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# Exclusive semileptonic decays of ground-state $cb$ baryons driven by a $c \to s, d$ quark transition

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We evaluate semileptonic decays of spin-1/2 and spin-3/2 doubly heavy $cb$ baryons. The decays are driven by a $c \to s, d$ transition at the quark level. We check our results for the form factors against heavy quark spin symmetry constraints obtained in the limit of very large heavy quark masses and near zero recoil.

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## 1. Semileptonic decay widths

The total decay width for semileptonic $c \to l$ transitions, with $l = s, d$, is given by

$$\Gamma = |V_{cl}|^2 \frac{G_F^2}{8\pi^4} \frac{M'^2}{M} \int \sqrt{\omega^2 - 1}\, \mathscr{L}^{\alpha\beta}(q) \mathscr{H}_{\alpha\beta}(P,P')\, d\omega, \tag{1.1}$$

where $|V_{cl}|$ is the modulus of the corresponding CKM matrix element for a semileptonic $c \to l$ decay ($|V_{cs}| = 0.97345$ and $|V_{cd}| = 0.2252$ [1]), $G_F = 1.16637(1) \times 10^{-11}$ MeV$^{-2}$ [1] is the Fermi decay constant, $P, M$ ($P', M'$) are the four-momentum and mass of the initial (final) baryon, $q = P - P'$ and $\omega$ is the product of the initial and final baryon four-velocities $\omega = v \cdot v' = \frac{P}{M} \cdot \frac{P'}{M'} = \frac{M^2 + M'^2 - q^2}{2MM'}$. In the decay, $\omega$ ranges from $\omega = 1$, corresponding to zero recoil of the final baryon, to a maximum value that, neglecting the neutrino mass, is given by $\omega = \omega_{\max} = \frac{M^2 + M'^2 - m^2}{2MM'}$, which depends on the transition and where $m$ is the final charged lepton mass. Finally $\mathscr{L}^{\alpha\beta}(q)$ is the leptonic tensor after integrating in the lepton momenta. It can be cast as

$$\mathscr{L}^{\alpha\beta}(q) = A(q^2)\, g^{\alpha\beta} + B(q^2)\, \frac{q^\alpha q^\beta}{q^2}, \tag{1.2}$$

where explicit expressions for the scalar functions $A(q^2)$ and $B(q^2)$ can be found in Eqs. (3) and (4) of Ref. [2].

The hadron tensor $\mathscr{H}_{\alpha\beta}(P,P')$ is given by

$$\mathscr{H}^{\alpha\beta}(P,P') = \frac{1}{2J+1} \sum_{r,r'} \left\langle B', r'\, \vec{P}' \middle| J_{cl}^{\alpha}(0) \middle| B, r\, \vec{P} \right\rangle \left\langle B', r'\, \vec{P}' \middle| J_{cl}^{\beta}(0) \middle| B, r\, \vec{P} \right\rangle^*, \tag{1.3}$$

with $J$ the initial baryon spin, $\left|B, r\ \vec{P}\right\rangle$ $\left(\left|B', r'\ \vec{P}'\right\rangle\right)$ the initial (final) baryon state with three-momentum $\vec{P}$ ($\vec{P}'$) and spin third component $r$ ($r'$) in its center of mass frame. Baryon states are normalized such that $\left\langle B, r'\ \vec{P}' \middle| B, r\ \vec{P}\right\rangle = 2E\,(2\pi)^3\, \delta_{rr'}\, \delta^3(\vec{P} - \vec{P}')$, with $E$ the baryon energy for three-momentum $\vec{P}$. Our states are constructed in Appendix A of Ref. [3]. Finally, $J_{cl}^{\mu}(0) = \overline{\Psi}_l(0)\gamma^\mu(1 - \gamma_5)\Psi_c(0)$ is the $c \to l$ charged weak current.

For the actual calculation of the decay width we parameterize the hadronic matrix elements in terms of form factors, which are functions of $\omega$ or equivalently of $q^2$. The different form factor decomposition that we use are given in the following.

