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Cosmological applications of Delta-Gravity

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We present a model of the gravitational field based on two symmetric tensors. The equations of motion of test particles are derived: Massive particles do not follow a geodesic but massless particles trajectories are null geodesics of an effective metric. Outside matter, the predictions of the model coincide exactly with General Relativity, so all classical tests are satisfied. In Cosmology, we get accelerated expansion without a cosmological constant. We also give a preliminary discussion of Dark Matter within the model.

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1. Introduction

General Relativity(GR) works very well at the macroscopic scales[1]. Its quantization has proved to be difficult, though. It is non renormalizable, which prevents its unification with the other forces of Nature. Trying to make sense of Quantum GR is the main physical motivation of String Theories [2]. Moreover, recent discoveries in Cosmology [6, 7] has revealed that most part of matter is in the form of unknown matter(dark matter,DM) and that the dynamics of the expansion of the Universe is governed by a mysterious component that accelerates the later stages of the expansion(dark energy,DE). Although GR is able to accommodate both DM and DE, the interpretation of the dark sector in terms of fundamental theories of elementary particles is problematic[8]. Although some candidates exists that could play the role of DM, none have been detected yet. Also, an alternative explanation based on the modification of the dynamics for small accelerations cannot be ruled out[9].

In GR, DE can be explained if a small cosmological constant(Λ) is present. At the later stages of the evolution of the Universe Λ will dominate the expansion, explaining the acceleration. Such small Λ is very difficult to generate in Quantum Field Theory (QFT) models, because in this models Λ is the vacuum energy, which is usually very large.

In recent years there has been various proposals to explain the observed acceleration of the universe. They involve the inclusion of some additional field like in quintessence, chameleon, vector dark energy or massive gravity; Addition of higher order terms in the Einsten-Hilbert action, like f(R) theories and Gauss-Bonnet terms; Modification of gravity on large scales by introduction of extra dimensions. For a review, see [10].

Less widely explored, but interesting possibilities, are the search for non-trivial ultraviolet fixed points in gravity (asymptotic safety[12]) and the notion of induced gravity[13]. The first possibility uses exact renormalization-group techniques[14] and lattice and numerical techniques such as Lorentzian triangulation analysis[15]. Induced gravity proposed that gravitation is a residual force produced by other interactions.

In recent papers, [16, 17] a field theory model explore the emergence of geometry by the spontaneous symmetry breaking of a larger symmetry where the metric is absent. Previous work in this direction can be found in [18], [19] and [20].

In this paper, we will review the results of [32, 24]. The main observation is that GR is finite on shell at one loop[21]. In [23, 22] we presented a type of gauge theories, δ gauge theories(DGT): The main properties of DGT are: 1) The classical equations of motion are satisfied in the full Quantum theory 2) They live at one loop. 3) They are obtained through the extension of the former symmetry of the model introducing an extra symmetry that we call δ symmetry, since it is formally obtained as the variation of the original symmetry. When we apply this prescription to GR we obtain δ gravity. Quantization of δ gravity is discussed in [24].

The impact of dark energy on cosmological observations can be expressed in terms of a fluid equation of state $p = w(R)\rho$, which is to be determined studying its influence on the large-scale structure and dynamics of the Universe.

In this paper we follow the same approach. So we will not include the matter dynamics, except by demanding that the energy-momentum tensor of the matter fluid is covariantly conserved. This is required in order to respect the symmetries of the model.

The main properties of this model at the classical level are: a)It agrees with GR, outside the sources and with adequate boundary conditions. In particular, the causal structure of delta gravity in vacuum is the same as in General Relativity. So all standard test are satisfied automatically. b) When we study the evolution of the Universe, it predicts acceleration without a cosmological constant or additional scalar fields. The Universe ends in a Big Rip, similar to the scenario considered in [25]. c) The scale factor agrees with the standard cosmology at early times and show acceleration only at late times. Therefore we expect that density perturbations should not have large corrections.

In section 2, we write the action defining the model and the corresponding symmetries. Section 3 discusses the motion of particles in the model. In section 4 we define proper time and distances. In section 5 we solve the equations of the model for Friedman-Robertson- Walker metric. In section 6, we find the red shift. In section 7, we define luminosity distance. In section 8, we fit the equations of section 6 to the Supernova Ia data. Section 9 contains a preliminary discussion of Dark Matter. Section 10 contains the conclusions and brief discussions of open problems. In Appendix A, we review δ-symmetries.

2. Definition of Delta gravity

In this section we define the action as well as the symmetries of the model and derive the equations of motion.

