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Chiral effective Field Theory for Nuclear Matter

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We review on a chiral power counting scheme for in-medium chiral perturbation theory with nucleons and pions as degrees of freedom [1]. It allows for a systematic expansion taking into account local as well as pion-mediated inter-nucleon interactions. Based on this power counting, one can identify classes of non-perturbative diagrams that require a resummation. We then calculate the nuclear matter energy density for the symmetric and purely neutron matter cases up-to-and-including next-to-leading order (NLO), in good agreement with sophisticated many-body calculations. Next, the neutron matter equation of state is applied to calculate the upper limit for neutron stars, with an upper bound around 2.3 solar masses, large enough to accommodate the most massive neutron star observed until now. We also apply our equation of state to constraint G_N in exceptionally large gravitational fields.

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1. Introduction

An interesting achievement in nuclear physics would be the calculation of atomic nuclei and nuclear matter properties from microscopic inter-nucleon forces in a systematic and controlled way. This is a non-perturbative problem involving the strong interactions. In the last decades, Effective Field Theory (EFT) has proven to be an indispensable tool to accomplish such aim. In this work we employ Chiral Perturbation Theory (CHPT) to nuclear systems [2, 3, 4], with nucleons and pions as the pertinent degrees of freedom. For the lightest nuclear systems with two, three and four nucleons, it has been successfully applied [5]. For heavier nuclei one common procedure is to employ the chiral nucleon-nucleon potential with standard many-body methods, sometimes supplied with renormalization group techniques [6], or in lattice calculations [7]. We have recently derived [1] a chiral power counting in nuclear matter that takes into account local multi-nucleon interactions simultaneously to pion-nucleon interactions. Many present applications of CHPT to nuclei and nuclear matter only consider meson-baryon chiral Lagrangians (see e.g. [5] for a summary), without constraints from free nucleon-nucleon scattering. Our novel power counting was applied in ref.[8], among other problems, to determine the nuclear matter energy per baryon. We elaborate on these results here. Next, we apply them to pure neutron matter case and study the upper limit of a neutron star mass. We see that our results can accommodate the recently observed neutron star with a mass $(1.97\pm0.04)\,M_\odot$ [9], the largest one confirmed until now. We also use our equation of state to constraint the running of the gravitational constant G_N with the gravitational field intensity, exceptionally large (around 2×10^{12} m/s^2) inside a neutron star of 2 solar masses. In this case we show that G_N cannot exceed its value on Earth by more than a 12% [10].

2. Chiral Power Counting

Ref.[11] establishes the concept of an "in-medium generalized vertex" (IGV). Such type of vertices result because one can connect several bilinear vacuum vertices through the exchange of baryon propagators with the flow through the loop of one unit of baryon number, contributed by the nucleon Fermi seas. At least one is needed because otherwise we would have a vacuum closed nucleon loop that in a low energy EFT is buried in the chiral higher order counterterms. It was also stressed in ref.[12] that within a nuclear environment a nucleon propagator could have a "standard" or "non-standard" chiral counting. To see this note that a soft momentum $Q\sim p$, related to pions or external sources can be associated to any of the vertices. Denoting by k the on-shell four-momenta associated with one Fermi sea insertion in the IGV, the four-momentum running through the j^{th} nucleon propagator can be written as $p_j = k+Q_j$. If $Q_j^0=\mathcal{O}(m_\pi)=\mathcal{O}(p)$ one has the standard counting so that the baryon propagator scales as $\mathcal{O}(p^{-1})$. However, if Q_j^0 is of the order of a kinetic nucleon energy in the nuclear medium then the nucleon propagator should be counted as $\mathcal{O}(p^{-2})$. This is referred as the "non-standard" case [12]. In order to treat chiral Lagrangians with an arbitrary number of baryon fields (bilinear, quartic, etc) ref.[1] considered firstly bilinear vertices like in refs.[11, 12], but now the additional exchanges of heavy meson fields of any type are allowed. The latter should be considered as merely auxiliary fields that allow one to find a tractable representation of the multi-nucleon interactions that result when the masses of the heavy mesons tend to infinity. These heavy meson fields are denoted in the following by H, and a

heavy meson propagator is counted as $\mathcal{O}(p^0)$ due to their large masses. On the other hand, ref.[1] takes the non-standard counting case from the start and any nucleon propagator is considered as $\mathcal{O}(p^{-2})$. In this way, no diagram whose chiral order is actually lower than expected if the nucleon propagators were counted assuming the standard rules is lost. In the following $m_\pi \sim k_F \sim \mathcal{O}(p)$ are taken of the same chiral order, and are considered much smaller than a hadronic scale Λ_χ of several hundreds of MeV that results by integrating out all other particle types, including nucleons with larger three-momentum, heavy mesons and nucleon isobars [4]. The final formula obtained in ref.[1] for the chiral order ν of a given diagram is

