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$B-\bar{B}$ mixing pameter using CPP_ν model

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The mixing parameters Δm_{B_q} of the neutral $B_q-\bar{B}_q$ systems have been studied. Based on standard model, Δm_{B_q} is related to the off-diagonal elements of the Hamiltonian corresponding to the neutral B-meson oscillation. Due to the large mass difference $m_{u,c} \ll m_t$ only top quark contribution becomes dominant in this type of mixing operators. Apart from the standard model parameters involved here, the bound state parameters like the decay constant (f_P) and the bound quark mass (m_b, m_{B_q}) are determined through a potential model description of the B_q mesons. We adopt coulomb plus power type of potential of the form $V(r) = -\frac{\alpha_c}{r} + Ar^\nu$ with ν varying from 0.1 to 1.0. It is clear from the present study that both spectroscopy and mixing properties of B_d and B_s mesons are well described with relatively shallow potential with $0.5 \le \nu \le .7$.

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1. Introduction

The neutral $B_q-\bar{B}_q$ mixing occurs through the second order weak interactions where dominant contributions from the internal virtual top-quark loops [1] make the oscillation frequency ΔM_q dependant on CKM matrix elements. It has fundamental importance such as testing the standard model, extention of standard model, study of top-quark physics etc. Precise experimental measurements for Δm_q for both B and B_s mesons are being planned [2, 3]. The relation between Δm_q and branching ratio of $B_q \to \mu^+\mu^-$ [4] makes it more interesting from the experimental point of view where Δm_q and $B_q \to \mu^+\mu^-$ will be simultaneously measured at Tevatron Run II and at LHC. Such precise measurements therefore important to understand the exact nature of the interquark interaction that forms the bound mesonic state.

2. Theory

The time evolution hamiltonian corresponding to the neutral B meson system is described by

$$H = M - i\frac{\Gamma}{2} = \begin{pmatrix} M - i\frac{\Gamma}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{12}^* - i\frac{\Gamma_{12}^*}{2} & M - i\frac{\Gamma}{2} \end{pmatrix} \tag{2.1}$$

The mass difference $\Delta M_{d,s}$ of two different eigenstates of hamiltonian of Eqn. 2.1 are functions of the off-diagonal elements $|M_{12}|$ and Γ_{12}. In the case of $B_q-\bar{B}_q$ systems we have the ratio $\Gamma_{12}/M_{12} \approx 10^{-3}$ [5] and hence neglecting it from the off-diagonal elements, we have $\Delta M_q = 2|M_{12}|$. This matrix element $|M_{1,2}|$ are related to dispersive part of the $\Delta B = 2$ transitions and is given by [5, 6].

$$|M_{12}| = \frac{G_F^2 m_t^2 M_{B_q} f_{B_q}^2}{12\pi^2} g(x_t)\eta_t |V_{tq}^* V_{tb}|^2 B \tag{2.2}$$

Where $\eta_t = 0.55$ is the gluonic correction [7], f_{B_q} is the model dependant decay constants, B is the bag parameter which is taken as 1.34 ± 0.12 from lattice simulations [8] for both B and B_s mesons. The function $g(x_t)$ where $x_t = m_W^2/m_t^2$ is given by

$$g(x_t) = \frac{1}{4} + \frac{9}{4(1-x_t)} - \frac{3}{2(1-x_t)^2} - \frac{3}{2}\frac{x_t^2}{(1-x_t)^3}\log x_t \tag{2.3}$$

The decay constants f_{B_q} are given by the Van Royen-Weiskopff formula with incorporating first order QCD correction [9],

$$f_{P/V}^2(nS) = \frac{3\left|R_{nP/V}(0)\right|^2}{\pi M_{nP/V}}\left(1 + \frac{\alpha_s}{\pi}\left[\frac{m_1-m_2}{m_1+m_2}\ln\frac{m_1}{m_2} - \delta^{V,P}\right]\right)^2 \tag{2.4}$$

Here $\delta^V = \frac{8}{3}$ and $\delta^P = 2$ [9]. $R_{nP/V}(0)$ is the radial wave function at zero separation of the vector (V) and pseudoscalar (P) mesons. Apart from the standard model parameters in Eqn. 2.2, the bound state parameters of Eqn. 2.4 are computed based on potential model description of the B_d, B_s mesons. In the limit of heavy quark mass $m_Q \to \infty$, B_q meson properties are governed by the dynamics of light degree of freedom similar to hydrogenlike system.
Phenomenologically, the interaction potential consists of a central term $V_c(r)$ and a spin dependent

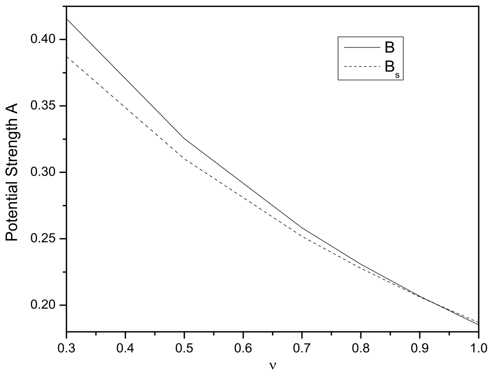

Figure 1: Potential strength A in $GeV^{1/\nu}$ against potential index ν

part V_{SD}. The central part $V_c(r)$ is expressed in terms of a vector (Coulomb) plus a scalar (confining) part given by

$$V_c(r) = V_V + V_S = -\frac{4}{3}\frac{\alpha_s}{r} + Ar^\nu \tag{2.5}$$

The present study with the choices of ν in the range $0.1 \leq \nu \leq 1.0$, is an attempt to understand the interquark interaction potential that explains both the spectra and decay properties. The running strong coupling constant appeared in the expression of potential $V(r)$ in turn is related to the quark mass parameter as

$$\alpha_s(\mu^2) = \frac{4\pi}{(11-\frac{2}{3}n_f)ln(\mu^2/\Lambda^2)} \tag{2.6}$$