We use the metric convention of [11]. The action of δ gravity is:

$$\begin{aligned} S(g,\tilde{g},\lambda) = \int d^dx\sqrt{-g}(-\frac{1}{2\kappa}R+\mathscr{L}_M)+ \\ \kappa_2\int\left[\left(R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R\right)+\kappa T_{\mu\nu}\right]\sqrt{-g}\tilde{g}^{\mu\nu}d^dx \quad + \\ \kappa_2\kappa\int\sqrt{-g}\,(\lambda^{\mu;\nu}+\lambda^{\nu;\mu})\,T_{\mu\nu}d^dx \end{aligned} \tag{2.1}$$

Here $\kappa=\frac{8\pi G}{c^4}$, κ_2 is an arbitrary constant and $T_{\mu\nu}:=-\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathscr{L}_M)}{\delta g^{\mu\nu}}$ is the energy-momentum tensor of matter.$R_{\mu\nu}$ is the Ricci's tensor and R is the curvature scalar of $g_{\mu\nu}$. $\tilde{g}^{\mu\nu}$ is a two-contravariant tensor under general coordinate transformations.

The action (2.1) is obtained by applying the prescription contained in [23, 22]. That is, we add to the action of general relativity, the variation of it and consider the variation $\delta g_{\mu\nu}=\tilde{g}_{\mu\nu}$ as a new field. Similarly, the symmetries we write below are obtained as variation of the infinitesimal general coordinate transformations where the variation of the infinitesimal parameter $\delta\xi_0^\rho=\xi_1^\rho$ is the infinitesimal parameter of the new transformation δ. The last term in (2.1) is needed to implement the condition $T^{\mu\nu}_{;\nu}=0$ as an equation of motion in order to implement the δ symmetry (2.2) off shell. This term is not needed in vacuum.

Action (2.1) is invariant under the following transformations(δ):

$$\begin{aligned} \delta g_{\mu\nu} &= g_{\mu\rho}\xi^\rho_{0,\nu}+g_{\nu\rho}\xi^\rho_{0,\mu}+g_{\mu\nu,\rho}\xi^\rho_0=\xi_{0\mu;\nu}+\xi_{0\nu;\mu} \\ \delta\tilde{g}_{\mu\nu}(x) &= \xi_{1\mu;\nu}+\xi_{1\nu;\mu}+\tilde{g}_{\mu\rho}\xi^\rho_{0,\nu}+\tilde{g}_{\nu\rho}\xi^\rho_{0,\mu}+\tilde{g}_{\mu\nu,\rho}\xi^\rho_0 \\ \delta\lambda_\mu &= -\xi_{1\mu}+\lambda_\rho\xi^\rho_{0,\mu}+\lambda_{\mu,\rho}\xi^\rho_0 \end{aligned} \tag{2.2}$$

From now on we will fix the gauge $\lambda_\mu = 0$. This gauge preserves general coordinate transformations but fixes completely the extra symmetry with parameter $\xi_{1\mu}$.

Equations of motion Varying $g_{\mu\nu}$ we get:

$$S^{\gamma\sigma} + \frac{1}{2}(R\tilde{g}^{\gamma\sigma} - g_{\mu\nu}\tilde{g}^{\mu\nu}R^{\gamma\sigma}) - \frac{1}{2}g^{\gamma\sigma}\frac{1}{\sqrt{-g}}\left(\sqrt{-g}\nabla_\nu \tilde{g}^{\mu\nu}\right)_{,\mu} + \frac{1}{4}g^{\gamma\sigma}\frac{1}{\sqrt{-g}}\left(\sqrt{-g}g^{\alpha\beta}\nabla_\beta(g_{\mu\nu}\tilde{g}^{\mu\nu})\right)_{,\alpha} = \kappa\frac{\delta T_{\mu\nu}}{\delta g_{\gamma\sigma}}\tilde{g}^{\mu\nu} \tag{2.3}$$

where $S^{\gamma\sigma} = (U^{\sigma\beta\gamma\rho} + U^{\gamma\beta\sigma\rho} - U^{\sigma\gamma\beta\rho})_{;\rho\beta}$, $U^{\alpha\beta\gamma\rho} = \frac{1}{2}\left[g^{\gamma\rho}(\tilde{g}^{\beta\alpha} - \frac{1}{2}g^{\alpha\beta}g_{\mu\nu}\tilde{g}^{\mu\nu})\right]$

Varying $\tilde{g}^{\mu\nu}$ we get Einstein equation:

$$\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \kappa T_{\mu\nu} = 0 \tag{2.4}$$

Varying λ_μ we get:$T^{\mu\nu}_{;\nu} = 0$

Covariant derivatives as well as raising and lowering of indices are defined using $g_{\mu\nu}$. Notice that outside the sources($T_{\mu\nu} = 0$), a solution of (2.3) is $\tilde{g}^{\mu\nu} = \lambda g^{\mu\nu}$, for a constant λ, since $g^{\mu\nu}_{;\rho} = 0$ and $R_{\mu\nu} = 0$. We will have $\tilde{g}^{\mu\nu} = g^{\mu\nu}$, assuming that both fields satisfy the same boundary conditions far from the sources.