1. $1/2 \to 1/2$ transitions.
   Here we take the commonly used decomposition in terms of three vector $F_1, F_2, F_3$ and three axial $G_1, G_2, G_3$ form factors

$$\begin{aligned}\left\langle B'(1/2), r'\ \vec{P}' \middle| J_{cl}^{\mu}(0) \middle| B(1/2), r\ \vec{P}\right\rangle = \bar{u}_{r'}^{B'}(\vec{P}')\Big\{ &\gamma^\mu \left[F_1(\omega) - \gamma_5 G_1(\omega)\right] + v^\mu \left[F_2(\omega) - \gamma_5 G_2(\omega)\right] \\ &+ v'^\mu \left[F_3(\omega) - \gamma_5 G_3(\omega)\right] \Big\} u_r^B(\vec{P}).\end{aligned} \tag{1.4}$$

   The $u_r$ are Dirac spinors normalized as $(u_{r'})^\dagger u_r = 2E\, \delta_{rr'}$.

2. $1/2 \to 3/2$ transitions.
   In this case we follow Llewellyn Smith [4] to write

$$\left\langle B'(3/2), r'\vec{P}' \,|\, \overline{\Psi}_l(0)\gamma^\mu(1-\gamma_5)\Psi_c(0)\,|\, B(1/2), r\vec{P}\right\rangle = \bar{u}^{B'}_{\lambda\, r'}(\vec{P}')\,\Gamma^{\lambda\mu}(P,P')\,u^B_r(\vec{P}),$$

$$\Gamma^{\lambda\mu}(P,P') = \left[\frac{C_3^V}{M}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C_4^V}{M^2}(g^{\lambda\mu}q\cdot P' - q^\lambda P'^\mu) + \frac{C_5^V}{M^2}(g^{\lambda\mu}q\cdot P - q^\lambda P^\mu) + C_6^V g^{\lambda\mu}\right]\gamma_5$$
$$+\left[\frac{C_3^A}{M}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) + \frac{C_4^A}{M^2}(g^{\lambda\mu}q\cdot P' - q^\lambda P'^\mu) + C_5^A g^{\lambda\mu} + \frac{C_6^A}{M^2}q^\lambda q^\mu\right]. \tag{1.5}$$

   Here $u^{B'}_{\lambda\, r'}$ is the Rarita-Schwinger spinor of the final spin 3/2 baryon normalized such that $(u^{B'}_{\lambda\, r'})^\dagger u^{B'\,\lambda}_r = -2E'\,\delta_{rr'}$, and we have four vector ($C^V_{3,4,5,6}(\omega)$) and four axial ($C^A_{3,4,5,6}(\omega)$) form factors. Within our model we shall have that $C_5^V(\omega) = C_6^V(\omega) = C_3^A(\omega) = 0$.

3. $3/2 \to 1/2$ transitions.
   Similar to the case before we use

$$\left\langle B'(1/2), r'\,\vec{P}' \,\middle|\, \overline{\Psi}_{l'}(0)\gamma^\mu(1-\gamma_5)\Psi_c(0)\,\middle|\, B(3/2), r\,\vec{P}\right\rangle =$$
$$(\bar{u}^B_{\lambda\, r}(\vec{P})\tilde{\Gamma}^{\lambda\mu}(P',P)u^{B'}_{r'}(\vec{P}'))^* = \bar{u}^{B'}_{r'}(\vec{P}')\gamma^0(\tilde{\Gamma}^{\lambda\mu}(P',P))^\dagger\gamma^0 u^B_{\lambda\, r}(\vec{P}),$$

$$\tilde{\Gamma}^{\lambda\mu}(P',P) = \Big(-\frac{C_3^V}{M'}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) - \frac{C_4^V}{M'^2}(g^{\lambda\mu}q\cdot P - q^\lambda P^\mu) - \frac{C_5^V}{M'^2}(g^{\lambda\mu}q\cdot P' - q^\lambda P'^\mu) + C_6^V g^{\lambda\mu}\Big)\gamma_5$$
$$+\Big(-\frac{C_3^A}{M'}(g^{\lambda\mu}\not{q} - q^\lambda\gamma^\mu) - \frac{C_4^A}{M'^2}(g^{\lambda\mu}q\cdot P - q^\lambda P^\mu) + C_5^A g^{\lambda\mu} + \frac{C_6^A}{M'^2}q^\lambda q^\mu\Big). \tag{1.6}$$

   Again, and within our model, we shall have that $C_5^V(\omega) = C_6^V(\omega) = C_3^A(\omega) = 0$.

4. $3/2 \to 3/2$ transitions.
   A form factor decomposition for $3/2 \to 3/2$ can be found in Ref. [5] where a total of 7 vector plus 7 axial form factors are needed. In this case we do not evaluate the form factors but work directly with the vector and axial matrix elements.

