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Particle Production at High Energy and Large Transverse Momentum

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We revisit the "hybrid formalism" for particle production used recently to study saturation effects in single hadron multiplicities at forward rapidities at RHIC and LHC. We point out that at leading twist there is an extra contribution to the formulae used so far, which corresponds to particle production via inelastic scattering of the projectile partons on the target fields. This contribution is expected to be small due to kinematics at very forward rapidities/very high transverse momenta, but should be significant at high momenta and very high energies. This contribution is expected to be most affected by saturation effects and is therefore an interesting object of study in the context of possible onset of saturation at high energies.

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1. Introduction

There has been a big improvement in interpreting the RHIC data within the Color Glass Condensate (CGC) formalism in the last decade. A strong suppression of particle production in deuteron - gold collisions at forward rapidities is observed at RHIC which can be interpreted as an evidence for the importance of saturation effects at RHIC. This effect appears also in [1]. Although the data is described very well, there are some peculiarities of the results of [1] that make one wonder. First a very large K-factor is required to fit the overall magnitude of the production of pions. Secondly and perhaps more worryingly, the suppression in the theoretical curves of [1] persists to extremely high transverse momenta, where one expects perturbation theory to be long applicable and R_{dA} to be equal to one. The calculations of [1] are based on the "hybrid formalism" of [2]. The question we would like to address is that if the formula derived in [2] takes into account all contributions at leading twist.

2. Gluon Production

In this section we derive the expression for the gluon contribution to hadron production in the hybrid formalism. The quark and antiquark contributions will be included in the following section.

We consider a process where an energetic projectile scatters off a static target. The wave function of the incoming projectile can be represented as

$$|\Psi\rangle_{in} = \Omega|v\rangle \tag{2.1}$$

where $|v\rangle$ is the zeroth order wave function and Ω is a unitary operator that diagonalizes the QCD Hamiltonian. The outgoing state after scattering is

$$|\Psi\rangle_{out} = S|\Psi\rangle_{in} \tag{2.2}$$

where S is the eikonal scattering matrix for the projectile partons which propagate through the static target fields. The number of produced gluons is then given by

$$\frac{dN}{d^2k dk^+} = \langle v|\Omega^\dagger S^\dagger \Omega a^\dagger(k,k^+) a(k,k^+) \Omega^\dagger S \Omega|v\rangle \tag{2.3}$$

In order to find the operator Ω, we start with the light-cone QCD Hamiltonian

$$H = \int_{k^+>0} \frac{dk^+}{2\pi} d^2x \left(\frac{1}{2} \Pi^-_a(k^+,x)\Pi^-_a(-k^+,x) + \frac{1}{4} G^{ij}_a(k^+,x) G^{ij}_a(-k^+,x) \right) \tag{2.4}$$

where the electric and magnetic pieces are

$$\begin{aligned} \Pi^-_a(x^-,x) &= -\frac{1}{\partial^+}(D^i \partial^+ A_i)^a(x^-,x) \\ G^{ij}_a(x^-,x) &= \partial_i A^a_j(x^-,x) - \partial_j A^a_i(x^-,x) - g f^{abc} A^b_i(x^-,x) A^c_j(x^-,x) \end{aligned} \tag{2.5}$$

We are working in the light cone gauge, $A^+ = 0$. The other light cone component of the vector potential A^- can be expressed via the solution of Maxwell's equations as $A^- = -\frac{1}{\partial^+}\partial_i A_i$. The transverse components of the vector potential A^i are expanded in terms of the gluon creation and annihilation operators

$$A_i^a(x^-,x) = \int_0^\infty \frac{dk^+}{2\pi}\frac{1}{\sqrt{2k^+}}\left\{a_i^a(k^+,x)e^{-ik^+x^-} + a_i^{a\dagger}(k^+,x)e^{ik^+x^-}\right\} \tag{2.6}$$

where the creation and annihilation operators satisfy the standard canonical commutation relations. Ω is a unitary operator and it can be defined in terms of a Hermitian operator G as $\Omega = e^{-iG}$. Since the eigenvalues of the Hamiltonian are the eigenvalues of the free Hamiltonian to first order in coupling constant, to this order the Hermitian operator G satisfies the following equation

$$i[H_0, G] = H_1 \tag{2.7}$$

where H_0 and H_1 are the zeroth order and first order Hamiltonians in the perturbative expansion of H_{QCD}. After some algebra one gets

