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# Failures of Nuclear Models of Deformed Nuclei

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In spite of the success of Bohr and Mottelson in giving a general description of deformed nuclei, there are many properties of these superfluid entities that continue to challenge us for adequate explanation. Inadequacies in present theoretical accounts become more and more obvious as experiments explore nuclear structure to higher spins and give much more information on excited levels away from the yrast line. An overview is given of the experimental data that does not currently have an adequate description and of the more promising current theoretical developments.

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## 1. Introduction

The Nilsson model [1] led the way in giving a very useful microscopic description of the single particle states to be found near the nuclear Fermi surface as a function of quadrupole deformation. Sophistications of the Nilsson model introducing pairing, cranking, tuned interactions, larger bases, asymptotically correct potentials, more complex shapes etc have had their successes. However the incorporation of collective motion of the deformed field has been fraught with difficulties, not only in the mathematics and required size of computing, but also in understanding how experiment can guide the underlying physics.

Clearly the shape of a nucleus is a dominant factor in determining its properties. This is graphically illustrated by the discoveries of fission isomers [2], superdeformed nuclei [3,4] and the properties of nuclei with permanent octupole deformation [5]. The phenomenon of shape coexistence has recently been extensively reviewed [6]. Distinctly different shapes for the same nucleus will bring different levels to the Fermi surface. But different intrinsic configurations having very similar shapes will only have minor adjustments to these levels.

Another dominant feature of nuclei is the effect of pairing forces [7]. These forces lower the energies of pairs of nucleons in time reversed orbits due to the large overlap of such wavefunctions and the strongly attractive nucleon-nucleon interaction. The most spectacular result of pairing forces is that all even-even nuclei have spin $0^+$ ground states. Also pairing forces in quadrupole deformed nuclei produce a gap between the intrinsic ground state and the next intrinsic configuration (rotational band head). This is because a pair has to be broken in order to form a new configuration of the nucleons. If the pairing energy per nucleon is ***Δ*** MeV then the gap will be about ***2Δ*** MeV. Any intrinsic states found within this gap have been traditionally ascribed to collective vibrations of the nuclear shape. This phenomenon is nicely illustrated in Fig. 1 which is a spectrum of tritons produced [8] in the proton stripping reaction $^{153}$Eu(α,t)$^{154}$Gd. Only three intrinsic states are observed in the gap between the ground state and the onset of a high density of particle-hole (p-h) states above 2.0 MeV.

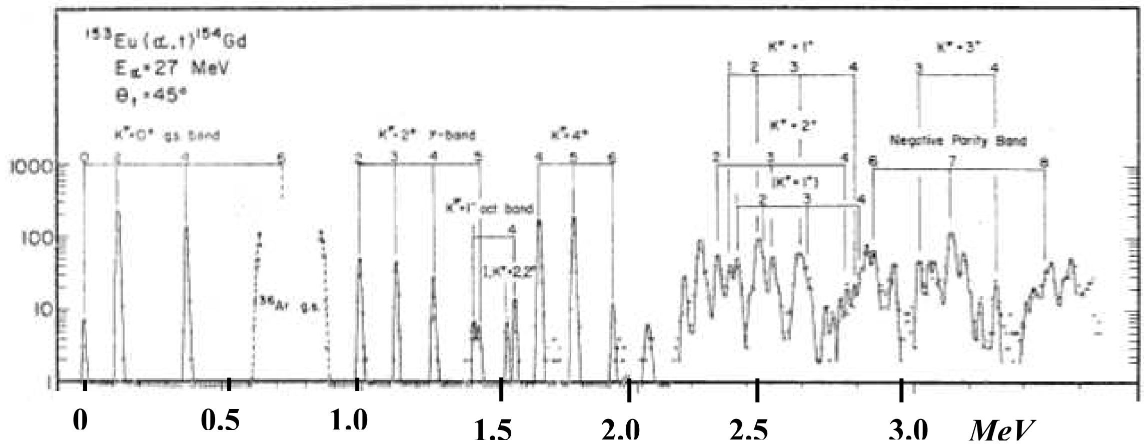


**Figure 1:** Spectrum of tritons [8] from the proton stripping reaction $^{153}$Eu(α,t)$^{154}$Gd. The pairing gap up to 2 MeV is clearly seen. The ground state band, the $K^\pi=2^+$ γ-band and a $K^\pi=4^+$ band lie within the pairing gap.

