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Anomalous AV^*V Green's function in soft-wall AdS/QCD

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In this talk we study the Green's function of two vector and one axial-vector currents within the soft-wall anti-de-Sitter (AdS) model of Quantum Chromodynamics (QCD), with a quadratic dilaton and chiral symmetry broken through a field X which gains a vacuum expectation value. We compare our predictions at high energies with the Operator Product Expansion both in the massless quark limit and for $m_q \neq 0$. The soft-wall model yields a zero magnetic susceptibility $\chi = 0$ and some problems are found in the case with $m_q \neq 0$. We also discuss the relation proposed by Son and Yamamoto between the AV^*V and $VV - AA$ correlators, which is not obeyed at high energies in soft wall AdS/QCD.

PoS(QNP2012)118

Sixth International Conference on Quarks and Nuclear Physics,
April 16-20 2012, Palaiseau, France.

This work was supported in part by the Universidad CEU Cardenal Herrera grants PRCEU-UCH15/10 and PRCEU-UCH35/11 and the MICINN-INFN fund AIC-D-2011-0818. I would like to thank the organizers for the kind scientific environment during the conference and all their help.

1. Introduction: AV^*V Green's function

The AV^*V Green's function was recently studied in the framework of soft-wall anti-de-Sitter (AdS) theories [1]. This analysis was motivated by a previous work by Son and Yamamoto [2] for holographic theories where chiral symmetry is broken through boundary condition [3]. In Ref. [2], the authors found an interesting relation between the $VV - AA$ correlator and the Green's function involving two vector currents $J_\mu = \bar{q}V\gamma_\mu q$ and $J_\sigma^{em} = \bar{q}\mathrm{Q}\gamma_\sigma \mathrm{q}$ and an axial-vector current $J_\nu^5 = \bar{q}A\gamma_\nu\gamma_5 q$, with V and A diagonal matrices and the electric charge matrix Q:

$$\begin{aligned} T_{\mu\nu}(q,k) &= i\int d^4x\, e^{iq\cdot x}\,\langle 0|\,T[J_\mu(x)J_\nu^5(0)]\,|\gamma(k,\varepsilon)\rangle \\ &= -\frac{i\,Q^2}{4\pi^2}\,\mathrm{Tr}\,[\mathrm{QVA}]\,P_\mu^{T\;\alpha}(q)\,\left\{P_\nu^{T\,\beta}(q)\,w_T(Q^2)+P_\nu^{L\,\beta}(q)\,w_L(Q^2)\right\}\;\tilde{f}_{\alpha\beta}\,, \qquad (1.1)\end{aligned}$$

with $k\to 0$ and related to the three-point Green's function $\langle 0|\,T[J_\mu(x)J_\nu^5(0)J_\sigma^{em}(y)]\,|0\rangle$. We use the notation $Q^2 \equiv -q^2$, $\tilde{f}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}f^{\alpha\beta}$ and $f^{\alpha\beta} = k^\alpha\varepsilon^\beta - k^\beta\varepsilon^\alpha$, and the transverse and longitudinal projectors, respectively, $P^T_{\mu\alpha}(q) = \eta_{\mu\alpha} - q_\mu q_\alpha/q^2$ and $P^L_{\mu\alpha} = q_\mu q_\alpha/q^2$.

At short-distance it is possible to use the Operator Product Expansion (OPE) for $m_q = 0$ [4, 5]:

$$w_L(Q^2) = \frac{2N_C}{Q^2}\,, \qquad w_T(Q^2) = \frac{N_C}{Q^2} + \frac{128\pi^3\alpha_s\,\chi\,\langle\bar{q}q\rangle^2}{9\,Q^6} + \mathscr{O}\left(\frac{\Lambda^6}{Q^8}\right). \qquad (1.2)$$

where the longitudinal component is completely fixed by the anomaly and does not receive any correction [4, 5, 6] and χ is defined by the condensate $\langle 0|\bar{q}\sigma^{\alpha\beta}q|\gamma\rangle = ie\chi\langle 0|\bar{q}q|0\rangle f^{\alpha\beta}$.