$$G = -gf^{abc}\int_{k,p,k^+,p^+>0}\frac{1}{\sqrt{2k^+p^+(k^++p^+)}}\frac{1}{\omega_{p+k}-\omega_p-\omega_k}\left\{\left[p_i - \frac{p^+}{k^+}k_i\right]a_i^b(k^+,k) \right. \tag{2.8}$$
$$\left. \times a_j^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p) + \frac{p^+}{p^++k^+}k_j a_i^b(k^+,k)a_i^c(p^+,p)a_j^{a\dagger}(k^++p^+,k+p)\right\} + h.c.$$

with $\omega(k) = k^2/2k^+$. The number of produced gluons to leading order in the coupling constant is given by

$$\frac{dN}{d^2kdk^+} = \frac{1}{(2\pi)^3}\langle v|\left[\,\left[\hat{S}^\dagger G - G\hat{S}^\dagger\right] a_k^{a\dagger}(k^+,k)a_k^a(k^+,k)\left[G\hat{S} - \hat{S}G\right]|v\rangle \tag{2.9}$$

Using the fact that the S matrix operator acts as a color rotation on all gluon creation and annihilation operators in coordinate space, i.e. $\hat{S}^\dagger a_i^a(q^+,v)\hat{S} = S^{ab}(v)a_i^b(q^+,v)$, and Eq. (2.8) one can see that

$$\frac{dN}{d^2kdk^+} \propto \delta(p^+-q^+)\langle v|a^\dagger(p^++k^+)a(p^++k^+)|v\rangle + \langle v|a^\dagger(p^++k^+)a^\dagger(q^+)a(p^+)a(q^++k^+)|v\rangle \tag{2.10}$$

The second term involves a two particle density in the state $|v\rangle$ which is suppressed in the leading twist "partonic" approximation. This term is neglected since it is beyond our approximation. Thus, keeping only the first term of Eq. (2.10) and assuming that the projectile state is color and rotationally invariant, after some algebra the single inclusive gluon spectrum reads

$$\begin{aligned}\frac{dN}{d^2kdk^+} = &\frac{\alpha_s}{2\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int_x^1\frac{d\xi}{\xi}\frac{1}{k^+}e^{ik(z-\bar{z})}\frac{2}{1-\xi}\left[(1-\xi)^2+\xi^2+(1-\xi)^2\xi^2\right]\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(v-z)_i}{(v-z)^2}\\ &\times tr\left\{\left[S^\dagger_{(1-\xi)v+\xi\bar{z}}T^aS_{(1-\xi)v+\xi\bar{z}} - S^\dagger_vT^aS_{\bar{z}}\right]\left[S^\dagger_{(1-\xi)v+\xi z}T^aS_{(1-\xi)v+\xi z} - S^\dagger_zT^aS_v\right]\right\}\\ &\times\frac{k^+}{2\pi\xi}\langle a_j^{\dagger b}\left(\frac{k^+}{\xi},(1-\xi)v+\bar{z}\right)a_j^b\left(\frac{k^+}{\xi},(1-\xi)v+z\right)\rangle\end{aligned} \tag{2.11}$$

where ξ is the longitudinal momentum fraction. Let us now consider the soft limit of this expression which corresponds to the situation when the longitudinal momentum of the observed gluon is much smaller than the momentum of the gluons in the valance state. Thus, taking the limit $\xi \to 0$ of Eq. (2.11) we get

$$\frac{dN}{d^2k dk^+} = \frac{\alpha_s}{\pi^2}\frac{1}{(2\pi)^2}\frac{N_c}{N_c^2-1}\int \frac{1}{k^+}e^{ik(z-\bar{z})}\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(z-v)_i}{(z-v)^2} tr\Big\{1-S_v^\dagger S_z - S_{\bar{z}}^\dagger S_v + S_{\bar{z}}^\dagger S_z\Big\}\langle a_j^{\dagger a}\left(\frac{k^+}{\xi},v\right) a_j^a\left(\frac{k^+}{\xi},v\right)\rangle \tag{2.12}$$

Assuming color neutrality of the hadronic state, the k_T factorized form reads ([4], [5] and [6])

$$\frac{dN}{d^2k dk^+} = \frac{\alpha_s}{\pi^2}\frac{1}{(2\pi)^2}\frac{1}{N_c^2-1}\int \frac{1}{k^+}e^{ik(z-\bar{z})}\frac{(v-\bar{z})_i}{(v-\bar{z})^2}\frac{(z-v)_i}{(z-v)^2} tr\Big\{S_{\bar{v}}^\dagger S_v - S_{\bar{v}}^\dagger S_z - S_{\bar{z}}^\dagger S_v + S_{\bar{z}}^\dagger S_z\Big\}\langle \rho_v^a \rho_{\bar{v}}^a\rangle \tag{2.13}$$