The introduction of pairing operators into the nuclear Hamiltonian leads to the concept of quasi-particles (qp) and the smearing of the Fermi surface [7,9]. This has its snags as particle number is no longer conserved and calculations have to eventually project their results on to good number and angular momentum. However the pairing concept has proved very successful in reproducing energy levels, spectroscopic factors from direct reactions and in the Back Bending phenomenon [10]. The latter is due to the Coriolis force caused by the rotation of the non-spherical nucleus. As the rotational frequency increases so does the Coriolis force on paired particles, being strongest on those in high-$j$ orbits. Finally the force is sufficient to break the pair so that their spins now align with the rotational axis [11,12].

It is fascinating to realise that classical concepts of rotations and Coriolis forces persist down to nuclear scales of $10^{-15}$ m.

However, the way pairing is used in most nuclear Hamiltonians makes it a fairly crude approximation. Schiffer and co-workers [13,14] have compiled data on the interaction of pairs of nucleons. They find that while the $0^+$ time reversed state of two identical $T=1$ nucleons in $m$ and $-m$ substrates is always lowest, there is a spread in level energies for other levels with finite total angular momentum $J$. This is illustrated in Fig.2 where the nucleon-nucleon interaction energy is plotted as a function of the angle $\theta_{12}$ between the individual nucleon spin vectors $j_1, j_2$ so that

$$\theta_{12} = \cos^{-1} \{ (j_1^2 + j_2^2 - J^2) / 2 j_1 j_2 \}$$

Because of this spread in energies for the states with $J \neq 0$ it is clear that the pairing approximation, where only the $J=0$ state has any extra binding energy, is not accurate to better than about 20%.

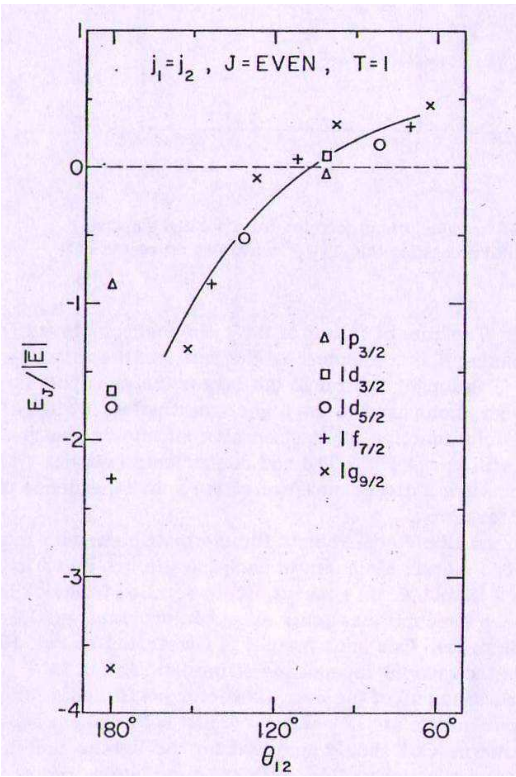


**Figure 2:** Normalised energies of states with two identical nucleons in the same $j$ orbit as a function of the angle $\theta_{12}$ between their angular momenta [14].

Most calculations stick with monopole pairing in which all states contribute equally to the pairing energy, except for an energy factor dependent on the distance of the level from the Fermi surface. Jerry Garrett et al. [15,16] pointed out long ago that this is just not the case. In the rare earth nuclei the first backbends in even-even nuclei are caused by the alignment of a pair of $\boldsymbol{i_{13/2}}$ neutrons. Rotational bands in odd neutron nuclei based on an odd neutron that is not involved in the alignment, backbend at a lower rotational frequency due to the reduction in the pairing energy caused by blocking by the odd neutron [17]. This is shown for rare earth isotopes in Fig.3. However it was observed that the bands based on the [505]11/2$^-$ high-K orbital, that is extruded from the lower shell by the deformation, backbend at the same frequency as the neighbouring even-even nuclei, Fig. 3. This observation demonstrates that this high-K orbital, up-sloping in the Nilsson diagram, does not partake in the pairing in the way that the down-sloping low-K orbitals do.

