The Local Dark Matter Density

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We present the recent robust determination of the value of the Dark Matter density at the Sun's location ($\rho_\odot$) with a technique that does not rely on a global mass-modeling of the Galaxy. The method is based on the local equation of centrifugal equilibrium and depends on local and quite well known quantities such as the angular Sun's velocity, the disk to dark contribution to the circular velocity at the Sun, and the thin stellar disk scale length. This determination is independent of the shape of the dark matter density profile, the knowledge of the rotation curve at any radius, and the very uncertain bulge/disk/dark-halo mass decomposition. The result is: $\rho_\odot = 0.43(0.11)(0.10)\,\text{GeV/cm}^3$, where the quoted uncertainties are due to the uncertainty a) in the slope of the circular-velocity at the Sun location and b) in the ratio between this radius and the exponential length scale of the stellar disk. The devised technique is also able to take into account any future improvement in the data relevant for the estimate.

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1. Introduction

Galaxy rotation curves (e.g. Rubin et al. 1980; Bosma et al. 1981) have unveiled the presence of a dark "mass component" in spirals. They are pillars of the paradigm of massive dark halos, composed of a still undetected kind of matter surrounding the luminous part of galaxies. The kinematics of spirals shows universal systematics (Persic, Salucci 1996; Salucci et al. 2007), which seems to be at variance with the predictions emerging from simulations performed in the Λ cold dark matter (ΛCDM) scenario, (e.g. Navarro et al. 1996), the current cosmological paradigm of galaxy formation (e.g. Gentile et al. 2004) (However, see Macciò et al. (2012)).

Dedicated searches of DM particle candidates have intensified in recent years: direct-detection experiments look for the scattering of DM particles inside the detectors, which clearly is proportional to the DM density in the Sun's region $\rho_\odot \equiv \rho_{DM}(R_\odot)$. Indirect-detection experiments searching for the secondary particles (e.g. neutrinos) produced by DM annihilation at the center of the Sun or Earth depend on the DM density inside these objects which in turn is driven, via a capture mechanism, by the same local DM density $\rho_\odot$. Thus, in both direct and indirect searches the knowledge of the the local density $\rho_\odot$ is very important for a precise prediction of the searched signal or to obtain reliable bounds on the DM particle cross-section.

A value of $\rho_\odot = 0.3\,\mathrm{GeV/cm^3}$ has been routinely quoted, but its origin is not clear, neither supported by data (see the introduction of Salucci et al. (2010)). An example is the work by Caldwell, Ostriker (1981) that devised what can be considered as the standard method to determine the value of $\rho_\odot$ from a variety of observations. Their resulting value $0.23^{+0.23}_{-0.12}\mathrm{GeV/cm^3}$ is however uncertain and moreover plagued by very outdated kinematics.

In general, it is quite simple to infer the distribution of dark matter in *spiral galaxies*. Spiral's kinematics, in fact, reliably traces the underlying gravitational potential (Persic, Salucci 1996; Salucci et al. 2007). Then, from co-added and/or individual RCs, we can build suitable mass models that include stellar and gaseous disks, along with a spherical bulge and a dark halo. More in detail, by carefully analyzing (high quality) circular velocity curves, with the help of relevant photometric and HI data, one can derive the halo density at different radii. The accuracy of the measurements and of the analysis is excellent and the results obtained are at the heart of the present debates on Galaxy formation (e.g. Gentile et al. 2004, 2005; DeBlok 2009; an innovative review on this issue is found at `www.sissa.it/ap/dmg/dmaw_presentation.html`)

To measure $\rho_\odot$, instead, is far from simple, because the MW kinematics, unlike that of external galaxies, does not trace the gravitational potential straightforwardly. We do not directly measure the circular velocity of stars and gas but rather the terminal velocity V_T of the rotating HI disk, and this only inside the solar circle (e.g. McClure-Griffiths, Dichey 2007). This velocity is related to the circular velocity $V(r)$, for $r < R_\odot$, by means of $V(r) = V_T(r) + V_\odot \, r/R_\odot$, where $R_\odot \simeq 8$ kpc is the distance of the Sun from the Galaxy center and $V_\odot$, the value of the circular velocity at the Sun's position. Both quantities are known within an uncertainty of 5% - 10% (e.g. McMillan, Binney 2009) which triggers a similar uncertainty in the derived amplitude and slope of the circular velocity. As a result, (see also Sofue 2009), the MW circular velocity from 2 kpc to 8 kpc is known within a not negligible uncertainty:

