep-ex]].

[2] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G G **37**, 075021 (2010).

[3] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).

[4] Y. .A. Simonov and J. A. Tjon, Phys. Rev. D **70**, 114013 (2004) [hep-ph/0409361].

[5] A. M. Badalian, Y. .A. Simonov and M. A. Trusov, Phys. Rev. D **77**, 074017 (2008) [arXiv:0712.3943 [hep-ph]].

[6] S. Coito, G. Rupp and E. van Beveren, Phys. Rev. D **84**, 094020 (2011) [arXiv:1106.2760 [hep-ph]].

[7] Z. -Y. Zhou and Z. Xiao, Phys. Rev. D **84**, 034023 (2011) [arXiv:1105.6025 [hep-ph]].

[8] J. -J. Wu, Q. Zhao and B. S. Zou, Phys. Rev. D **75**, 114012 (2007) [arXiv:0704.3652 [hep-ph]].

[9] F. E. Close and E. S. Swanson, Phys. Rev. D **72**, 094004 (2005) [hep-ph/0505206].

[10] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D **68**, 054006 (2003) [hep-ph/0305025].

[11] A. Faessler, T. Gutsche, V. E. Lyubovitskij and Y. -L. Ma, Phys. Rev. D **76**, 114008 (2007) [arXiv:0709.3946 [hep-ph]].

[12] F. -K. Guo, P. -N. Shen and H. -C. Chiang, Phys. Lett. B **647**, 133 (2007) [hep-ph/0610008].

[13] X. -H. Liu and Q. Zhao, Phys. Rev. D **81**, 014017 (2010) [arXiv:0912.1508 [hep-ph]].

[14] Y. -J. Zhang, G. Li and Q. Zhao, Phys. Rev. Lett. **102**, 172001 (2009) [arXiv:0902.1300 [hep-ph]].

[15] X. Liu, B. Zhang and X. Q. Li, Phys. Lett. B **675**, 441 (2009) [arXiv:0902.0480 [hep-ph]].

[16] J. -J. Wu, X. -H. Liu, Q. Zhao and B. -S. Zou, Phys. Rev. Lett. **108**, 081803 (2012) [arXiv:1108.3772 [hep-ph]].

[17] Q. Wang, X. -H. Liu and Q. Zhao, Phys. Rev. D **84**, 014007 (2011) [arXiv:1103.1095 [hep-ph]].

[18] X. -G. Wu and Q. Zhao, Phys. Rev. D **85**, 034040 (2012) [arXiv:1111.4002 [hep-ph]].

[19] A. Manohar and H. Georgi, Nucl. Phys. B **234**, 189 (1984).

[20] D. O. Riska and G. E. Brown, Nucl. Phys. A **679**, 577 (2001) [nucl-th/0005049].

[21] X. -H. Zhong and Q. Zhao, Phys. Rev. D **78**, 014029 (2008) [arXiv:0803.2102 [hep-ph]].

[22] X. -H. Zhong and Q. Zhao, Phys. Rev. D **81**, 014031 (2010) [arXiv:0911.1856 [hep-ph]].