If we allow $m_q \neq 0$, the OPE yields corrections proportional to the quark mass at one loop [4]:

$$w_L(Q^2) - 2w_T(Q^2) = \mathscr{O}\left(\frac{\Lambda^4}{Q^6}\right), \quad w_T(Q^2) = \frac{N_C}{Q^2}\left[1 + \frac{2m_q^2}{Q^2}\ln\frac{m_q^2}{Q^2} - \frac{8\pi^2 m_q\langle\bar{q}q\rangle\chi}{N_C Q^2} + \mathscr{O}\left(\frac{\Lambda^4}{Q^4}\right)\right]. \qquad (1.3)$$

2. The holographic setup in AdS/QCD

We will consider a gauged $U(n_f)_R\otimes U(n_f)_L$ chiral symmetry and the AdS line element $ds^2 = g_{MN}dx^M dx^N = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2)$, with the coordinate indices $M,N = 0,1,2,3,5$, $\eta_{\mu\nu}$ =diag$(+1,-1,-1,-1)$, R the AdS curvature radius (set to unity from now on) and the 5D coordinate being in the range $0^+ \leq z < +\infty$. The 5D Yang-Mills action describing the fields $\mathscr{A}^M_{L,R}$ dual to the left and right currents $J^\mu_{L,R}$, as well as the scalar-pseudoscalar field X, is given by

$$S_{YM} = \frac{1}{k_{YM}}\int d^5x\sqrt{g}e^{-\Phi}Tr\left\{|DX|^2 - \mathscr{V}(X) - \frac{1}{4g_5^2}(F_L^2+F_R^2)\right\}\;, \qquad (2.1)$$

with the field strength tensors $F^{MN}_{L,R} = F^{MNa}_{L,R}T^a$, T^a the $U(n_f)$ group generators, the X field potential $\mathscr{V}(X)$ and g the determinant of the metric tensor g_{MN}. We take the quadratic dilaton background $\Phi(z) = (cz)^2$, chosen in order to recover linear Regge trajectories for vector resonances, and k_{YM} is a parameter included to provide canonical $4d$ dimensions for the fields. The covariant derivative acting on X is defined as $D^M X = \partial^M X - i\mathscr{A}^M_L X + iX\mathscr{A}^M_R$. The gauge fields $\mathscr{A}^M_{L,R}$ are usually combined into a vector field $V^M = \frac{\mathcal{A}_L^M + \mathcal{A}_R^M}{2}$ and an axial-vector field $A^M = \frac{\mathcal{A}_L^M - \mathcal{A}_R^M}{2}$. The study of the vector and scalar correlators at high energies allows one to fix the constants in the Yang-Mills action: $k_{YM} = \frac{16\pi^2}{N_c}$ and $g_5^2 = \frac{3}{4}$ [7].

In this kind of approaches [7], one introduces a spinless field X which is dual to the quark bifundamental operator $\bar{q}_R^\alpha q_L^\beta$. This field gains the v.e.v. $X = \frac{v(y)}{2} e^{2i\pi}$ [7]. Chiral symmetry becomes broken when $v(y) \neq 0$, as the left and right sectors of the theory get connected to each other. Moreover, a phase-shift π is induced for the v.e.v. in the bulk when the parallel axial-vector source is switched on: π gets coupled to $A^{\|}$ in the equations of motion (EoM). Thus, for the bulk to boundary (B-to-b) propagators one finds the EoM, within the gauge $V_z = A_z = 0$,

$$\partial_y \left(\frac{e^{-y^2}}{y} \, \partial_y V_\perp \right) - \tilde{Q}^2 \frac{e^{-y^2}}{y} V_\perp = 0 \,, \qquad \partial_y \left(\frac{e^{-y^2}}{y} \, \partial_y A_\perp \right) - \tilde{Q}^2 \frac{e^{-y^2}}{y} A_\perp - \frac{g_5^2 v^2(y) e^{-y^2}}{y^3} A_\perp = 0$$
$$\partial_y \left(\frac{e^{-y^2}}{y} \, \partial_y A_{\|} \right) + \frac{g_5^2 v^2(y) e^{-y^2}}{y^3} (\pi - A_{\|}) = 0 \,, \qquad \tilde{Q}^2 (\partial_y A_{\|}) + \frac{g_5^2 v^2(y)}{y^2} \partial_y \pi = 0 \,, \tag{2.2}$$

with $y \equiv cz$ and $\tilde{Q}^2 \equiv Q^2/c^2$. In momentum space the 5D fields $\tilde{\phi}(q,y) = -i \frac{q^\mu}{q^2} \tilde{A}_\mu^{\|}(q,y)$ and $\tilde{\pi}(q,y)$ are respectively related to the B-to-b propagators $A_{\|}(q,y)$ and $\pi(q,y)$ [1].