In partonic approximation there is only a small number of gluons in the hadron and there are no correlations between different gluons. Therefore, for a color singlet hadronic state, color charge correlator can be written as

$$\langle \rho_v^a \rho_{\bar{v}}^a\rangle = \delta^2(v-\bar{v})N_c\langle\int \frac{dp^+}{2\pi}a_i^{\dagger a}(p^+,v)a_i^a(p^+,v)\rangle \tag{2.14}$$

Hence, in the leading twist approximation the soft limit of the "hybrid formula" is as the partonic limit of the k_T factorized formula. In the soft limit the color charge correlation function and scattering amplitude can be expressed in terms of the projectile and target gluon momentum distributions as

$$\langle \rho_v^a \rho_{\bar{v}}^a\rangle = \frac{1}{8\pi\alpha_s}\int_p e^{ip\cdot(v-\bar{v})}p^2\phi_P(p,b) \quad , \quad tr[1-S_v^\dagger S_{\bar{v}}] = 2\pi\alpha_s N_c\int_p e^{ip\cdot(v-\bar{v})}\frac{1}{p^2}\phi_T(p,b) \tag{2.15}$$

Then, the single inclusive gluon spectrum can be written as

$$\frac{dN}{d^2k d\eta d^2b} = \frac{\alpha_s N_c}{N_c^2-1}\int_l \left[\frac{1}{(l+k)^2}+\frac{1}{(l+k)^2}\frac{l^2}{k^2}+2\frac{1}{(l+k)^2}\frac{l.k}{k^2}\right]\phi_T(l+k)\phi_P(l) \tag{2.16}$$

In the limit of large momentum of the produced gluon $k \gg Q_s, \Lambda_{QCD}$ the momentum integral in Eq. (2.16) is dominated by two regions. In the first region $l \ll k$. In this kinematics the incoming projectile gluon has a small transverse momentum and scatters with a large transverse momentum transfer from the target. We refer to this contribution as elastic :

$$\left[\frac{dN}{d^2k d\eta}\right]_{elastic} = \frac{\alpha_s N_c}{N_c^2-1}\frac{1}{k^2}\phi_T(k)\int_{l<Q\sim k}\phi_P(l) \tag{2.17}$$

The second contribution comes from the momentum range $l = k+q$ with $q \ll k$. This contribution corresponds to a projectile gluon coming in with a large transverse momentum and scattering with a small momentum transfer. We refer to this contribution as inelastic:

$$\left[\frac{dN}{d^2k d\eta}\right]_{inelastic} = \frac{\alpha_s N_c}{N_c^2-1}\frac{1}{k^2}\phi_P(k)\int_{q<Q\sim k}\phi_T(q) \tag{2.18}$$

At high p_T, both contributions are of the same order of magnitude. The probability of finding a low p_T gluon in the projectile is of order unity, but the probability of scattering this low p_T gluon with a large momentum transfer is of order α_s. Whereas for the inelastic contribution, the probability of finding a high p_T parton in the incoming wave function is of order α_s, but scattering of this high p_T parton with a small momentum exchange is of order unity. Assuming a perturbative behavior, $\phi = \mu^2/p^2$ for both the projectile and the target, one can write

$$\left[\frac{dN}{d^2kd\eta}\right]_{elastic} = \alpha_s\mu_P^2\mu_T^2 \ln\frac{p^2}{\Lambda_{QCD}^2} \quad , \qquad \left[\frac{dN}{d^2kd\eta}\right]_{inelastic} = \alpha_s\mu_P^2\mu_T^2 \ln\frac{p^2}{Q_s^2} \tag{2.19}$$

where we have assumed perturbative behavior for the target above the saturation momentum, Q_s. It is clear that at parametrically large transverse momentum the two contributions are comparable and both must be kept.

If we now go back from soft limit to "hybrid formalism", it is easy to identify the same process. Elastic contribution is related to the case when all the transverse momentum of the produced gluon originates from the momentum transfer from the target. In the case of inelastic scattering, the large transverse momentum of the gluon in the final state can only arise from a large relative momentum between the two splitted gluons in the incoming projectile wave function.Taking into account both elastic and inelastic contributions, also including the gluon splitting functions and gluon fragmentation functions, one arrives to the final formula for the gluon production