Expressions relating form factors to weak current matrix elements can be found in Appendix B of Ref. [3].

Heavy Quark Spin Symmetry (HQSS) imposes constraints on the form factors. These constraints have been deduced in Ref. [3] using the Trace Formalism [6, 7] by requiring invariance under separate bottom and charm spin rotations. Though these relations are strictly valid in the limit of very large heavy quark mass and near zero recoil of the final baryon they turn out to be reasonable accurate for the whole available phase space.

## 2. Results

The results we obtain for the semileptonic decay widths of *cb* baryons are presented in Tables 1 ($c \to s$ decays) and 2 ($c \to d$ decays). We show between parentheses the results obtained ignoring configuration mixing in the spin-1/2 *cb* initial baryons. Due to the finite value of the heavy quark masses, the hyperfine interaction between the light quark and any of the heavy quarks can admix both $S$=0 and 1 components into the wave function for total spin-1/2 states. Thus, the actual physical spin-1/2 $cb$ baryons that we call $\Xi_{cb}^{(1)}$, $\Xi_{cb}^{(2)}$ and $\Omega_{cb}^{(1)}$, $\Omega_{cb}^{(2)}$, and that were obtained in Ref. [8], are admixtures of the $\Xi_{cb}, \Xi'_{cb}$ ($\Omega_{cb}, \Omega'_{cb}$) states where the $c$ and $b$ quarks are coupled to well defined total spin $S$=1,0. While masses are not very sensitive to hyperfine mixing, it was pointed out by Roberts and Pervin [9] that hyperfine mixing could greatly affect the decay widths of doubly heavy $cb$ baryons. This assertion was checked in Ref. [10] where Roberts and Pervin found that hyperfine mixing in the $cb$ states has a tremendous impact on doubly heavy baryon $b \to c$ semileptonic decay widths. These results were qualitatively confirmed by our own calculation in Ref. [8]. We further investigated the role of hyperfine mixing in electromagnetic transitions [11] finding again large corrections to the decay widths. A similar study was conducted by Branz et al. in Ref. [12]. We expected configuration mixing should also play an important role for $c \to s, d$ semileptonic decay of $cb$ baryons. Indeed, we find that configuration mixing has an important effect when the two light quarks in the final state couple to total spin 0.

| | $\Gamma$ [$10^{-14}$ GeV] | | | |
|---|---|---|---|---|
| | This work | [13] | [14] | [15] |
| $\Xi_{cbu}^{(1)+} \to \Xi_b^0 e^+ \nu_e$ | 3.74 (3.45) | (3.4) | | |
| $\Xi_{cbu}^{(2)+} \to \Xi_b^0 e^+ \nu_e$ | 2.65 (2.87) | | | |
| $\Xi_{cbu}^{(1)+} \to \Xi_b'^0 e^+ \nu_e$ | 3.88 (1.66) | | $2.44 \div 3.28^{\dagger}$ | |
| $\Xi_{cbu}^{(2)+} \to \Xi_b'^0 e^+ \nu_e$ | 1.95 (3.91) | | | |
| $\Xi_{cbu}^{(1)+} \to \Xi_b^{*0} e^+ \nu_e$ | 1.52 (3.45) | | | |
| $\Xi_{cbu}^{(2)+} \to \Xi_b^{*0} e^+ \nu_e$ | 2.67 (1.02) | | | |
| $\Xi_{cbu}^{(2)+} \to \Xi_b^0 e^+ \nu_e + \Xi_b'^0 e^+ \nu_e + \Xi_b^{*0} e^+ \nu_e$ | 7.27 (7.80) | | | $(9.7 \pm 1.3)^*$ |
| $\Xi_{cbu}^{*+} \to \Xi_b^0 e^+ \nu_e$ | 4.08 | | | |
| $\Xi_{cbu}^{*+} \to \Xi_b'^0 e^+ \nu_e$ | 0.747 | | | |
| $\Xi_{cbu}^{*+} \to \Xi_b^{*0} e^+ \nu_e$ | 5.03 | | | |

| | $\Gamma$ [$10^{-14}$ GeV] |
|---|---|
| $\Omega_{cbs}^{(1)0} \to \Omega_b^- e^+ \nu_e$ | 7.21 (3.12) |
| $\Omega_{cbs}^{(2)0} \to \Omega_b^- e^+ \nu_e$ | 3.49 (7.12) |
| $\Omega_{cbs}^{(1)0} \to \Omega_b^{*-} e^+ \nu_e$ | 2.98 (6.90) |
| $\Omega_{cbs}^{(2)0} \to \Omega_b^{*-} e^+ \nu_e$ | 5.50 (2.07) |
| $\Omega_{cbs}^{*0} \to \Omega_b^- e^+ \nu_e$ | 1.35 |
| $\Omega_{cbs}^{*0} \to \Omega_b^{*-} e^+ \nu_e$ | 10.2 |