The vector EoM can be analytically solved [1], but for the remaining EoM one needs to specify the v.e.v. $v(y)$. Its asymptotic behaviour close to the UV brane ($y \to 0$ in our choice of coordinates) is related to the explicit (quark mass m_q) and spontaneous chiral symmetry breaking (quark condensate $\sigma \propto \langle \bar{q} q \rangle$ in massless QCD):

$$v(y) \stackrel{y \to 0}{=} \frac{m_q}{c} y + \frac{\sigma}{c^3} y^3 + \mathcal{O}(y^4) \,. \tag{2.3}$$

where the first terms of its power expansion in y determine the behaviour of $w_{T,L}$ at high-energies [1].

The QCD chiral anomaly will be provided by the Chern-Simons action and, more precisely, the AV^*V amplitude studied here will be provided by the piece [1]

$$S_{\rm CS}\Big|_{AV^*V} = 3\kappa_{CS} \, \varepsilon_{ABCDE} \int d^5x \; \mathrm{Tr} \Big[A^A \{ F_{(V)}^{BC}, F_{(V)}^{DE} \} \Big] = 48 \kappa_{CS} \, d^{ab} \, \tilde{F}_{em}^{\mu\nu} \int d^5x \, A_\nu^b \, \partial_z V_\mu^a \,, \tag{2.4}$$

with the group factor $d^{ab} = \mathrm{Tr}[\mathrm{Q}\{\mathrm{T^a},\mathrm{T^b}\}]$. This yields the structure functions

$$w_{L\,(T)}(Q^2) = -\frac{2N_C}{Q^2} \int_0^\infty dy \, A_{\|\,(\perp)}(Q^2,y) \, \partial_y V_\perp(Q^2,y) \,. \tag{2.5}$$

The global normalization is fixed *a posteriori* through $\kappa_{CS} = -\frac{N_C}{96\pi^2}$ in the case with $m_q = 0$.

3. $w_{T,L}$ results for $m_q = 0$ and $m_q \neq 0$

In the massless quark limit one can demonstrate that $A^{\|}(Q^2,y) = 1$ [1]. The perpendicular B-to-b propagators can be solved perturbatively in $1/\tilde{Q}^2$ in the form $A^\perp(Q^2,y) = \sum_{n=0}^\infty A_n^\perp(t)(1/\tilde{Q}^2)^n$ and $V^\perp(Q^2,y) = \sum_{n=0}^\infty V_n^\perp(t)(1/\tilde{Q}^2)^n$, with $t \equiv yQ/c$. This yields the high-energy expansion

$$w_L(Q^2) = \frac{2N_C}{Q^2} \,, \qquad w_T(Q^2) = \frac{N_C}{Q^2} \left[1 - \frac{3\tau\sigma^2}{2Q^6} + \mathcal{O}\left(\frac{\Lambda^8}{Q^8} \right) \right] , \tag{3.1}$$

with $\tau \simeq 2.7$ defined by the integral of Bessel functions provided in Ref. [1]. The parallel B-to-b propagator $A_{\|} = 1$ ensures the recovery of the OPE prediction for w_L, which becomes fully determined by the boundary conditions. Conversely, the QCD dynamics is contained in w_T. The comparison with the OPE (1.3) leads to a vanishing prediction for the magnetic susceptibility $\chi = 0$.

In the case with $m_q \neq 0$, all the B-to-b propagators can be solved perturbatively in the way we did for Eq. (3.1), gaining corrections proportional to the quark mass and leading to the amplitudes

$$w_L(Q^2) = \frac{2N_C}{Q^2}\left[1 - (1-\pi(Q^2,0))\frac{3m_q^2}{8Q^2} + \mathcal{O}\left(\frac{m_q\Lambda^3}{Q^4}\right)\right],$$
$$w_T(Q^2) = \frac{N_C}{Q^2}\left[1 - \frac{m_q^2}{4Q^4} + \mathcal{O}\left(\frac{m_q\Lambda^3}{Q^4}\right) + \mathcal{O}\left(\frac{\Lambda^6}{Q^6}\right)\right]. \tag{3.2}$$

As $A^{\|}$ and π EoMs are coupled, the perturbative solutions for $Q^2 \to \infty$ depend on the UV boundary condition $\pi(Q^2,0)$. The comparison of the NLO term proportional to m_q with the OPE (1.3) yields again a vanishing magnetic susceptibility $\chi = 0$. The m_q^2 terms is more cumbersome since the recovery of the finite OPE log $m_q^2 \ln\frac{m_q^2}{Q^2}$ in $w_L(Q^2)$ requires a logarithmic dependence on Q^2 of the UV boundary condition $\pi(Q^2,0)$. The transverse component of the amplitude is even more problematic as the holographic model generates an m_q^2/Q^2 term without logs and it is impossible to recover the finite logarithms from the OPE without including any further ingredient to the theory.