The equation for $\tilde{g}^{\mu\nu}$ is linear and of second order in the derivatives.

3. Particle motion in the gravitational field

We are aware of the presence of the gravitational field through its effects on test particles. For this reason, here we discuss the coupling of a test particle to a background gravitational field, such that the action of the particle is invariant under (2.2).

In δ gravity we postulate the following action for a test particle:

$$S_p = -m\int dt\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} + \kappa_2'\int d^ny\sqrt{-g}\mathscr{T}_{\mu\nu}(\tilde{g}^{\mu\nu} + \lambda^{\mu;\nu} + \lambda^{\nu;\mu})$$

where $\mathscr{T}_{\mu\nu}$ is the energy momentum tensor of the test particle:

$$\mathscr{T}_{\mu\nu}(y) = \frac{m}{2\sqrt{-g}}\int dt\frac{\dot{x}_\mu\dot{x}_\nu}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}\delta(y-x)$$

$\kappa_2' = \kappa_2\kappa$ is a dimensionless constant.

That is:

$$S_p = m\int\frac{dt}{\sqrt{-g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}}\left(g_{\mu\nu} + \frac{{\kappa_2}'}{2}\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu \tag{3.1}$$

were $\bar{g}_{\mu\nu} = \tilde{g}_{\mu\nu} + \lambda^{\mu;\nu} + \lambda^{\nu;\mu}$. Notice that S_p is invariant under (2.2) and t-parametrizations.

From now on we work in the gauge $\lambda_\mu = 0$.

Since far from the sources, we must have free particles in Minkowski space,i.e $g_{\mu\nu} \sim \eta_{\mu\nu}, \tilde{g}_{\mu\nu} \sim \eta_{\mu\nu}$, it follows that we are describing the motion of a particle of mass $m' = m(1 + \frac{\kappa_2}{2}')$

Since in vacuum $\tilde{g}^{\mu\nu} = g^{\mu\nu}$, the equation of motion for test particles is the same as Einstein's. Moreover, the equation of motion is independent of the mass of the particle.

In order to include massless particles, we prefer to use the action [26]:

$$L = \frac{1}{2}\int dt \left(vm^2 - v^{-1}\left(g_{\mu\nu} + \kappa_2'\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu + \frac{m^2 + v^{-2}\left(g_{\mu\nu} + \kappa_2'\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu}{2v^{-3}g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}\left(m^2 + v^{-2}g_{\lambda\rho}\dot{x}^\lambda\dot{x}^\rho\right)\right) \tag{3.2}$$

This action is invariant under reparametrizations:

$$x'(t') = x(t); dt'v'(t') = dtv(t); t' = t - \varepsilon(t) \tag{3.3}$$

The equation of motion for v is:

$$v = -\frac{\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}}{m} \tag{3.4}$$

Replacing (3.4) into (3.2), we get back (3.1).

Let us consider first the massive case. Using (3.3) we can fix the gauge $v = 1$. Introducing $mdt = d\tau$, we get the action:

$$L_1 = \frac{1}{2}m\int d\tau \left(1 - \left(g_{\mu\nu} + \kappa_2\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu + \frac{1 + \left(g_{\mu\nu} + \kappa_2\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu}{2g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta}\left(1 + g_{\lambda\rho}\dot{x}^\lambda\dot{x}^\rho\right)\right) \tag{3.5}$$

plus the constraint obtained from the equation of motion for v:

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1 \tag{3.6}$$

From L_1 the equation of motion for massive particles is derived. We define:$\bar{\mathfrak{g}}_{\mu\nu} = g_{\mu\nu} + \frac{\kappa_2}{2}'\bar{g}_{\mu\nu}$.

$$\frac{d(\dot{x}^\mu\dot{x}^\nu\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^\beta g_{\alpha\beta} + 2\dot{x}^\beta\bar{\mathfrak{g}}_{\alpha\beta})}{d\tau} - \frac{1}{2}\dot{x}^\mu\dot{x}^\nu\bar{\mathfrak{g}}_{\mu\nu}\dot{x}^\beta\dot{x}^\gamma g_{\beta\gamma,\alpha} - \dot{x}^\mu\dot{x}^\nu\bar{\mathfrak{g}}_{\mu\nu,\alpha} = 0 \tag{3.7}$$

We will discuss the motion of massive particles elsewhere.