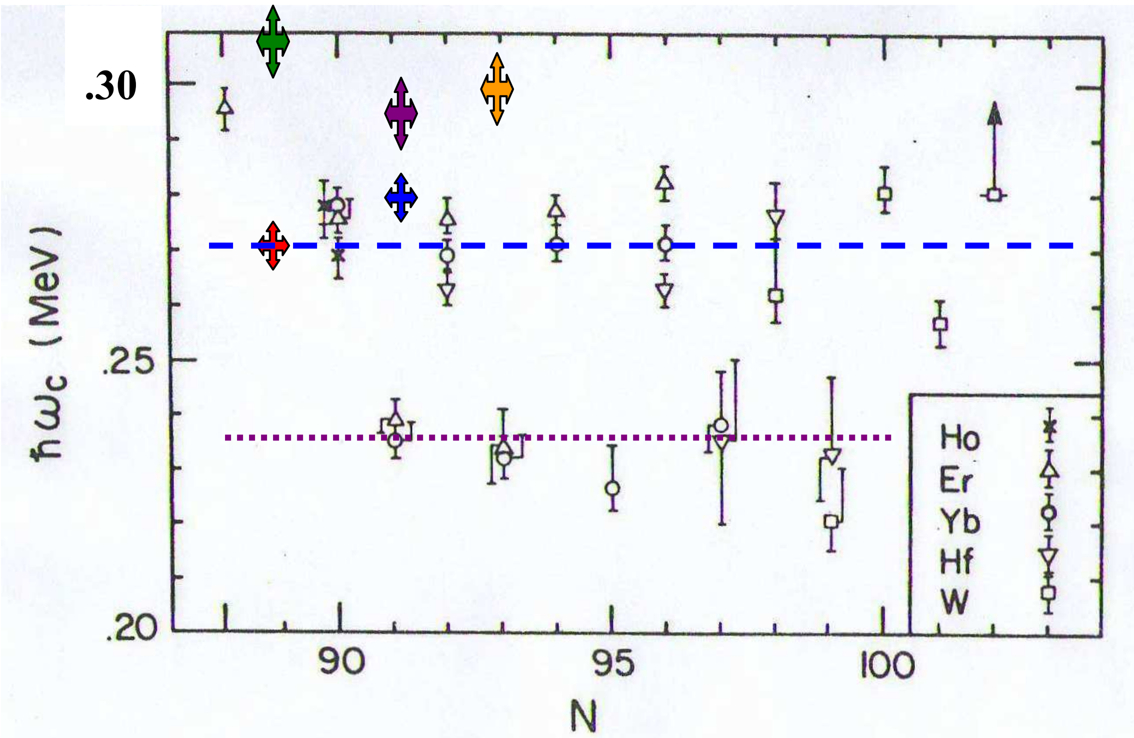


**Figure 3.** Alignment frequencies $\hbar\omega_c$ for $\boldsymbol{i_{13/2}}$ neutrons in rare earth nuclei [15]. The dashed (blue) line indicates the average for even number of neutrons and the dotted (purple) line the average for odd number of neutrons. The reduction in frequency is due to the blocking of pairing by the odd neutron. The coloured crosses show the alignment frequency for the high-K [505]11/2$^-$ bands in odd neutron nuclei demonstrating that this oblate orbital is not involved in the normal pairing; $^{153}$Gd green, $^{155}$Dy red, $^{157}$Dy purple, $^{159}$Er blue, $^{161}$Er orange.

The initial semi-classical way [18] of representing quadrupole oscillations of the shape of a deformed nucleus was in terms of volume conserving changes in the radius as;

$\boldsymbol{\delta R \propto (3\cos^2\theta - 1)\cos\omega_\beta t}$ for β-vibrations along the symmetry *z*-axis and

$\boldsymbol{\delta R \propto \sin^2\theta\, \cos(2\square \pm \omega_\gamma t)}$ for γ-vibrations in the (*x,y*) directions, perpendicular to the *z*-symmetry axis.

In the rotation-vibration model [19-21] the energies of the states that arise from these collective vibrations are given, in an obvious notation, by;

$$E(n_\beta n_\gamma IK) = \hbar\omega_\beta(n_\beta + ½) + \hbar\omega_\gamma(2n_\gamma + ½|K| + 1) + [I(I+1) - K^2]\,\hbar^2/2\mathscr{I} \qquad (1)$$

If $n_\beta = 1$ and $n_\gamma = 0$ in equ.(1), then a rotational band exists with its band-head at an excitation energy $E_x = \hbar\omega_\beta$ , spin and parity $I^\pi = 0^+$ and spin projection on the symmetry axis $K^\pi = 0^+$. This band has always been assumed to be the lowest excited $0_2^+$ state and is referred to as the "β-vibrational band". It is identified with β deformation oscillations of the nuclear shape along the symmetry axis. Unfortunately there is little evidence that these $0_2^+$ states have the properties of a β-vibration [22,23]. {The ground state is the first $0^+$ state in even-even nuclei and is usually referred to as $0_1^+$.}

Most text books manage to miss the $½|K|$ term in Equ. (1), giving the impression that the first $K^\pi=2^+$ band has $n_\gamma= 1$. But Equ. (1) shows that the traditional $K^\pi=2^+$ γ-band is not a band containing a quantum in the γ direction but has $n_\gamma= 0$ and a bandhead excitation energy given by $E_x= \hbar\omega_\gamma + \hbar^2/\mathscr{I}$. In the rotation-vibration model there is a strong coupling between rotations and γ-vibrations, physically expressing the fact that rotations with non-vanishing K become possible only in the presence of dynamical triaxiality [20,21]. Any model having the γ degree of freedom will have zero-point fluctuations and a similar origin for $K=2^+$ bands.