$$V(r) = (215 \pm 30)\ \mathrm{km/s} \tag{1.1}$$

$$d\log V(r)/d\log r \equiv \alpha(r) = 0.0 \pm 0.15\,. \tag{1.2}$$

The variations in equation (1.1) are due to a mix of observational errors in the kinematics, to uncertainties in the values of $R_\odot$ and $V_\odot$ and to actual radial variations of V. The first two trigger also part of the possible range of the velocity slope (1.2). Data show that the radial variations of $\alpha(r)$ are, instead, quite small: $d\alpha(r)/dr \simeq 0 \pm 0.03/\mathrm{kpc} \simeq 0$; in the disk region the MW RC can be approximated by a straight line, whose slope however is mildly uncertain.

The outer (out to 60 kpc) MW "effective" circular velocity $V(r) = (GM(r)/r)^{1/2}$ is even more uncertain and it depends on the assumptions made on dynamical and structural properties of its estimators. It appears to decline with radius, with quite an uncertain slope $d\log V(r)/d\log r = -0.20^{+0.05}_{-0.15}$ ($R_\odot < r < 60\,\mathrm{kpc}$) (Battaglia 2005; Xue et al. 2008; Brown et al. 2009). These uncertainties, combined with the intrinsic "flatness" of the RC in the region specified above (that complicates the mass modeling even in the case of a high-quality RC (Tonini, Salucci 2004)), make it very difficult to obtain a reliable bulge/disk/halo mass model and consequently an accurate estimate of $\rho_\odot$.

A different approach was devised by Salucci et al. (2010); the idea is to resort to the equation of centrifugal equilibrium (see Fall Efstathiou 1980 for details) and to use recent results found in external galaxies (Salucci et al. 2007). Let us start with

$$V^2/r = a_H + a_D + a_B, \tag{1.3}$$

where a_H, a_D, and a_B are the radial accelerations generated by the halo, stellar disk, and bulge. Taking first the (quite good) approximation of spherical DM halo, we have $a_H \propto r^{-2} \int_0^r \rho_H(r)\, r^2 dr$. A similar relation holds for the bulge. Therefore, by differentiating equation (1.3), we obtain the DM density at any radius in terms of the local angular velocity $\omega(r) = V/r$, the RC slope $\alpha(r)$, the disk-to-dynamical mass ratio $\beta(r)$ (see later), and the bulge density $\rho_B(r)$:

$$\rho_H(r) = \omega(r)^2 [F_{tot}(\alpha(r)) - F_D(\beta(r))] - \rho_B(r). \tag{1.4}$$

with F_{tot} and F_D known functions.

In external spirals, equation (1.4) is not useful for determining the DM density at different radius because 1) it collapses for $r < R_D$ where $F_{tot} \simeq F_D$ and where the bulge density is dominating, $\rho_B \gg \rho_H$, and 2) the radial variations of $\alpha(r)$ have non-negligible observational uncertainties; 3) the quantity ω is known with far less accuracy than V, the main kinematical observational quantity.

Instead, when estimating $\rho_\odot$, i.e. the density of the MW DM halo at a *specific* radius (the Sun position), the above drawbacks disappear, infact: 1) we have that $F_{tot}(R_\odot) \gg F_D(R_\odot)$ because $R_\odot > 3R_D$, so equation (1.4) does not collapse and, as a bonus, the most uncertain term of its r.h.s. turns out to be very small 2) $\omega_\odot$ is very precisely measured; and 3) at the Sun's position the bulge density $\rho_B(R_\odot)$ is totally negligible, $< \rho_H/50$ (e.g. Sofue et al. 2009).

The method is clearly simpler for a spherically symmetric DM halo, and if we assume an infinitesimally thin disk for the distribution of stars in the Galaxy. Nevertheless, in Salucci et al. (2010) the effects of a possible halo oblateness and disk thickness were dealt with.

Our claim (Salucci et al. 2010) is that by means of this method we to obtain an independent and reliable determination of the local DM halo density and of its intrinsic uncertainty.