Where, $n_f = 4$ is the number of flavours, μ is renormalization scale related to the constant quark masses and Λ is the QCD scale which is taken as $0.150\ GeV$ by fixing $\alpha_s = 0.118$ at the Z-boson mass (91 GeV). For computing the mass difference between different degenerate meson states, we consider the spin dependent part of the usual one gluon exchange potential (OGEP) given by [10, 11]. Accordingly, the spin-dependent part, $V_{SD}(r)$ is taken as,

$$V_{SD}(r) = \frac{\nabla^2 V_V}{3m_Q m_q}\left[S(S+1) - \frac{3}{2}\right] \tag{2.7}$$

The spin average masses of $B^* - B$ and $B_s^* - B_s$ mesons are computed using the experimental values of $M_B = 5.280\ GeV$, $M_B^* = 5.325\ GeV$, $M_{B_s} = 5.366$ and $M_{B_s}^* = 5.415\ GeV$ respectively [12]. We employ the numerical approach as given by [13] to find eigen values and radial wave functions of the respective Schroedinger equation. The potential parameter A is made to vary with ν, keeping the quark mass parameter fixed for each choices of $Q\bar{q}$ systems. For the $Q\bar{q}$ system, $m_1 = m_Q$ and $m_2 = m_{\bar{q}}$. We re-examine the predictions of the decay constants f_P and f_V under different potential schemes (by the choices of different ν) with and without the QCD correction expressed as the bracketed quantities in Eqn. 2.4.

Table 1: Decay constants ($f_{P/V}$) for ground state B and B_s mesons in GeV and Δm_q in ps^{-1}

	B meson		B_s meson		Mixing Parameters	
	$^3S_1$	$^1S_0$	$^3S_1$	$^1S_0$	Δm_d	Δm_s
0.1	0.122	0.122	0.145	0.145	0.19	5.84
0.3	0.166	0.165	0.196	0.195	0.34	10.54
0.5	0.195	0.194	0.230	0.229	0.47	14.51
0.7	0.217	0.215	0.256	0.254	0.57	17.82
0.8	0.226	0.224	0.266	0.264	0.62	19.24
0.9	0.234	0.232	0.275	0.273	0.67	20.56
1.0	0.241	0.239	0.284	0.282	0.71	21.92
[15, 14]	0.238	0.196	0.272	0.216		
[16]	0.219	0.189	0.251	0.218		
[17]	0.200	0.182		0.216		
[18]	0.225	0.204	0.313	0.281		
[19]	0.196	0.179	0.229	0.204		
[20]	0.219	0.189	0.251	0.218		
[21]		0.195		0.232		
Expt.					0.51 ± 0.02[2]	17.77 ± 0.10 [3]

3. Results and Conclusion

The hyperfine splitting of the 1^3S_1 and 1^1S_0 states are found to be very sensitive to the choices of quark mass parameter and potential strength A. The most suitable values of the quark mass parameter are found to be $m_b = 4.4\ GeV$, $m_u = 0.330\ GeV$ and $m_s = 0.500\ GeV$. The corresponding A values are obtained from the $1S$ fitting and are plotted in Fig. 1 against the potential exponent ν of B and B_s systems. Our computed values of decay constants for B_q mesons are listed in Table 1 against potential exponent ν. Other theoretical model predictions are also tabulated for comparison. Our predicted decay constants agrees with other theoretical model predictions for choices of potential exponent, $0.5 \leq \nu \leq 1.0$. However, latest predicted values of f_P from lattice QCD [21] agrees for potential exponent $\nu \sim 0.5$.The decay constants are then employed to compute the mixing parameter $\Delta m_{d,s}$ and the resultant values are tabulated in Table 1 along with the decay constants. Our computed values of Δm_q agree with the experimental data of $\Delta m_d = 0.51 \pm 0.02\ ps^{-1}$ [2] and of $\Delta m_s = 17.77 \pm 0.10\ ps^{-1}$ [3] for the choices of potential exponent $0.5 \leq \nu \leq 0.7$ for both the cases of B and B_s mesons.

Using expression relating Δm_q with branching ratio of $B_q \to \mu^+\mu^-$ given by model with minimal flavour violation (MFV) [22] and employing our predicted values of Δm_q for the potential exponent $\nu = 0.7$ result into $BR(B_d \to \mu^+\mu^-) = 1.21 \times 10^{-10}$ and $BR(B_s \to \mu^+\mu^-) = 3.91 \times 10^{-9}$ which are in good agreement with more accurately predicted values of $BR(B_d \to \mu^+\mu^-) = (1.00 \pm 0.14) \times 10^{-10}$ and $BR(B_s \to \mu^+\mu^-) = (3.42 \pm 0.54) \times 10^{-9}$ [4].

Hence, we can conclude that both the spectroscopy and mixing properties of B_d and B_s mesons are well described with relatively shallow potential with ν lying in the range $0.5 \leq \nu \leq 0.7$.

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