It is very clear that the **γ** degree of freedom, in describing the shapes of deformed nuclei, is indispensible. A nice illustration of this is the self-consistent relativistic mean field plus BCS calculations of the München group and colleagues [24,25]. Fig. 4 shows that for the deformed nuclei $^{148}$Nd and $^{150}$Nd, strong minima with oblate shapes seen in the calculations using only β deformation [24], turn out to be saddle points on a very γ-soft total energy surface when the **γ** degree of freedom is included in the calculations [25].

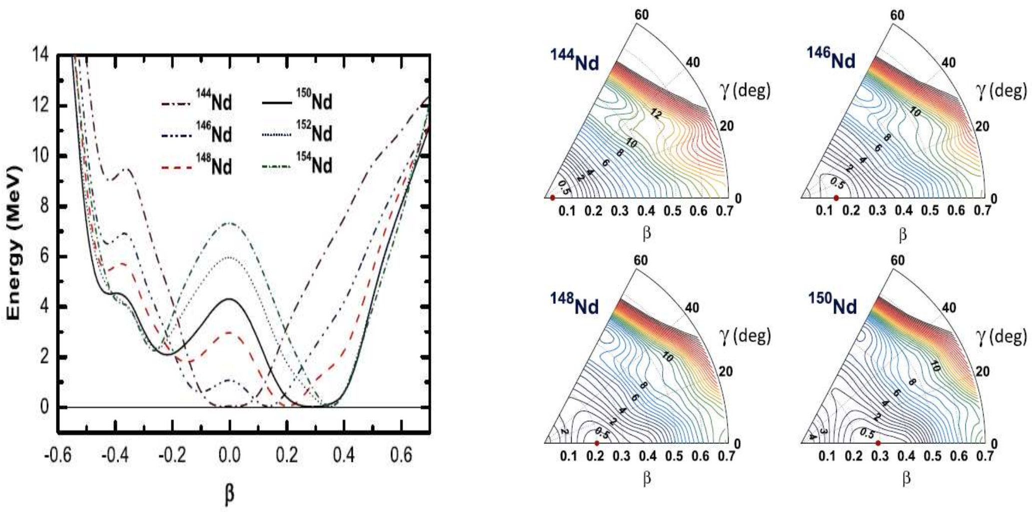


**Figure 4.** Self-consistent relativistic mean field plus BCS calculations for the even Nd isotopes. On the left are the ground state energies calculated [24] varying only the axial β degree of freedom. To the right are the total energy surfaces calculated [25] when the axial-symmetry breaking γ degree of freedom is included. These calculations show that the oblate minima on the left are really saddle points associated with the deeper prolate minima due to γ softness.

Initial microscopic descriptions of the collective vibrations of deformed nuclei [26-30] start with the assumption there are indeed β and γ vibrations of the nuclear mass distribution and therefore shape. These they represent by Bosons or Phonons that have the required quantum numbers. The second quantization formalism is an elegant way of dealing with such objects, so Boson or Phonon operators are set up, with the appropriate commutation algebra, to operate on a vacuum. This vacuum is taken to be the ground state of the nucleus. Clearly such vacua are different for each individual even-even nucleus due to the changes in the numbers of nucleons.

The Boson/Phonon operators are then expanded, in some convenient basis, in terms of single particle-single hole (p-h) excitations, as these are the lowest energy excitations of any vacuum. This may be a spherical basis, an oscillator basis or the set of Nilsson states with or without pairing. To generate such operators an interaction has to be found or postulated which is then usually expanded in terms of spherical harmonics. Invariably, after suitable approximations, variational minimisations and the like, some unknown parameters, such as the strength(s) of the interaction(s), have to be determined by fitting to some data.

The Achilles Heel of this approach is that it is almost invariably assumed that the lowest experimentally observed excited $0_2^+$ state is the one boson/phonon $K^\pi= 0^+$ excitation and that the lowest $K^\pi= 2^+$ state is the $Y_{2,2}$ vibration. This contribution is to point out that the first assumption is wrong and that the second assumption is probably wrong.