The action for massless particles is:

$$L_0 = \frac{1}{4}\int dt \left(-v^{-1}\left(g_{\mu\nu} + \kappa_2\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu\right) \tag{3.8}$$

In the gauge $v = 1$, we get:

$$L_0 = -\frac{1}{4}\int dt \left(g_{\mu\nu} + \kappa_2\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu \tag{3.9}$$

plus the equation of motion for v evaluated at $v = 1$: $\left(g_{\mu\nu} + \kappa_2'\bar{g}_{\mu\nu}\right)\dot{x}^\mu\dot{x}^\nu = 0$

So, the massless particle moves in a null geodesic of $\mathfrak{g}_{\mu\nu} = g_{\mu\nu} + \kappa_2'\bar{g}_{\mu\nu}$.

4. Distances and time intervals

In this section, we define the measurement of time and distances in the model.

In GR the geodesic equation preserves the proper time of the particle along the trajectory. Equation(3.7) satisfies the same property: Along the trajectory $\dot{x}^\mu \dot{x}^\nu g_{\mu\nu}$ is constant.Therefore we define proper time using the original metric $g_{\mu\nu}$,

$$d\tau = \sqrt{-g_{\mu\nu}dx^\mu dx^\nu} = \sqrt{-g_{00}}dx^0 (dx^i = 0) \tag{4.1}$$

Following [27], we consider the motion of light rays along infinitesimally near trajectories and (4.1) to get the three dimensional metric:

$$\begin{aligned} dl^2 &= \gamma_{ij} dx^i dx^j, \\ \gamma_{ij} &= \frac{g_{00}}{\mathfrak{g}_{00}}(\mathfrak{g}_{ij} - \frac{\mathfrak{g}_{0i}\mathfrak{g}_{0j}}{\mathfrak{g}_{00}}) \end{aligned} \tag{4.2}$$

That is, we measure proper time using the metric $g_{\mu\nu}$ but the space geometry is determined by both metrics. In this model massive particles do not move on geodesics of a four dimensional metric. Only massless particles move on a null geodesic of $\mathfrak{g}_{\mu\nu}$. So, delta gravity is not a metric theory.

5. Friedman-Robertson-Walker(FRW) metric

In this section, we discuss the equations of motion for the Universe described by the FRW metric. We use spatial curvature equal to zero to agree with cosmological observations.

Here we will deal only with a perfect fluid, since rotational and translational invariance implies that the energy-momentum tensor of the Universe has this form.The energy momentum tensor for a perfect fluid is [11]:

$$T_{\mu\nu} = p g_{\mu\nu} + (p+\rho)U_\mu U_\nu, g^{\mu\nu}U_\mu U_\nu = -1 \tag{5.1}$$

Then:

$$\frac{\delta T_{\mu\nu}}{\delta g_{\gamma\sigma}}\tilde{g}^{\mu\nu} = p\tilde{g}^{\gamma\sigma} + \frac{1}{2}(p+\rho)(U^\gamma U_\nu \tilde{g}^{\sigma\nu} + U^\sigma U_\nu \tilde{g}^{\gamma\nu}) \tag{5.2}$$

In this case, assuming flat three dimensional metric:

$$\begin{aligned} -ds^2 &= dt^2 - R(t)^2 \left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\} \\ -d\tilde{s}^2 &= \tilde{A}(t)dt^2 - \tilde{B}(t)\left\{dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right\} \end{aligned}$$

Using (3.7, 4.1), we can check that these are co-mobile coordinates and the proper time interval $d\tau$ for a co-moving clock is just dt, so t is the time measured in the rest frame of a co-moving clock. Equations (2.3, 5.2) give:

$$\begin{aligned} -\dot{R}\dot{\tilde{B}} - \frac{1}{2}pR\tilde{B} + \frac{1}{2}R^{-1}\dot{R}^2\tilde{B} - \frac{1}{6}\rho R^3\tilde{A} + \frac{3}{2}R\dot{R}^2\tilde{A} = 0 \\ -p\tilde{B} - 2\ddot{\tilde{B}} - R^{-2}\dot{R}^2\tilde{B} + 2R^{-1}\ddot{R}\tilde{B} + 2R^{-1}\dot{R}\dot{\tilde{B}} + \\ \rho\ R^2\tilde{A} + \dot{R}^2\tilde{A} + 2R\dot{R}\dot{\tilde{A}} + 2R\tilde{A}\ddot{R} = 0 \end{aligned} \tag{5.3}$$

Einstein's equations are:

$$\frac{3\ \left(\frac{d}{d\ t}\ R\right)^2}{R^2}=\kappa\rho\quad ,2\ R\ \left(\frac{d^2}{d\ t^2}\ R\right)+\left(\frac{d}{d\ t}\ R\right)^2=-\kappa R^2 p$$

We use the equation of state $p=w\rho$, to get, for $w\neq -1$:

$$\begin{aligned} &R=R_0 t^{\frac{2}{3(1+w)}}, \tilde{A}=3wl_2 t^{\left(\frac{w-1}{w+1}\right)}, \\ &\tilde{B}=R_0^2 l_2 t^b, b=\frac{4}{3w+3}+\frac{w-1}{w+1} \end{aligned} \tag{5.4}$$

l_2 is a free parameter.