**Table 1:** $\Gamma$ decay widths for $c \to s$ decays. Results where configuration mixing is not considered are shown in between parentheses. In this latter case the $\Xi_{cb}^{(1)}$, $\Xi_{cb}^{(2)}$ baryons in the table should be interpreted respectively as the $\Xi'_{cb}$, $\Xi_{cb}$ states. The result with a $\dagger$ corresponds to the decay of the $\widehat{\Xi}_{cb}$ state (see main text). The result with an $*$ is our estimate from the total decay width and the branching ratio given in [15]. Similar results are obtained for decays into $\mu^+ \nu_\mu$.

In Fig. 1 we check that our calculation respects the constraints on the form factors deduced in Ref. [3] using HQSS. Those constraints have been deduced for the $\widehat{B}_{cb} = -\frac{\sqrt{3}}{2} B'_{cb} + \frac{1}{2} B_{cb}$ and $\widehat{B}'_{cb} = \frac{1}{2} B'_{cb} + \frac{\sqrt{3}}{2} B_{cb}$, where the spins of the $c$ and light quark couple to total spin 1 and 0 respectively.

| | $\Gamma$ [$10^{-14}$ GeV] | | $\Gamma$ [$10^{-14}$ GeV] |
|---|---|---|---|
| $\Xi_{cbu}^{(1)+} \to \Lambda_b^0 e^+ \nu_e$ | 0.219 (0.196) | $\Omega_{cbs}^{(1)0} \to \Xi_b^- e^+ \nu_e$ | 0.179 (0.164) |
| $\Xi_{cbu}^{(2)+} \to \Lambda_b^0 e^+ \nu_e$ | 0.136 (0.154) | $\Omega_{cbs}^{(2)0} \to \Xi_b^- e^+ \nu_e$ | 0.120 (0.133) |
| $\Xi_{cbu}^{(1)+} \to \Sigma_b^0 e^+ \nu_e$ | 0.198 (0.0814) | $\Omega_{cbs}^{(1)0} \to \Xi_b'^- e^+ \nu_e$ | 0.169 (0.0702) |
| $\Xi_{cbu}^{(2)+} \to \Sigma_b^0 e^+ \nu_e$ | 0.110 (0.217) | $\Omega_{cbs}^{(2)0} \to \Xi_b'^- e^+ \nu_e$ | 0.0908 (0.182) |
| $\Xi_{cbu}^{(1)+} \to \Sigma_b^{*0} e^+ \nu_e$ | 0.0807 (0.184) | $\Omega_{cbs}^{(1)0} \to \Xi_b^{*-} e^+ \nu_e$ | 0.0690 (0.160) |
| $\Xi_{cbu}^{(2)+} \to \Sigma_b^{*0} e^+ \nu_e$ | 0.147 (0.0556) | $\Omega_{cbs}^{(2)0} \to \Xi_b^{*-} e^+ \nu_e$ | 0.130 (0.0487) |
| $\Xi_{cbu}^{*+} \to \Lambda_b^0 e^+ \nu_e$ | 0.235 | $\Omega_{cbs}^{*0} \to \Xi_b^- e^+ \nu_e$ | 0.196 |
| $\Xi_{cbu}^{*+} \to \Sigma_b^0 e^+ \nu_e$ | 0.0399 | $\Omega_{cbs}^{*0} \to \Xi_b'^- e^+ \nu_e$ | 0.0336 |
| $\Xi_{cbu}^{*+} \to \Sigma_b^{*0} e^+ \nu_e$ | 0.246 | $\Omega_{cbs}^{*0} \to \Xi_b^{*-} e^+ \nu_e$ | 0.223 |

**Table 2:** $\Gamma$ decay widths for $c \to d$ decays. In between parentheses we show the results without configuration mixing. Similar results are obtained for decays into $\mu^+ \nu_\mu$.