## 2. Experimental Data

It has been convincingly shown by Paul Garrett [22] that the $0_2^+$ states in nuclei do not meet the criteria required to be identified as the β-vibrations postulated by Bohr and Mottelson [18]. The nearest properties that he could find that could belong to a β-vibration were for the $0_2^+$ state in $^{154}$Gd. It has been shown [23] that this $0_2^+$ state in $^{154}$Gd, at an excitation energy of 681 keV, is actually a 2p-2h neutron state lowered into the pairing gap by the configuration dependent pairing postulated by Griffin, Jackson and Volkov [31]. Such states can exist in the pairing gap when there is a high-K Nilsson oblate orbital that has been extruded to the Fermi surface by the deformation. This orbital does not contribute to the normal pairing [15 and Fig. 3] as it is decoupled from the high density of low-K prolate orbitals that are driving the deformation. This decoupling is due to the oblate-prolate pairing force $G_{op}$ being significantly weaker than the oblate-oblate $G_{oo}$ and prolate-prolate $G_{pp}$ pairing forces. Central to this model is the paucity of oblate Nilsson levels near the Fermi surface. This decoupling of the polar and equatorial orbitals leads to the oblate pairing energy $\Delta_o$ , and hence the oblate quasi-particle energy, being reduced and permitting the existence of low-lying $0^+$ states. Also the two neutron transfer cross-section to these states is no longer reduced by the normal pairing effects. Ragnarsson and Broglia [32] coined the term "Pairing Isomers" for such $0^+$ levels. This very simple concept is illustrated in Fig. 5 for N = 90 nuclei, for example $^{154}$Gd$_{90}$. The relationship between the experimental excitation energies of $0_2^+$ states in nuclei with even proton number Z and neutron numbers N = 96 - 98 and the excitation energies of the intruder [505]11/2$^-$ Nilsson states in the neighbouring odd neutron nuclei, is shown in Fig. 6. This relationship between extruded orbitals and low-lying $0_2^+$ states has been commented on in many previous publications [33-36].