6. Red Shift

To make the usual connection between redshift and the scale factor, we consider light waves traveling from $r=r_1$ to $r=0$,along the r direction with fixed θ,ϕ. Photons moves on a null geodesic of $\mathfrak{g}$:

$$0=-(1+\kappa_2'\tilde{A})dt^2+(R^2+\kappa_2'\tilde{B})(dr^2+r^2 d\theta^2+r^2\sin^2\theta d\phi^2) \tag{6.1}$$

So,

$$\int_{t_1}^{t_0} dt\sqrt{\frac{1+\kappa_2' tA}{R^2+\kappa_2' tB}}=r_1 \tag{6.2}$$

A typical galaxy will have fixed r_1,θ_1,ϕ_1. If a second wave crest is emitted at $t=t_1+\delta t_1$ from $r=r_1$, it will reach $r=0$ at $t_0+\delta t_0$, where

$$\int_{t_1+\delta t_1}^{t_0+\delta t_0} dt\sqrt{\frac{1+\kappa_2' tA}{R^2+\kappa_2' tB}}=r_1$$

Therefore, for $\delta t_1,\delta t_0$ small, which is true for light waves, we have:

$$\delta t_0\sqrt{\frac{1+\kappa_2' tA}{R^2+\kappa_2' tB}}(t_0)=\delta t_1\sqrt{\frac{1+\kappa_2' tA}{R^2+\kappa_2' tB}}(t_1) \tag{6.3}$$

Introduce:

$$\tilde{R}(t)=\sqrt{\frac{R^2+\kappa_2' tB}{1+\kappa_2' tA}}(t)$$

We get: $\frac{\delta t_0}{\delta t_1}=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. A crucial point is that, according to equation (4.1), δt measure the change in proper time. That is: $\frac{\nu_1}{\nu_0}=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$, where ν_0 is the light frequency detected at $r=0$ corresponding to a source emission at frequency ν_1. Or in terms of the redshift parameter z, defined as the fractional increase of the wavelength λ:

$$z=\frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}-1=\frac{\lambda_0-\lambda_1}{\lambda_1} \tag{6.4}$$

We see that $\tilde{R}$ replaces the usual scale factor R in the computation of z.

7. Luminosity distance

Let us consider a mirror of radius b that is receiving light from a distant source. The photons that reach the mirror are inside a cone of half-angle ε with origin at the source.

Let us compute ε.The light path of rays coming from a far away source at $\vec{x}_1$ is given by $\vec{x}(\rho) = \rho\hat{n} + \vec{x}_1$, $\rho > 0$ is a parameter and $\hat{n}$ is the direction of the light ray.The path reaches us at $\vec{x} = 0$ for $\rho = |\vec{x}_1| = r_1$. Write $\hat{n} = -\hat{x}_1 + \vec{\varepsilon}$. Since $\hat{n}, \hat{x}_1$ have modulus 1, $\varepsilon = |\vec{\varepsilon}| << 1$ is precisely the angle between $-\vec{x}_1$ and $\hat{n}$ at the source.The impact parameter is the proper distance of the path from the origin, when $\rho = |\vec{x}_1|$. The proper distance is determined by the 3-dimensional metric (4.2). That is $b = \tilde{R}(t_0) r_1 \theta = \tilde{R}(t_0) r_1 \varepsilon$, i.e. $\varepsilon = \frac{b}{\tilde{R}(t_0) r_1}$.

Then the solid angle of the cone is $\pi\varepsilon^2 = \frac{A}{r_1^2 \tilde{R}(t_0)^2}$, where $A = \pi b^2$ is the proper area of the mirror.The fraction of all isotropically emitted photons that reach the mirror is $f = \frac{A}{4\pi r_1^2 \tilde{R}(t_0)^2}$. Each photon carries an energy $h\nu_1$ at the source and $h\nu_0$ at the mirror. Photons emitted at intervals δt_1 will arrive at intervals δt_0. We have $\frac{\nu_1}{\nu_0} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}, \frac{\delta t_0}{\delta t_1} = \frac{\tilde{R}(t_0)}{\tilde{R}(t_1)}$. Therefore the power at the mirror is $P_0 = L \frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2} f$, where L is the luminosity of the source. The apparent luminosity is $l = \frac{P_0}{A} = L \frac{\tilde{R}(t_1)^2}{\tilde{R}(t_0)^2} \frac{1}{4\pi r_1^2 \tilde{R}(t_0)^2}$. In Euclidean space, the luminosity decreases with distance d according to $l = \frac{L}{4\pi d^2}$.This permits to define the luminosity distance:$d_L = \sqrt{\frac{L}{4\pi l}} = \tilde{R}(t_0)^2 \frac{r_1}{\tilde{R}(t_1)}$. Using (6.2) we can write this in terms of the red shift:

$$d_L = (1+z)\int_0^z \frac{dz'}{\tilde{H}(z')}, \tilde{H} = \frac{\dot{\tilde{R}}}{\tilde{R}} \tag{7.1}$$

8. Supernova Ia data

The supernova Ia data gives, m (apparent or effective magnitude) as a function of z. This is related to distance d_L by $m = M + 5log(\frac{d_L}{10pc})$. Here M is common to all supernova and m changes with d_L alone.