$$\nu = 4 - E + \sum_{i=1}^{V_\pi} (n_i + \ell_i - 4) + \sum_{i=1}^{V} (d_i + v_i + \omega_i - 2) + V_\rho \ . \tag{2.1}$$

where E is the number of external pion lines, n_i is the number of pion lines attached to a vertex without baryons, ℓ_i is the chiral order of the latter with V_π its total number. In addition, d_i is the chiral order of the i^{th} vertex bilinear in the baryonic fields, v_i is the number of mesonic lines attached to it, ω_i that of only the heavy lines, V is the total number of bilinear vertices and V_ρ is the number of IGVs. It is important to stress that ν given in eq.(2.1) is bounded from below [1]. Because of the last term in eq.(2.1) adding a new IGV to a connected diagram increases the counting at least by one unit. The number ν given in eq.(2.1) represents a lower bound for the actual chiral power of a diagram, μ, so that $\mu \geq \nu$. The real chiral order of a diagram might be different from ν because the nucleon propagators are counted always as $\mathcal{O}(p^{-2})$ in eq.(2.1), while for some diagrams there could be propagators that follow the standard counting. Eq.(2.1) implies the following conditions for augmenting the number of lines in a diagram without increasing the chiral power by adding i) pionic lines attached to mesonic vertices, $\ell_i = n_i = 2$, ii) pionic lines attached to meson-baryon vertices, $d_i = v_i = 1$ and iii) heavy mesonic lines attached to bilinear vertices, $d_i = 0$, $\omega_i = 1$.

3. Nuclear matter energy density

The energy per baryon in nuclear matter at NLO in the counting of Eq. (2.1) requires to evaluate the set of diagrams shown in Fig. 1. Diagram 1 represents the kinetic energy from the Fermi sea of nucleons. Diagram 2 arises from the nucleon self-energy summed for all the nucleons. Finally, diagrams 3 correspond to the NN interactions in the nuclear medium, 3.1 is for the direct part and 3.2 for the crossed one.

A detailed derivation of the final result is given in Ref. [8]. We reproduce here the final expression for the diagrams 3, $\mathscr{E}_3$, given by

$$\mathscr{E}_3 = 4 \sum_{I,J,\ell,S} \sum_{\alpha_1,\alpha_2} (2J+1) \chi(S\ell I)^2 \int \frac{d^3a}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \theta(\xi_{\alpha_1} - |\vec{\alpha} + \mathbf{q}|) \theta(\xi_{\alpha_2} - |\mathbf{a} - \mathbf{q}|)$$
$$\times \left[-T^{i_3}_{JI}(\ell, \ell, S; \mathbf{q}^2, \vec{\alpha}^2, \mathbf{q}^2) + \widetilde{g}_0 \Sigma_{\infty\ell} - m \int \frac{d^3p}{(2\pi)^3} \left\{ \frac{1 - \theta(\xi_{\alpha_1} - |\vec{\alpha} + \mathbf{p}|) - \theta(\xi_{\alpha_2} - |\vec{\alpha} - \mathbf{p}|)}{\mathbf{p}^2 - \mathbf{q}^2 - i\varepsilon} \Sigma_{p\ell} - \frac{\Sigma_{\infty\ell}}{\mathbf{p}^2} \right\} \right] . \tag{3.1}$$

where an expansion in NN partial waves in the nuclear medium is used. For further details in the notation we refer to Ref. [8]. The only quantity not determined from the NN scattering in vacuum

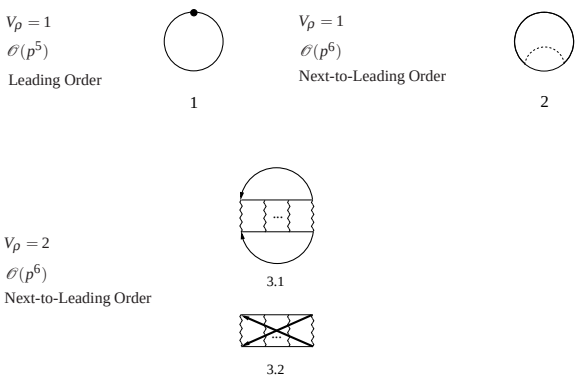

Figure 1: Contributions to the nuclear matter energy up to NLO or $\mathcal{O}(p^6)$. The wiggly lines correspond to the one-pion exchange plus the *NN* contact interactions from the $\mathcal{O}(p^0)$ quartic nucleon Lagrangian [3, 4].