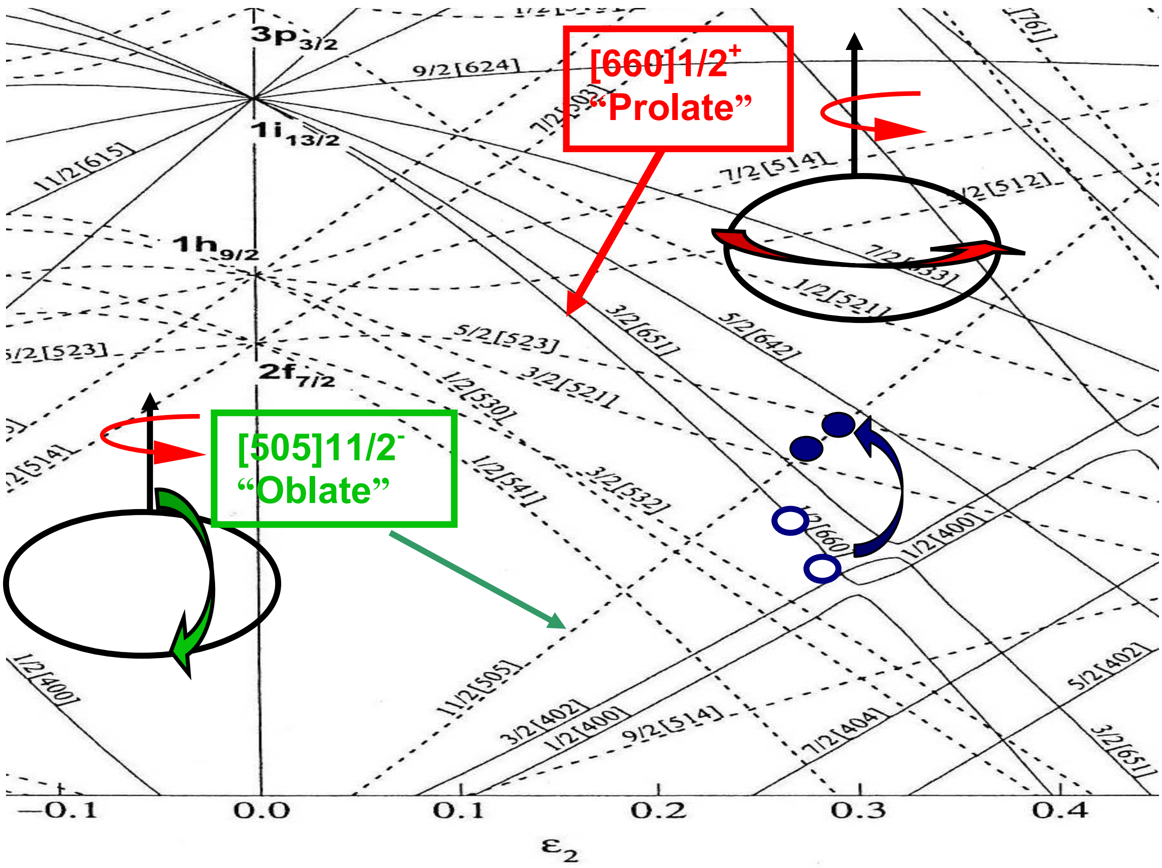


**Figure 5.** Nilsson diagram illustrating the configuration of the $0_2^+$ states in N = 90 nuclei, e.g. $^{154}Gd_{90}$. Two neutrons are taken out of a down-sloping prolate orbital and put in the up-sloping [505]11/2$^-$ oblate orbital from the $h_{11/2}$ shell. Pairing is configuration dependent, decoupling the high density of down-sloping prolate orbitals from the low density of up-sloping oblate orbitals.

This mechanism was first pointed out [31] to explain the $0_2^+$ states in actinide nuclei, observed by Maher [37], which did not have the properties of a β-vibration. But the mechanism is not confined to the actinides or to the rare earths. It has to be the case when any intruder high-K orbital is extruded to the Fermi surface by the deformation and has a lower density of fellow oblate orbitals than the deformation driving prolate orbitals. One example is the spherical to deformed shape change near A=100 [6]. In this case the extruded oblate orbit is the neutron [404]9/2$^+$ from the $\boldsymbol{g_{9/2}}$ shell [38,39]. Another example is in $^{128}$Ce where a "β-band" has been observed [40] to high spin. In this case the extruded oblate orbital is the proton [404]9/2$^+$ from the $\boldsymbol{g_{9/2}}$ shell. It should be stressed that the properties of these $0_2^+$ states is not the same as "shape coexistence" [6] as the $0_2^+$ states have very similar deformations as their ground $0_1^+$ states. Hence they have the same Nilsson orbitals at the Fermi surface. The occurrence of these $0_2^+$ states, low down in the pairing gap of even-even nuclei, has to be caused by a distinct mechanism. It is not due to a potential barrier between different shapes.

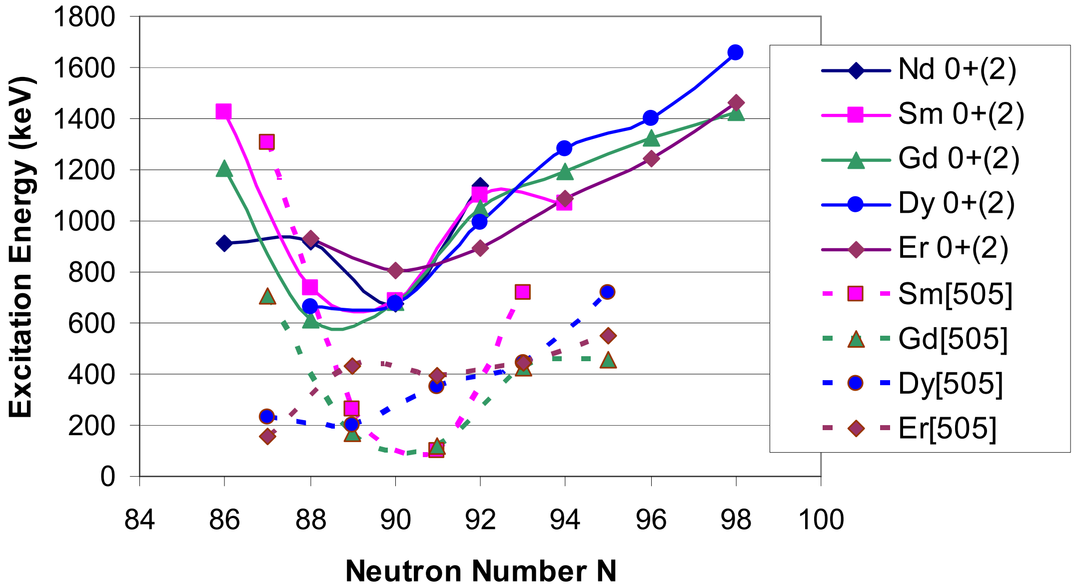


**Figure 6.** Systematics of the excitations energies of $0_2^+$ states in even-even nuclei, continuous lines, and of the [505]11/2$^-$ neutron states in the neighbouring odd-N nuclei, dotted lines.

