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Constraining the properties of K_{l3} form factors with analyticity and unitarity

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We investigate the vector and scalar $K\pi$ form factors at low energies by the method of unitarity bounds which makes use of perturbative QCD for a suitable correlator and information on the phase and modulus along the elastic region of the unitarity cut. Using the values of the form factors at $t = 0$ we obtain stringent constraints on the slope and curvature for the two form factors. The use of the value of the scalar form factor at the Callan-Treiman point $(M_K^2 - M_\pi^2)$ improves the constraints significantly.

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1. Introduction

Form factors are fundamental quantities for testing the nonperturbative regime of QCD. These quantities could be calculated if hadron structure could be solved. In the absence of such a solution, form factors provide an excellent meeting ground between experimental measurements and theoretical approaches. We investigate the shape parameters of the semileptonic K_{l3}-decays where a kaon decays to a pion, a charged lepton, and a neutrino. K_{l3}-decays provide the most accurate determination of Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{us}. These processes can be used to derive potentially stringent constraints on new physics scenarios. Bounds on violations of CKM unitarity and lepton universality and deviations from the $V-A$ structure provide a significant test for the physics beyond Standard Model (SM). Significant progress on both the experimental and theoretical sides has motivated us to investigate the shape parameters of the semileptonic kaon decays. A detailed analysis of precise tests of the SM with semileptonic kaon decays has been reviewed in [1].

2. Formalism

We apply the formalism of the unitarity bounds [2] which is based on the general principles of analyticity and unitarity and is model independent. Our primary motivation is to study the parameters of the Taylor expansion of the K_{l3} form factors at zero momentum transfer. These parameters have crucial importance in the study of Chiral Perturbation Theory (ChPT), the low-energy effective theory of QCD, and provide a solid ground to test QCD at low energies. We also find constraints on its value at the special point $(M_\pi^2 - M_K^2)$, details may be found in [3].

Kaon decay is characterized by the vector and scalar form factors. The kinematically allowed region is $m_l^2 \leq t \leq (M_K - M_\pi)^2$. The expansion of the vector form factor about $t = 0$ is

$$f_+(t) = f_+(0)\left(1 + \lambda'_+ \frac{t}{M_\pi^2} + \frac{1}{2}\lambda''_+ \frac{t^2}{M_\pi^4} + \cdots\right), \tag{2.1}$$

where λ'_+ and λ''_+ are slope and curvature parameters respectively and $f_+(0) = 0.964$ comes from Lattice[4]. Analogous expansion for the scalar form factor can be defined.

The formalism is based on the fact that an integral involving the modulus squared of the form factor can be bounded along the unitarity cut using a dispersion relation satisfied by a QCD correlator. The QCD correlator for the vector form factor is

$$\chi_1(Q^2) = \frac{1}{\pi}\int_0^\infty dt\, \frac{t\,\mathrm{Im}\Pi_1(t)}{(t+Q^2)^3}, \tag{2.2}$$

$$\mathrm{Im}\Pi_1(t) \geq \frac{3}{2}\frac{1}{48\pi}\frac{[(t-t_+)(t-t_-)]^{3/2}}{t^3}|f_+(t)|^2, \tag{2.3}$$

with $t_\pm = (M_K \pm M_\pi)^2$ is positive definite. We can write a similar expression for the scalar form factor. The vector correlator in pQCD ($\overline{MS}$ scheme) when $Q \gg \Lambda_{\mathrm{QCD}}$ is [5, 6]

$$\chi_1(Q^2) = \frac{1}{8\pi^2 Q^2}\left(1 + \frac{\alpha_s}{\pi} - 0.062\alpha_s^2 - 0.162\alpha_s^3 - 0.176\alpha_s^4\right). \tag{2.4}$$

Using a conformal mapping $t \to z(t)$

$$z(t) = \frac{\sqrt{t_+} - \sqrt{t_+ - t}}{\sqrt{t_+} + \sqrt{t_+ - t}}, \tag{2.5}$$

the dispersion relation is brought into a standard form

$$\frac{1}{2\pi}\int_0^{2\pi} d\theta |g(\exp(i\theta))|^2 \leq I \ , \tag{2.6}$$

where

$$g(z) = F(t(z))w(z) \ , \tag{2.7}$$

and I is the value of the QCD correlator. Here $w(z)$ is an outer function, i.e., a function analytic and without zeros in $|z| < 1$. Square integrability allows us to write above equation in the following form

$$\sum_{n=0}^{\infty} |g_n|^n \leq I \ . \tag{2.8}$$

Bounds on the shape parameters are obtained by truncating the series at a desired order. Bounds can be improved if the phase information along the unitarity cut is known from an independent source. We use Omnès function for the implementation of the phase

$$\mathcal{O}(t) = \exp\left(\frac{t}{\pi}\int_{t_+}^{\infty} dt \frac{\delta(t')}{t'(t'-t)}\right), \tag{2.9}$$

where $\delta(t)$ is the $I = 1/2$ elastic P-wave $K\pi$ scattering phase, in the elastic region and arbitrary Lipschitz continuous above t_{in}, see [3]. The important feature of this formalism is its independence of the phase information above t_{in}. Bounds can be further improved if the modulus of the form factor is known along the unitarity cut. In this case, contribution from t_+ to t_{in} needs to be removed from pQCD value which is now the input for the bound given below

$$I' = \chi_1(Q^2) - \frac{1}{32\pi^2}\int_{t_+}^{t_{\text{in}}} dt \, \frac{[(t-t_+)(t-t_-)]^{3/2}|f_+(t)|^2}{t^2(t+Q^2)^3} \ . \tag{2.10}$$

We obtain a problem identical to the above, but for functions analytic in the t-plane cut for $t > t_{\text{in}}$. The low-energy integral for the vector as well as scalar form factor is estimated using the Breit-Wigner parameterizations of $|f_+(t)|$ and $|f_0(t)|$ respectively in terms of the resonances given by the Belle Collaboration for fitting the rate of $\tau \to K\pi\nu_\tau$ decay [7]. From Eq. 2.10 we finally obtain constraints on the slope and curvature.