2. A new local method for determining $\rho_\odot$

We model the Galaxy as composed by a stellar exponential thin disk plus an unspecified spherical DM halo with density profile $\rho_H(r)$. For the present work we can neglect the thick stellar and

the HI disk because their surface density, between $2\,$kpc and $R_\odot$, are 100 to 5 times smaller than the stellar surface density (Nakanishi, Sofue 2003). Similarly, we neglect the stellar bulge because, as mentioned above, its spatial density at $R_\odot$ is virtually zero (e.g. Sofue et al. 2009). It is worth noticing that the standard method of galaxy modeling and its variants cannot take these very simplifying assumptions because the global modeling involves all these mass components in a crucial way.

Let us rewrite the equation of centrifugal equilibrium by subtracting the disk component from the total acceleration. From its radial derivative we then find

$$\rho_H(r) = \frac{X_q}{4\pi G r^2} \frac{d}{dr}\left[r^2 \left(\frac{V^2(r)}{r} - a_D(r) \right) \right], \tag{2.1}$$

where X_q is a factor correcting the spherical Gauss law used above in case of oblateness q of the DM halo (Salucci et al. 2010). The X_q correction is very small: $X_q \simeq 1.00\text{--}1.05$.

The disk component can be reliably modeled as a Freeman stellar exponential thin disk of length scale $R_D = (2.5 \pm 0.2)$ kpc (Picaud, Robin 2004; M. Jurić et al. 2008; Robin et al. 2008; Reylé 2009). The stellar surface density is then: $\Sigma(r) = (M_D/2\pi R_D^2)\, e^{-r/R_D}$. Also, the disk can be considered infinitesimally thin. In fact, its thickness z_0 is small, $z_0 \sim 250\,$pc (M. Jurić et al. 2008) and moreover $z_0 \ll R_D < R_\odot$. We can thus write $a_D(r) = \frac{GM_D r}{R_D 3}(I_0 K_0 - I_1 K_1)\, X_{z_0}$, where I_n and K_n are the modified Bessel functions computed at $r/2R_D$, and $X_{z_0} \simeq 0.95$ accounts for the nonzero disk thickness.

Since only the first derivative of the circular velocity $V(r)$ enters in (2.1) and in any case this function in the solar neighborhood is almost linear, we can write

$$V(r) = V_\odot[1 + \alpha_\odot\,(r - R_\odot)/R_\odot], \tag{2.2}$$

where $\alpha_\odot = \alpha(R_\odot)$ is the velocity slope at the Sun's radius. Then equation (2.1) becomes

$$\rho_H(r) = \frac{X_q}{4\pi G}\left[\frac{V^2(r)}{r^2}(1 + 2\alpha_\odot) - \frac{GM_D}{R_D^3} H(r/R_D) X_{z_0} \right], \tag{2.3}$$

with $2H(r/R_D) = (3I_0 K_0 - I_1 K_1) + (r/R_D)(I_1 K_0 - I_0 K_1)$. Equation (2.3) holds at any radius outside the bulge region and measures $\rho_H(R_\odot) \equiv \rho_\odot$ by subtracting the density of the stellar component from the one of the whole gravitating matter.

The disk mass can be parametrized (Persic, Salucci 1990) by $M_D = \beta\ 1.1\ G^{-1}V_\odot^2 R_\odot$, with $\beta = V_D^2/V^2|_{R_\odot}$, i.e. the fraction of the disc contribution to the circular velocity at the Sun. Finally, by exploiting the fact that the quantity $V/R|_{R_\odot} \equiv \omega = (30.3 \pm 0.3)$ km/s/kpc is measured with very high accuracy and much better than $V_\odot$ and $R_\odot$ separately (McMillan, Binney 2009; Reid 2009), we obtain

$$\rho_\odot = 1.2 \times 10^{-27} \frac{\text{g}}{\text{cm}^3} \left(\frac{\omega}{\text{km/s\,kpc}} \right)^2 X_q \left[(1 + 2\alpha_\odot) - 1.1\,\beta\,f(r_{\odot D})\,X_{z_0} \right], \tag{2.4}$$

where $r_{\odot D} \equiv R_\odot/R_D$ and $f(r_{\odot D}) = r_{\odot D}^3 H(r_{\odot D})$.