If the $0_2^+$ states were true collective states composed of many p-h components, with no individual p-h configuration dominating, then in the neighbouring odd nuclei ALL the single particle states should couple equally to the collective $0_2^+$ state with only minor deviations from the excitation energy of the collective state. A nice illustration of this is the $^{157}$Gd(p,t)$^{155}$Gd two neutron pick-up reaction [41] shown in Fig. 7. The ground state of $^{157}$Gd is a [521]3/2$^-$ neutron which is also the ground state configuration of $^{155}$Gd. Hence the (p,t) reaction populates the ground state band in $^{155}$Gd and the $K^\pi = 3/2^-$ band formed by coupling the [521]3/2$^-$ neutron to the $0_2^+$ state in $^{154}$Gd. This coupling has also been observed in (n,γ) spectroscopy [42]. However the coupling of the high-K [505]11/2$^-$ intruder neutron orbital to the $0_2^+$ state in $^{154}$Gd is missing. This shows [43] that the $0_2^+$ state has two time reversed neutrons in the [505]11/2$^-$ orbit as a major component of its wave function. This blocking of the coupling of [505]11/2$^-$ neutron orbitals to $0_2^+$ states is found to be a general property of odd neutron nuclei near N=90.

Unlike the $0_2^+$ states in even-even nuclei the lowest $K^\pi = 2^+$ "γ-vibrational" bands are sometimes excited in single particle transfer reactions. This can be seen in Fig. 1 where the "γ-vibrational" band is populated in the $^{153}$Eu(α,t) $^{154}$Gd proton stripping reaction. This is a relatively rare case as the transfer is into the [411]1/2$^+$ proton orbital which then couples to the [413]5/2$^+$ ground state proton of $^{153}$Eu to give an allowed value of ΔK = 2. More often there are no available orbitals for the ΔK = 2 rule to be obeyed and no single particle transfer to the "γ-band" is observed. The properties of "γ-bands" were extensively discussed [44] at last year's Bormio Conference, Bormio 2011. It is clear that the $K^\pi = 2^+$ bands are real collective phenomena unlike the ephemeral β-vibrations. It is not yet clear to me if the "γ-vibrational" bands are just a $\boldsymbol{K^\pi=2^+}$ projection of the zero point motion on the symmetry axis, or if they are more of a traditional Boson or Phonon? What the experimental data suggest is that there is a γ-vibration built on every intrinsic state.