We compare δ gravity to General Relativity(GR) with a cosmological constant:

$$H^2 = H_0^2(\Omega_m(1+z)^3 + (1-\Omega_m)), \Omega_\Lambda = 1 - \Omega_m$$

Notice that $\tilde{A} = 0$ for $w = 0$ in (5.4). So, it seems that we cannot fit the supernova data. However $w = 0$ is not the only component of the Universe. The massless particles that decoupled earlier still remain. It means that the true w is between $0 \leqslant w < \frac{1}{3}$, but very close to $w = 0$. So, we will fit the data with $w = 0.1, 0.01, 0.001$ and see how sensitive the predictions are to the value of w.

Using data from Essence[29], we notice that R^2 test changes very little for the chosen sequence of w's. Each fit determines the best l_2 for a given w. In this way we see that l_2 scales like $l_2 \sim \frac{a}{3w}$, a being independent of w. As an approximation to the limit $w = 0$, we get:

$$\tilde{R}(t) = R(t)\frac{\sqrt{a}}{\sqrt{a-t}} \tag{8.1}$$

$\sqrt{\frac{1}{3w}}$renormalizes the derivative of $\tilde{R}$ at $t = 0$. It is not divergent, because for $t \to 0$, $w \to \frac{1}{3}$. a is a free parameter determined by the best fit to the data.

Of course, the complete model must include the contribution of normal matter($w = 0$) plus relativistic matter $(w = \frac{1}{3})$. But, at later times, the data should tend to (8.1). The exact solution of the model with two fluids is found in [28].

Let us fit the data to the simple scaling model (8.1).

We get:

$\Omega_m = 0.22 \pm 0.03, M = 43.29 \pm 0.03$, $\chi^2(perpoint) = 1.0328$, General Relativity

$a = 2.21 \pm 0.12, M = 43.45 \pm 0.06$, $\chi^2(perpoint) = 1.0327$, Delta Gravity

δ-gravity with non-relativistic(NR) matter alone give a fit to the data as good as GR with NR matter plus a cosmological constant.

According to the fit to data, a Big Rip will happen at $t = 2.21049$ in unities of t_0(today). It is a similar scenario as in [25].

Finally, we want to point out that since for $t \to 0$, we have $w \to \frac{1}{3}$, then $\tilde{R}(t) = R(t)$. Therefore the accelerated expansion is slower than (8.1) when we include both matter and radiation in the model.

9. Dark Matter

Several years ago, the astronomers were able to measure the speed of individual stars around the center of the galaxies[3]. Surprisingly, such speeds $v(r)$as a function of the distance r to the galactic center, did not follow Kepler law. Most of the galactic mass was assumed to be in the form of stars, which concentrate near the galactic center. So, the expectation was that the speed of rotation of stars far from the center will decrease as $r^{-1/2}$. The observation shows that rotation curves(RC) follow a different pattern [4].

A natural way to explain the observed velocities was to assume the existence of extra mass that cannot be seen but interact gravitationally (Dark Matter,DM).

When in the late 1970s the phenomenon of DM was discovered a few truly flat RCs were highlighted in order to rule out the claim that non Keplerian velocity profiles originate from a faint baryonic component distributed at large radii. At that time a large part of the evidence for DM was provided by extended, low-resolution HI RCs of very luminous spirals whose velocity profile did show small radial variations. The increase in the quality of the RCs though soon leads to the conclusion that baryonic (dark) matter was not a plausible candidate for the cosmological DM and that the RCs did show variation with radius, even at large radii. Later numerical simulations in the Cold Dark Matter scenario also predicted asymptotically declining RCs. The flat RC paradigm was hence dismissed in the 1990s[5]

Additional support for the existence of DM comes from the study of galaxy stability against gravitational collapse: The form of the galaxy that we can see (luminous part) is not gravitationally stable unless we assume the existence of a spherically symmetric halo that we cannot see.

From observation, we get that 80-90% of the galactic mass is DM.

However, the physical nature of Dark Matter is not known yet[6].