Let us to focus on the advantages of this technique : a) it does not require assuming a particular DM halo density profile or the dynamical status of some distant tracers of the gravitational field; b) it is independent of the poorly known values of $V_\odot$ and of the RC slope at different radii $\alpha(r)$; c) it does not depend on the structural properties of the bulge, which in the mass modeling, leads to a

degeneracy with the values of the mass of stellar disk and of the DM halo. d) it only mildly depends on the ratio $r_{\odot D}$, as well as, on the disk mass parameter β. Let us also note that the determination depends on the RC slope at the Sun $\alpha_\odot$, but in a well specified way.

To proceed further we discuss the uncertainties on the parameters appearing in equation (2.4). Our determination depends on the ratio $r_{\odot D} \equiv R_\odot/R_D$. For this we adopt the reference value and uncertainty $r_{\odot D} = 3.4 \pm 0.5$, as suggested by the values of R_D discussed above and by the average of values of $R_\odot$ obtained recently: $R_\odot = 8.2 \pm 0.5$ kpc (Ghez et al. (2008); Gillessen et al. (2008); Bovy et al. (2009)). This leads to $f(r_{\odot D}) \simeq 0.42 \pm 0.20$, whose uncertainty propagates only mildly into the determination of $\rho_\odot$, because the second term of the r.h.s. of equation (2.4) is only one third of the first.

Present data constrain the slope of the circular velocity at the Sun to a central value of $\alpha_\odot = 0$ and within a fairly narrow range $-0.075 \leq \alpha_\odot \leq 0.075$. The uncertainty of $\alpha_\odot$ is the main source of the uncertainty of the present determination of $\rho_\odot$, see for instance (Olling, Merrifield 2001).

In equation (2.4), β is the only quantity that is not observed and therefore intrinsically uncertain. We can, however, constrain it by computing the maximum value β^M for which the disk contribution at $2.2 R_D$ (where it has its maximum) totally accounts for the circular velocity. With no assumption on the halo density profile one gets $\beta^M = 0.85$, independently of $V_\odot$ and $R_\odot$ (Persic, Salucci 1990). However, this is really an absolute maximal value and it corresponds, out to $R_\odot$, to a solid body halo profile: $V_h \propto R^{\alpha_h}$ with $\alpha_h = 1$. Instead, mass modeling performed so far for the MW and for external galaxies tend to find a lower value: $\alpha_h(3R_D) \leq 0.8$, which yields $\beta^M = 0.77$. We can also set a lower limit for the disk mass, i.e. β^m: first, the microlensing optical depth to Baade's Window constrains the baryonic matter within the solar circle to be greater than $3.9 \, 10^{10} M_\odot$ (McMillan, Binney 2009). Second, the MW disk B-band luminosity $L_B = 2 \times 10^{10} L_\odot$ and the very reasonable value $M_D/L_B = 2$ again imply $M_D \simeq 4 \, 10^{10} M_\odot$. All this leads to $\beta^m = \beta^M/1.3 \simeq 0.65$. We thus take $\beta = 0.72^{+0.05}_{-0.07}$ as reference range.

Using the reference values and expanding around their central values, we find

$$\rho_\odot = 0.43 \frac{\text{GeV}}{\text{cm}^3} \left[1 + 2.9 \, \alpha_\odot - 0.64 \left(\beta - 0.72 \right) + 0.45 \left(r_{\odot D} - 3.4 \right) \right.$$
$$\left. - 0.1 \left(\frac{z_0}{\text{kpc}} - 0.25 \right) + 0.10 \left(q - 0.95 \right) + 0.07 \left(\frac{\omega}{\text{km/s kpc}} - 30.3 \right) \right] . \quad (2.5)$$

This equation estimates the DM density at the Sun's location in an analytic way, in terms of the involved observational quantities at their present status of knowledge. The equation is written in a form such that, for the present reference values of these quantities, the term in the square brackets on the r.h.s equals 1, and the central result is $\rho_\odot = 0.43 \, \text{GeV/cm}^3$. As such, the determination is ready to account for future improved measurements by simply inserting them in the r.h.s. of (2.5).

The next step is to estimate the uncertainty in the present determination of $\rho_\odot$. From equation (2.5) and the allowed range of values discussed above, we realize that the main sources of uncertainty are $\alpha_\odot$, β and $r_{\odot D}$. The other parameters give at most variations of 2-3%, and can be neglected in the following.