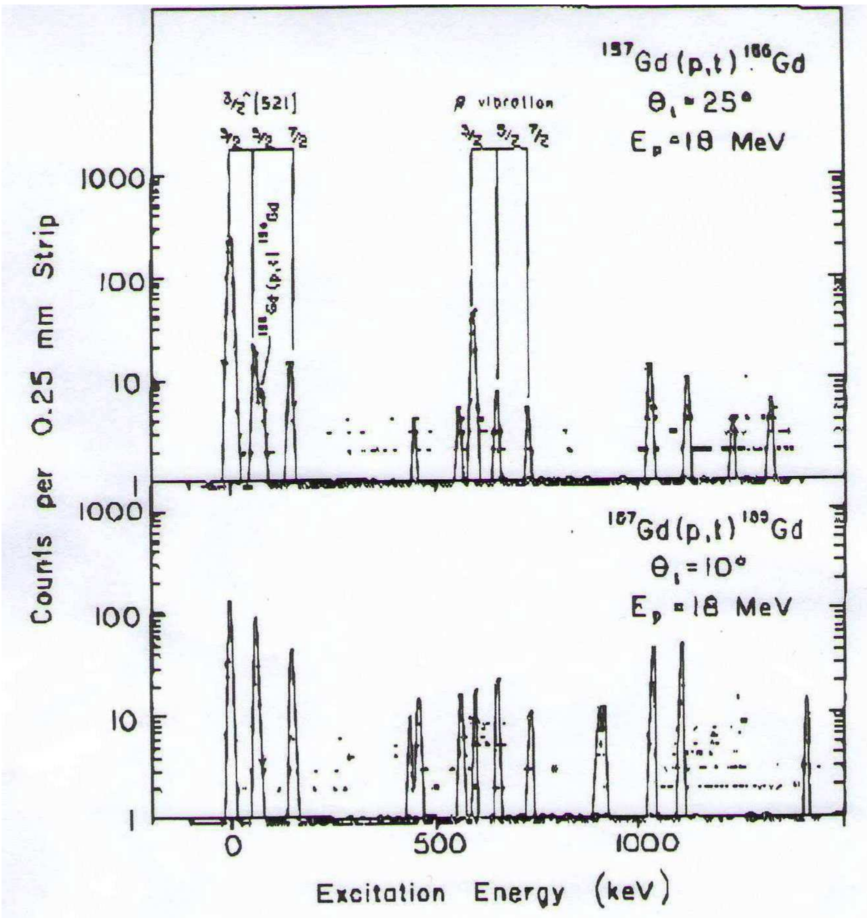


**Figure 7.** Triton spectra from the $^{157}$Gd(p,t)$^{155}$Gd reaction [41]. Members of the ground state [521]3/2$^-$ band and this neutron coupled to the $0_2^+$ state in $^{154}$Gd (β-vibrational band) are observed.

## 3. Discussion

It becomes rather obvious that macroscopic theories, such as the Interacting Boson Model (IBM), in its various forms, and models based on the Bohr Hamiltonian cannot address many of the pertinent features of the experimental data that we now have for deformed nuclei. The IBM was originally put forward [45] as a way of truncating the huge shell model spaces required to describe medium mass nuclei, A≈100, and to give a good account of the apparent vibrational structure of the lowest levels. As the microscopic fermion structure of the nucleus is lost, so is the Pauli principle, the ability to describe the alignment of the high-***j*** orbitals that causes back-bending in deformed nuclei, the ability to calculate particle transfer spectroscopic factors, any hope of calculating M1 strengths, etc. In particular, claims are made for achieving "good" fits to ground-state bands, by adjusting the symmetry of a Hamiltonian, while ignoring the established fact that the strength of the mixing of the ground-state configuration with the aligned "S-band" can influence the energies of the yrast levels down to very low spins.

As discussed above, expansions in terms of p-h states of Bosons [26-28] and/or Phonons [29,30], even ones that preserve the Pauli Principle [46], suffer from the assumption that the lowest K=0 excitation in their model can be identified with the $0_2^+$ state observed experimentally in deformed nuclei. As experiments, over many years, have shown that these are 2p-2h hole states, or 4qp states, it is not surprising that these models have not been all that successful!

However, Random Phase Approximation (RPA) calculations, which also represent vibrations in terms of p-h states, have had their successes. A classic example is the success of Takashi Nakatsukasa et al. [47] in describing the structure of excited Superdeformed bands in the Hg isotopes in terms of $Y_{3,2}$ octupole vibrational states.

Quasi-particle RPA (QRPA) calculations have been carried out by Zawisha, Speth and Pal [48] for deformed nuclei in the rare earths and actinides. They use a deformed Wood-Saxon potential for their single particle basis and a zero-range density dependent residual interaction. They calculate both low-lying "β-vibrations" and "γ-vibrations" as well as giant resonances. They conclude;

⇒**"…it is more reasonable to identify the high-lying $K^\pi = 0^+$ *and* $2^+$ giant quadrupole resonances with the classical β- and γ-vibrations."**

⇒ **"This is due to the fact that the energies of the low-lying states are mainly given by the details of the single-particle structure at the Fermi surface whereas the high-lying states are of real collective nature."**

⇒ **"…the microscopic wave vector of the low-lying $K^\pi = 0^+$ states is predominantly of the pairing-vibrational type in agreement with the enhanced two-particle transfer cross sections."**

As any pairing forces have to come out of their residual interaction, they are not confined by the usual approximations of monopole pairing. In the face of results like these, it is rather amazing that the nuclear structure community continues to interpret the low-lying spectroscopy of deformed nuclei in terms of β- and γ-vibrations or worse, interacting bosons.

Another very promising theoretical approach is the Triaxial Projected Shell Model (TPSM) pioneered by Kenji Hara [49] together with Javid Sheikh and Yang Sun. The TPSM uses a triaxial deformed Nilsson basis with a quadrupole-quadrupole interaction and both monopole and quadrupole pairing. This gives mixed K states which, when projected to good angular momentum, results in bands with spins 0, 2, 4 … Their calculations [50] of $0_n^+$ states in $^{158}$Er results in the conclusion that these states are mixed qp states coupled to vibrations. The calculations of $K^\pi = 2^+$ bands in the Er isotopes [51] and $K^\pi = 4^+$ bands [52] are equally promising.

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