Analogously the scalar form factor $f_0(t)$ may be analysed and constrained. Furthermore, in this case, there are two low energy theorems, namely, soft pion theorem

$$f_0(M_K^2 - M_\pi^2) = F_K/F_\pi + \Delta_{CT} \tag{2.11}$$

$\Delta_{CT} \simeq 0$ to two-loops in chiral perturbation theory. and a soft-kaon theorem

$$f_0(M_\pi^2 - M_K^2) = F_\pi/F_K + \overline{\Delta}_{CT} \tag{2.12}$$

$\overline{\Delta}_{CT} = 0.03$ is one-loop in chiral perturbation theory and not known at two-loops. For details, see [3] and references therein. Bounds on shape parameters can be improved using above values of form factors . Since higher order corrections to soft kaon theorem are not known, we do not use it in our analysis. Nevertheless we predict a very narrow range for higher order corrections to soft-kaon theorem. We use recent determination of decay constants $F_K/F_\pi = 1.193 \pm 0.006$ [8].

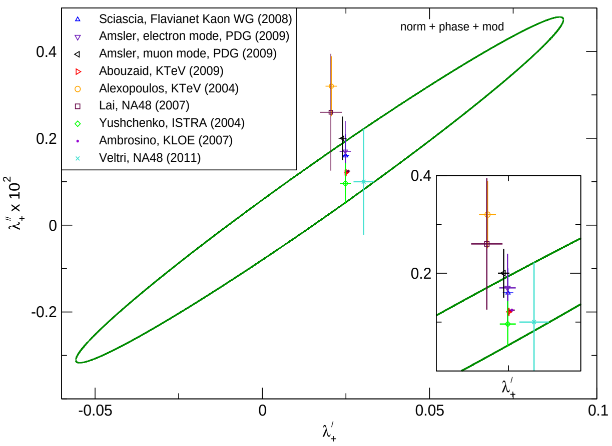

Figure 1: The best constraints for the slope and curvature of the vector form factor when we include phase and modulus information.

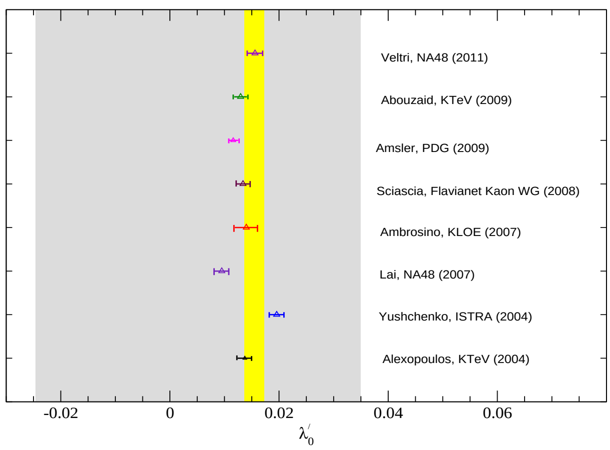

Figure 2: Slope of the scalar form factor, when we include the phase, modulus and the Callan-Treiman constraint.

3. Results and Discussion

In Fig. 1, we present results for vector form factor. In this case phase and modulus information are used along with the value of $f_+(0)$. Our constraints are satisfied by all the available data except the results from NA48 and KLOE, which have curvatures slightly larger than the allowed values. We note also that the theoretical predictions $\lambda'_+ = (24.9 \pm 1.3) \times 10^3$, $\lambda''_+ = (1.6 \pm 0.5) \times 10^3$ obtained from ChPT to two loops, and $\lambda'_+ = (26.05^{+0.21}_{-0.51}) \times 10^{-3}$, $\lambda''_+ = (1.29^{+0.01}_{-0.04}) \times 10^{-3}$ [10], and $\lambda'_+ = (25.49 \pm 0.31) \times 10^{-3}$, $\lambda''_+ = (1.22 \pm 0.14) \times 10^{-3}$ [11] obtained from dispersion relations are consistent with our constraint. For more information, see, [3].

In Fig. 2, we show allowed range for the slope of the scalar form factor. The large grey band shows the slope without phase and modulus information while the yellow band corresponds to including the phase and modulus along with the soft pion theorem. Our constraints are satisfied by all the available data as well as the most recent result from NA48 [9] which is accomodated by our constraints. This new NA48 analysis is based on the form factor measurements of $K^{\pm}_{\mu 3}$-decays using a sample of 3.4×10^6 events.

The theoretical prediction of ChPT to two loops $\lambda_0' = (13.9^{-0.4}_{+1.3} \pm 0.4)\times 10^{-3}$, $\lambda_0'' = (8.0^{-1.7}_{+0.3})\times 10^{-4}$ is consistent within errors with our constraint. The same is true for the theoretical prediction $\lambda_0' = (16.00\pm 1.00)\times 10^{-3}$, $\lambda_0'' = (6.34\pm 0.38)\times 10^{-4}$ obtained from dispersion relations [12].

Lastly, this formalism can also be extended to the study of zeros of the form factors which have useful phenomenological implications, see for discussions on the $K\pi$ zeros in [3].

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