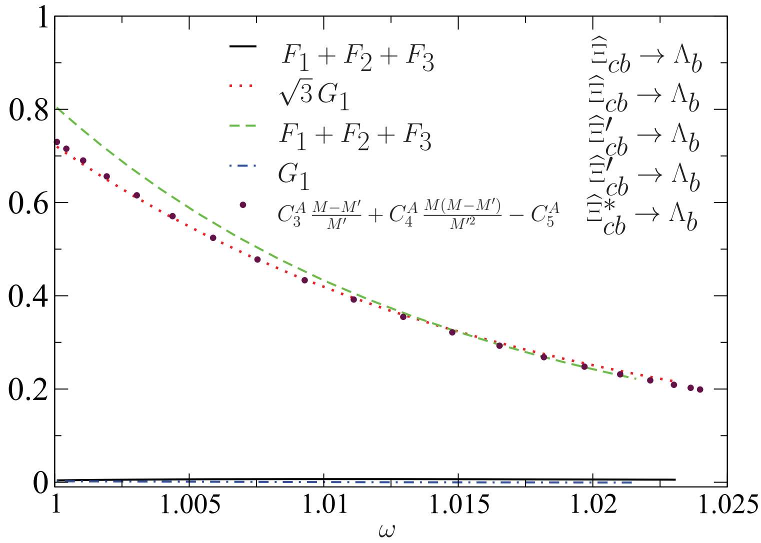


**Figure 1:** Test of HQSS constraints: Different combinations of form factors obtained in this work for several transitions with a $\Lambda_b$ in the final state ($S'_{\rm light} = 0$). For the calculation we have taken the masses of the $\widehat{\Xi}_{cb}, \widehat{\Xi}'_{cb}$ to be the masses of the physical states $\Xi_{cb}^{(1)}, \Xi_{cb}^{(2)}$. For very large heavy quark masses, HQSS predicts that the combination of form factors in the second, third and fourth lines should be equal, while for the first and fifth line they should be zero. For other transitions see Ref. [3].

These hatted states are very close to our physical $B_{cb}^{(1)}$ and $B_{cb}^{(2)}$ states. One sees deviations at the 10% level near zero recoil. Those deviation stem from corrections in the inverse of the heavy quark masses. In fact the constraints are satisfied to that level of accuracy over the whole $\omega$ range accessible in the decays. We found similar deviations in our recent study of the $c \to s, d$ decays of double charmed baryons in Ref. [2], where we explicitly showed these discrepancies tend to disappear when the mass of the heavy quark is made arbitrarily large.

Besides, in Ref. [3], with the use of the HQSS relations and assuming $M_{B_{cb}} = M_{B'_{cb}} = M_{B^*_{cb}}$ and

$M_{B_b} = M_{B'_b} = M_{B^*_b}$, we have made model independent, though approximate, predictions for ratios of $c \to s, d$ decay widths of *cb* doubly heavy baryons. Our values for those ratios agree with the HQSS motivated predictions at the level of 10% in most of the cases. We expect those predictions to hold to that level of accuracy in other approaches.

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## References

[1] K. Nakamura et al. (Particle Data Group), J. Phys. G **37**, 075021 (2010).

[2] C. Albertus, E. Hernandez and J. Nieves, Phys. Lett. B **704**, 499 (2011).

[3] C. Albertus, E. Hernández, J. Nieves Phys. Rev. D 85, 094035 (2012)

[4] C. H. Llewellyn Smith, Phys. Rept. **3**, 261 (1972).

[5] A. Faessler *et al.* Phys. Rev. D **80**, 034025 (2009).

[6] A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. B343, 1 (1990).

[7] A.V. Manohar and M.B. Wise, *Heavy Quark Physics* (Cambridge University Press, Cambridge, England, 2000), ISBN 0-521-64241-8.

[8] C. Albertus, E. Hernandez and J. Nieves, Phys. Lett. B **683**, 21 (2010).

[9] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **23**, 2817 (2008).

[10] W. Roberts and M. Pervin, Int. J. Mod. Phys. A **24**, 2401 (2009).

[11] C. Albertus, E. Hernandez, J. Nieves, Phys. Lett. B690, 265 (2010).

[12] T. Branz, A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij and B. Oexl, Phys. Rev. D **81**, 114036 (2010). [arXiv:1005.1850 [hep-ph]].

[13] M. A. Sanchis-Lozano, Nucl. Phys. B **440**, 251 (1995).

[14] A. Faessler *et al.* Phys. Lett. B **518**, 55 (2001).

[15] V. V. Kiselev and A. K. Likhoded, Phys. Usp. **45**, 455 (2002).