Most people think that DM is made of particles that interact weekly with normal matter. Until recently the standard cosmological scenario was the so called ΛCDM model. That is the evolution of the Universe is governed by a cosmological constant Λ that produces the accelerated expansion of the Universe(Dark Energy)[7, 30] and non relativistic particles (cold DM) that were the seeds of the galaxies. However, recent simulations of the neighborhood of the Milky Way[31] have challenged the CDM paradigma. They proposed instead that DM particles are warm, with a rest mass of 1Kev.

There is an alternative to DM that is gaining some support: MOND[9]. The main idea of MOND involves a modification of Newton Second Law for accelerations below a critical acceleration a_0. In this way the constant speed v_0 of individual stars far from the galactic center is explained. Therefore, according to MOND, DM particles do not exist.

Since DM particles have not been detected yet and even their existence is challenged in some models, in this section we want to explore a different scenario to understand the properties of galaxies. Preliminary studies of the solutions of DG in vacuum have shown that it contains extra degrees of freedom that produces an additional newtonian potential far from the sources.

In fact, far from a source the gravitational field correspond to the Schwarzschild solution: pointlike source, spherically symmetric.

The exact solution is:

$$g_{\mu\nu} = \begin{pmatrix} -(1-\frac{a}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{a}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\, sin(\theta)^2 \end{pmatrix} \tag{9.1}$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} -(1-\frac{a}{r}+\frac{ba}{r}) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{a}{r}} - \frac{ab}{r(1-\frac{a}{r})^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\, sin(\theta)^2 \end{pmatrix} \tag{9.2}$$

Boundary condition:$g_{\mu\nu} \sim \eta_{\mu\nu}\tilde{g}^{\mu\nu} \sim \eta^{\mu\nu}$ for $r \to \infty$.Notice that still there are 2 arbitrary constants.

Newtonian potential for Massive Particles

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, h_{00} = \frac{a}{r}, a = 2M$$
$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}, \tilde{h}_{00} = \frac{a(1-b)}{r}, a(1-b) = 2M'$$

The Newtonian potential is:

$$\phi = -((\frac{1}{1+\frac{\kappa_2}{2}'} - \frac{1}{2})h_{00} + \frac{\frac{\kappa_2}{2}'}{1+\frac{\kappa_2}{2}'}\tilde{h}_{00}) = -\frac{M_T}{r}$$

So the total mass of the source is:

$$M_T = M - \frac{\kappa_2' bM}{1+\frac{\kappa_2}{2}'}$$

So, the dark matter mass is:

$$M_{\mathrm{DM}} = -\frac{\kappa_2' bM}{1+\frac{\kappa_2}{2}'} \tag{9.3}$$

M is the mass coming from the fluid density in Einstein equations. b is a new constant to accomodate DM.

Photons and Gravitational Lensing:

The photon trajectory is given by:

$$\left[-(1-\frac{a}{r})-\kappa_2'(1-\frac{a}{r}+\frac{ba}{r})\right]dt^2+\left[\frac{1}{1-\frac{a}{r}}+\kappa_2'(\frac{1}{1-\frac{a}{r}}-\frac{ab}{r(1-\frac{a}{r})^2})\right]dr^2=0$$

$$\left[-1+\frac{1}{r}(a-\frac{\kappa_2' ba}{1+\kappa_2'})\right]dt^2+\left[1+\frac{1}{r}(a-\frac{\kappa_2' ba}{1+\kappa_2'})\right]dr^2=0$$

So, according to photons:

$$M_T = M - \frac{\kappa_2' bM}{1+\kappa_2'} \tag{9.4}$$

Notice that photons and massive particles see different M_T, but since κ_2' is very small, this difference is hard to detect.

To determine if δ Gravity can describe Dark Matter, we must be able to compute the speed of stars rotating around the center of galaxies. This is work in progress.

10. Conclusions and Open Problems

Delta Gravity agrees with GR when $T_{\mu\nu}=0$, imposing same boundary conditions for both tensor fields. In particular, the causal structure of delta gravity in vacuum is the same as in GR, since in this case the action (3.1) is proportional to the geodesic action in GR.

In a homogeneous and isotropic universe, we get accelerated expansion without a cosmological constant or additional scalar fields.

Notice that equation (5.4) implies that $\tilde{R}=R$ at the beginning of the Universe, when $w=\frac{1}{3}$, corresponding to ultrarelativistic matter. That is, the accelerated expansion started at a later time, which is needed if we want to recover the observational data of density perturbations and growth of structures in the Universe. An earlier acceleration of the expansion would prevent the growth of density perturbations.

Work is in progress to compute the growth of density perturbations and the anisotropies in the CMB. The comparison of these calculations with the considerable amount of astronomical data that will be available in the near future will be a very stringent test of the present gravitational model.