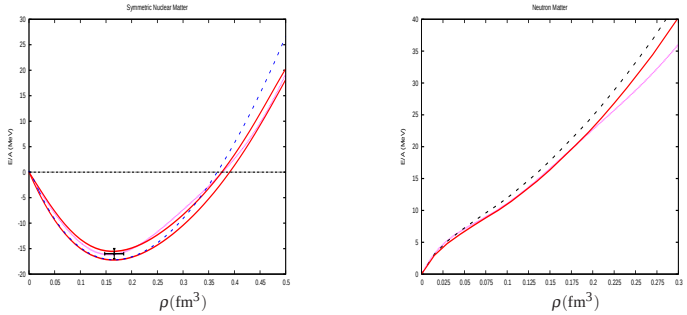

Figure 2: Energy per baryon for symmetric nuclear matter (left) and pure neutron matter (right).

is the constant $\widetilde{g}_0$. However, one can determine its natural size from the way it is introduced [8], around $-mm_\pi/4\pi \sim 0.510 m_\pi^2$. The resulting curves for the energy per baryon are shown in Fig. 2, left panel for symmetric nuclear matter and right one for purely neutron matter. The preferred final values of $\widetilde{g}_0$ are around $-1\, m_\pi^2$ for the former and $-0.5\, m_\pi^2$ for the latter. We see a good comparison with sophisticated many body calculations [13] shown by the dotted lines. The experimental point for the symmetric nuclear matter panel is indicated by the crossed. In this case we also are able to reproduce perfectly the nuclear matter incompressibility $K = 259$ MeV, to be compared with experiment $K = 250 \pm 25$ MeV [14].

4. Application to neutron stars. Constraining G_N.

We want to address two question by employing the energy per baryon (neutron matter equation of state at $T = 0$) obtained in the previous section. The first question is to know whether this equation of state, calculated from first principles, is able to account for the large mass of 1.97(4) $M_\odot$ of the recently observed pulsar J1614-2230 [9]. The second question is to know whether the Newtonian constant G_N has the same value as in the Earth for extremely large gravitational fields like those found in the previous pulsar, with its estimated acceleration on the surface around 2×10^{12} m/s^2 [10]. These questions were addressed in Ref. [10] and it was found that our equation of state

for neutron matter is able to accommodate such massive neutron stars, with an upper bound for a neutron star mass of around 2.3 $M_\odot$. This is shown in the left panel of Fig. 3.

Regarding G_N the procedure was to use the equation of state of Ref. [8] up to Fermi momenta of $450-600$ MeV (leaving this interval as a source of error) and then employ the hardest possible equation of state (with a sound velocity equal to the speed of light). The idea is that if the equation of state is harder this makes G_N to increase in order to provide enough gravitational attraction. Of course, G_N cannot grow indefinitely because otherwise the upper limit value for the mass of the neutron star would decrease too much, being in disagreement with the experimental determination for the pulsar J1614-2230 [9]. Taking into account all these considerations for the intense gravitational field in such neutron star, G_N cannot exceed 12% of its value on Earth at 95% confidence level. In the right panel of Fig. 3 we show this determination and others from other sources.

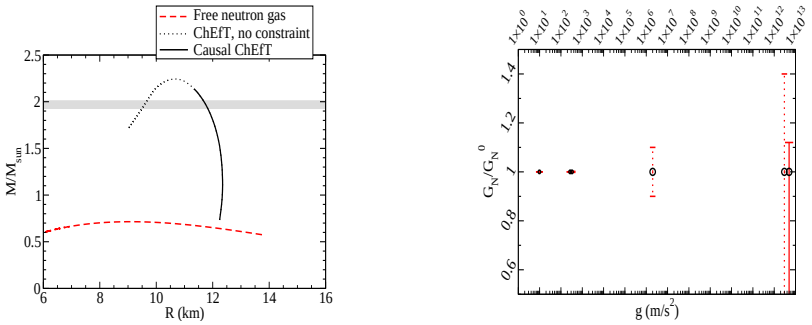

Figure 3: (Color online) Left panel: The neutron star mass as a function of the radius. Right panel: Newtonian constant normalized by its accepted value $6.6738(8)N(m/kg)^2$. From left to right: laboratory on Earth; orbital determinations of binary pulsars; white dwarf structure; neutron stars with 1.4 solar masses; neutron star with 1.97(4) solar masses.

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