4. Checking the Son-Yamamoto relation

This work was motivated by the relation proposed by Son and Yamamoto for $m_q = 0$ [2] in the kind of model where chiral symmetry is broken through boundary conditions [3]:

$$w_T(Q^2) - \frac{N_C}{Q^2} = \frac{N_C}{F_\pi^2}\Pi_{VV-AA}(Q^2). \tag{4.1}$$

Actually, although this kind of models fulfills this relation for any energy, the left-hand and right-hand sides of (4.1) do not obey the expected OPE short distance behaviour [2]: $w_T(Q^2) - \frac{N_C}{Q^2} = \mathcal{O}(e^{-Q})$, $\Pi_{VV-AA}(Q^2) = \mathcal{O}(e^{-Q})$.

In the type of models where chiral symmetry is broken through a scalar-pseudoscalar field X that gains a v.e.v. [7], one gets the right $1/Q^6$ behaviour for the $VV-AA$ correlator but the subleading corrections in the AV^*V Green's function do not start at the expected orders [2, 1]:

$$w_T(Q^2) - \frac{N_C}{Q^2} = -\frac{3N_C\sigma^2\tau}{2Q^8} + \mathcal{O}\left(\frac{\Lambda^8}{Q^{10}}\right), \qquad \Pi_{VV-AA}(Q^2) = -\frac{N_C\sigma^2}{10\pi^2 Q^6} + \mathcal{O}\left(\frac{\Lambda^8}{Q^8}\right). \tag{4.2}$$

Hence, Son-Yamamoto relation (4.1) is not fulfilled in this kind of models at high energies [1, 2].

It is worthy to mention an interesting result: if we saturate the two Weinberg sum-rules for $w_T(Q^2) - N_C/Q^2$ stemming from the OPE [4, 5] through the lightest multiplet of vector and axial-vector resonances one gets the minimal hadronical approximation (MHA) [8],

$$w_T(Q^2)\Big|_{\rm MHA} - \frac{N_C}{Q^2} = -\frac{N_C M_V^2 M_A^2}{Q^2(M_V^2+Q^2)(M_A^2+Q^2)} = \frac{N_C}{F^2}\Pi_{VV-AA}(Q^2)\Big|_{\rm MHA}, \tag{4.3}$$

which fulfills the Son-Yamamoto relation (4.1). Although the MHA may lead to inaccurate short-distance determinations it provides a fair estimate of the low-energy constants [9]. This may explain the reasonable agreement for the low-energy relation $C_{22}^W = -\frac{N_C}{32\pi^2 F^2} L_{10}$ [10].

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5. Conclusions

We have studied the AV^*V Green's function in the soft-wall [1]. When $m_q = 0$ one has the B-to-b propagators $\pi = A_\parallel = 1$. This ensures the exact recovery of the longitudinal structure amplitude $w_L(Q^2) = 2N_C/Q^2$ prescribed by QCD [4, 5, 6]. On the other hand, the transverse component corrections predicted in the soft-wall model start at $\mathcal{O}(1/Q^8)$, producing a zero magnetic susceptibility χ. This hints the need for further ingredients in our holographic description like, e.g., the inclusion of a five-dimensional field B^{MN} dual to the tensor operator $\bar{q}\sigma^{\alpha\beta}q$ [11].

The case $m_q \neq 0$ brings further problems. One needs to specify the value of $\pi(Q^2, y)$ at $y \to 0$ and the study of the subleading terms in the OPE proportional to $m_q\sigma$ yields again $\chi = 0$. Thus, the problem of the m_q corrections needs further understanding which might be obtained from the longitudinal part of the $\Pi_{AA}(Q^2)$ correlator.

We have also tested the Son-Yamamoto relation between the AV^*V Green's function and the $VV - AA$ correlator [2]. The hard and soft-wall models show problems at high energies and the OPE is not well recovered [1, 2]. However, the low-energy relation between even and odd-sector low-energy constants $C_{22}^W = -\frac{N_C}{32\pi^2 F^2} L_{10}$ seems to be reasonably well satisfied [10].

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