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11. Appendix A Review of δ-symmetries

Assume we have a group of transformations acting on the variables y with infinitesimal parameters ε. That is:

$$\delta y^i = \Lambda^i_\alpha(y)\varepsilon^\alpha \tag{11.1}$$

We define the δ transformation by:

$$\delta\bar{y}^i = \Lambda^i_\alpha(y)_{,j}\bar{y}^j\varepsilon^\alpha + \Lambda^i_\alpha(y)\bar{\varepsilon}^\alpha \tag{11.2}$$

$k_{,i} = \frac{\partial k}{\partial y^i}$.

Notice that, we have introduced a new field $\bar{y}^i$ and a new transformation with parameter $\bar{\varepsilon}^\alpha$.

It is easy to see that (11.1, 11.2) form a closed algebra.

An invariant action under the extended symmetry is built in the same way. We assume that $S(y)$ is invariant under (11.1):

$$\frac{\delta S}{\delta y^i}\Lambda^i_\alpha(y) = 0, \forall y, \text{all } \alpha \tag{11.3}$$

Then:

$$\bar{S}(y,\bar{y}) = S(y) + \frac{\delta S}{\delta y^i}\bar{y}^i$$

is invariant under (11.1, 11.2).

Proof:

$$\begin{aligned}\delta\bar{S}(y,\bar{y}) = \frac{\delta S}{\delta y^i}\Lambda^i_\alpha(y)\varepsilon^\alpha + \frac{\delta^2 S}{\delta y^i \delta y^j}\Lambda^j_\alpha(y)\varepsilon^\alpha\bar{y}^i + \frac{\delta S}{\delta y^i}(\Lambda^i_\alpha(y)_{,j}\bar{y}^j\varepsilon^\alpha + \Lambda^i_\alpha(y)\bar{\varepsilon}^\alpha) = \\ 0 + (\frac{\delta^2 S}{\delta y^i \delta y^j}\Lambda^j_\alpha(y)\bar{y}^i + \frac{\delta S}{\delta y^i}\Lambda^i_\alpha(y)_{,j}\bar{y}^j)\varepsilon^\alpha + 0\bar{\varepsilon}^\alpha = \\ (\frac{\delta^2 S}{\delta y^i \delta y^j}\Lambda^i_\alpha(y) + \frac{\delta S}{\delta y^i}\Lambda^i_\alpha(y)_{,j})\varepsilon^\alpha\bar{y}^j = \left\{\frac{\delta}{\delta y^j}(\frac{\delta S}{\delta y^i}\Lambda^i_\alpha(y))\right\}\varepsilon^\alpha\bar{y}^j = 0\end{aligned}$$

Last equality follows from equation (11.3).

Being careful with signs of permutations, these results are true for anti-commuting y, ε as well. In particular, super-symmetric transformations can be generalized to a δ- symmetry.

Other generalizations are possible. Suppose we have canonical transformations generated by $\varepsilon(x,p)$:

$$\begin{aligned}\delta F = (\varepsilon, F) \quad , \\ \delta\bar{F} = (\varepsilon,\bar{F}) + (\bar{\varepsilon},F)\end{aligned} \tag{11.4}$$

equations (11.1, 11.2) are particular cases of (11.4). (A,B) is the Poisson bracket. Now we can prove the closure of the algebra in a more general context.

$$\begin{aligned}\left[\delta_\beta,\delta_\alpha\right]F = (\delta_\beta(\alpha,F) - \alpha \leftrightarrow \beta) = (\alpha,(\beta,F)) - (\beta,(\alpha,F)) = (F,(\beta,\alpha)) = ((\alpha,\beta),F) = \delta_{(\alpha,\beta)}F \\ \left[\delta_\beta,\delta_\alpha\right]\bar{F} = (\delta_\beta(\alpha,\bar{F}) - \alpha \leftrightarrow \beta) = (\alpha,(\beta,\bar{F})) - (\beta,(\alpha,\bar{F})) = (\bar{F},(\beta,\alpha)) = ((\alpha,\beta),\bar{F}) = \delta_{(\alpha,\beta)}\bar{F} \\ \left[\delta_\alpha,\delta_{\bar{\beta}}\right]F = 0 \\ \left[\delta_\alpha,\delta_{\bar{\beta}}\right]\bar{F} = (\delta_\alpha(\bar{\beta},F) - \delta_{\bar{\beta}}(\alpha,\bar{F}) = (\bar{\beta},(\alpha,F)) - (\alpha,(\bar{\beta},F)) = (F,(\alpha,\bar{\beta})) = \delta_{(\bar{\beta},\alpha)}F \\ \left[\delta_{\bar{\alpha}},\delta_{\bar{\beta}}\right]\bar{F} = \delta_{\bar{\alpha}}(\bar{\beta},F) - \bar{\alpha} \leftrightarrow \bar{\beta} = 0\end{aligned}$$

Replacing Poisson bracket by commutators is the realization of the algebra we used in [23].