$$\frac{dN}{d^2kd\eta} = \int_{x_F}^1 \frac{dz}{z^2} D_{h/g}(z,Q)\Bigg[x_1 f_g(x_1,Q^2)N_A(x_2,\frac{k}{z},b=0) \tag{2.20}$$
$$+\frac{\alpha_s}{\pi^2}\frac{N_c^2}{N_c^2-1}\frac{z^4}{k^4}\int_{x_1}^1\frac{d\xi}{\xi}\left[1-\xi+\xi^2\right]P_{g/g}(\xi)x_1 f_g\left(\frac{x_1}{\xi},Q^2\right)\int_{p^2<Q^2}\frac{d^2p}{(2\pi)^2}p^2N_F(x_2,p,b=0)\Bigg]$$

where

$$N_A\left(k,b=\frac{\bar{z}+z}{2}\right) = \frac{1}{N_c^2-1}\int d^2(z-\bar{z})e^{ik\cdot(z-\bar{z})}tr\left[S_A^\dagger(\bar{z})S_A(z)\right] \tag{2.21}$$
$$N_F\left(k,b=\frac{\bar{z}+z}{2}\right) = \frac{1}{N_c}\int d^2(z-\bar{z})e^{ik\cdot(z-\bar{z})}tr\left[S_F^\dagger(\bar{z})S_F(z)\right] \tag{2.22}$$

with the longitudinal momentum fractions are

$$x_F = \frac{k}{\sqrt{s_{NN}}}e^\eta \quad , \quad x_1 = \frac{x_F}{z} \quad , \quad x_2 = x_1 e^{-2\eta} \tag{2.23}$$

3. Including Quarks

We derive Eq. (2.20) for hadron production that includes both elastic and inelastic contribution in a theory that does not contain quarks. This is obviously not a good approximation to reality especially at forward rapidities, where the quark contribution must be the leading one. Thus, one has to include the quark contributions. After some cumbersome calculation in the same spirit as gluon production one arrives to the final formula

$$\frac{dN_i}{d^2kd\eta} = \frac{1}{(2\pi)^2}\int_{x_F}^1\frac{dz}{z^2}\left[x_1 f_g(x_1,Q^2)N_A(x_2,\frac{k}{z}D_{h/g}(z,Q)+\Sigma_q x_1 f_q(x_1,Q^2)N_F(x_2,\frac{k}{z})D_{h/q}(z,Q)\right]$$
$$+\int_{x_F}^1\frac{dz}{z^2}\frac{\alpha_s}{2\pi^2}\frac{z^4}{k^4}\int\frac{d^2p}{(2\pi)^2}p^2N_F(p,x_2)x_1\int_{x_1}^1\frac{d\xi}{\xi}\Sigma_j\omega_{i/j}(\xi)P_{i/j}(\xi)f_j\left(\frac{x_1}{\xi},Q\right)D_{h/q}(z,Q) \tag{3.1}$$

where the inelastic weights ω_i are defined in [7].

4. Discussion

We presented the complete leading twist expression for inclusive hadron production in the hybrid formalism that was derived in [7]. It was shown that in addition to elastic scattering terms first derived in [2], there are also terms that correspond to inelastic scattering of the projectile partons on low momentum components of the target field.

The final states that correspond to the inelastic process are dihadron pairs where both hadrons are emitted at forward rapidity and have strong back to back correlation. Since both produced hadrons have large rapidity, such pairs with large transverse momentum are kinematically allowed only at large collision energy. Thus one might expect this contribution not be of great importance in RHIC kinematics, but it may be sizeable at LHC.

However the effect of inelastic contributions to single inclusive hadron production in proton-proton and proton-nucleus collisions at RHIC and LHC are investigated in [3]. It is shown that including the effects of inelastic contributions not only gives a good description of RHIC data but it also leads to a sharper increase of the nuclear modification factor R_{pA} with increasing p_T.

References

[1] J. Albacete and C. Marquet , Phys. Lett. B 687: 174-179 (2010). [arXiv:hep-ph/1001.1378]

[2] A. Dumitru, A. Hayashigaki and J. Jalilian-Marian , Nucl. Phys. A 765 (2006). [arXiv:hep-ph/0506308]

[3] J. Jalilian-Marian and A.H. Rezaeian, Phys. Rev. D **85**, 014017 (2012). [arXiv:hep-ph/1110.2810].

[4] A. Kovner and M. Lublinsky , JHEP 0611:083,2006. [arXiv:hep-ph/0609227] .

[5] Y. V. Kovchegov and K. Tuchin , Phys. Rev. D **65**, 074026 (2002). [arXiv:hep-ph/0111362].

[6] R. Baier, A. Kovner, M. Nardi and U.A. Weidemann , Phys. Rev. D **72**, 094013 (2005). [arXiv:hep-ph/0506126].

[7] T. Altinoluk and A. Kovner, Phys. Rev. D **83**, 105004 (2011). [arXiv:hep-ph/1102.5327].

PoS(QNP2012)162