Then, first, it is illustrative to consider $\alpha_\odot$, β and $r_{\odot D}$ as independent quantities. We thus have:

$$\rho_\odot = \left(0.43 \pm 0.094_{(\alpha_\odot)} \mp 0.016_{(\beta)} \pm 0.096_{(r_{\odot D})} \right) \frac{\text{GeV}}{\text{cm}^3} , \quad (2.6)$$

where $A_{(x)}$ means that A is the total effect due to the possible span of the quantity x.

At this point, we can go one step further, and assume that the MW is a typical spiral, and using recent results for the distribution of matter in external galaxies, namely that DM halos around spirals are self similar (Salucci et al. 2007) so that the fractional amount of stellar matter β *directly* dictates the value of rotation curve slope $\alpha_{\odot}$ (Persic, Salucci 1990):

$$\beta = 0.72 - 0.95\,\alpha_{\odot}\,. \tag{2.7}$$

Using this relation in equation (2.5) we find (neglecting the irrelevant q and z_0 terms)

$$\rho_{\odot} = 0.43\frac{\text{GeV}}{\text{cm}^3}\left[1 + 3.5\,\alpha_{\odot} + 0.45\left(r_{\odot D} - 3.4\right) + 0.07\left(\frac{\omega}{\text{km/s kpc}} - 30.3\right)\right]. \tag{2.8}$$

From the current known uncertainties, with the estimated range of $\alpha_{\odot}$, we finally arrive to

$$\rho_{\odot} = \left(0.430 \pm 0.113_{(\alpha_{\odot})} \pm 0.096_{(r_{\odot D})}\right)\frac{\text{GeV}}{\text{cm}^3}\,. \tag{2.9}$$

Its uncertainty mainly reflects our poor knowledge of the velocity slope $\alpha_{\odot}$ and the uncertainty in the galactocentric Sun distance.

3. Discussion and conclusion

We have described here a local determination of $\rho_{\odot}$, which relies directly on the equation of centrifugal equilibrium, by estimating the difference between the 'total' needed density and that of the stellar component.

The method leads to a very reliable kinematical local determination of $\rho_{\odot}$, avoiding model-dependent and dubious tasks, mandatory with the standard method, i.e., a) to assume a particular DM density profile and a specific dynamical status for the tracers of the gravitational potential, b) to deal with the non-negligible uncertainties of the global MW kinematics, c) to uniquely disentangle the flattish RC into the different bulge/disk/halo components.

This estimate of $\rho_{\odot}$ also shows that any kinematical determination using the Galactic rotation ultimately depends at least on three local quantities, the slope of the circular velocity at the Sun, the fraction of its amplitude due to the DM, and the ratio between the Sun galactocentric distance and the disk scale-length, whose uncertainty unavoidably propagates in the result. Two of these three quantities can be related by noting that the MW is a typical Spiral and using the relations available for these kind of galaxies (Salucci et al. 2007), so that the final uncertainty can be slightly reduced.

The resulting local DM density that we find is $\rho_{\odot} = (0.43 \pm 0.11_{(\alpha_{\odot})} \pm 0.10_{(r_{\odot D})})\,\text{GeV/cm}^3$. We remark that it is free from theoretical assumptions and can be easily updated by means of equation (2.5) as the relevant observational quantities will become better known.

Let us comment on other determinations in the literature. By applying a global modeling method with a refined statistical analysis to a large set of observational data, Catena, Ullio (2009) claimed a very precise measure, $\rho_{\odot} = (0.389 \pm 0.025)\,\text{GeV/cm}^3$, in agreement with our results but at variance with our estimated uncertainty (see Salucci et al. (2010) and also Weber, de Boer (2009)).

Finally, it is worth discussing the claim by Bidin et al. (2012). They, by means of the complex and not unambiguously accepted technique of exploiting the vertical velocity dispersion of old

tracer stars of the thick disk at the solar neighborhood, claimed the absence of any DM at the solar position, in disagreement with our (and others) result. However, as pointed out by Bovy and Tremaine (2012) their result is plagued by arbitrary assumptions: different z dependence of the tracers circular velocity and/or different kinematical status and spatial distribution can trigger a large number of different possible mass models, some of them clear of DM, others with proportions larger than those we estimate in (Salucci et al. 2010). As a result, the Bidin et al. (2012) method needs a series of independent checks and increased precision/statistics before being able to deliver a robust determination of $\rho_{\odot}$.

In conclusion, we believe that our technique provides the most trustable estimate of the local dark matter density and of its uncertainties.

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