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Isocurvature perturbations in dark radiation

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Recent cosmological observations such as measurements of the CMB anisotropy indicate the existence of non-interacting relativistic particles, dubbed the “extra radiation" in the Universe beyond the standard three neutrino species. In this paper we explore the possibility that the dark radiation, the mixture of the extra radiation and neutrino, has isocurvatrue fluctuations. A general formalism to evaluate isocurvature perturbations in the dark radiation is provided in the mixed inflaton-curvaton system, where the dark radiation is produced by the decay of both scalar fields. We also derive constraints on the abundance of the dark radiation and the amount of its isocurvature perturbation. These constraints are applied to some particle physics motivated models. Besides, we discuss the non-Gaussianity of the dark radiation isocurvature perturbation and forecast the constraint on it in the future CMB experiments.

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1. Introduction

Some recent experiments suggest that the effective number of neutrino species $N_{\rm eff}$ might be larger than 3.046 [1] by $\Delta N_{\rm eff} \approx 1$. For example, the constraint from the WMAP measurement of the cosmic microwave background (CMB) anisotropy combined with observations of the baryon acoustic oscillation (BAO) and the Hubble parameter (H0) is $N_{\rm eff} = 4.34^{+0.86}_{-0.88}$ (68%C.L.)[1]. Such results imply that there might be non-interacting energy component other than neutrino. Hereafter we call such a fluid “extra radiation" and the mixture of it and neutrino “dark radiation"(DR).

There are also theoretical motivations to consider the extra radiation in the particle physics beyond the Standard Model. For example, axion, a light scalar filed introduced to solve the strong CP problem, is a good candidate of the extra radiation. In theoretical papers treating the extra radiation, it has been so far (implicitly) assumed that the extra radiation has the adiabatic perturbation. However, the property of the fluctuation of the extra radiation depends on its production mechanism. For example, there are scenarios that the decay of a scalar field nonthermally produces the extra radiation. Axion produced by decay of saxion, the scalar partner of axion in the supersymmetric axion models, is among such scenarios. In such a case, it succeeds the fluctuation of the source scalar field, which arises as a quantum fluctuation during inflation. Therefore, the extra radiation can have an independent perturbation from the adiabatic one, an isocurvature perturbation. Investigating the isocurvature perturbation in the extra radiation may have a potential to distinguish the model of the extra radiation, if future observations further confirm $\Delta N_{\rm eff} \approx 1$.

Therefore, in this paper we derive constraints on the extra radiation from the currently available datasets, considering the effect of the extra radiation. In addition to its isocurvature perturbation, which affects the CMB anisotropy similarly to the neutrino isocurvature perturbation, its energy density itself affects the CMB through the change of the Hubble expansion rate. Due to these effects, we can constrain $N_{\rm eff}$ and the amplitude of the isocurvature perturbation in the DR S_{DR} using the CMB data. Besides, the DR isocurvature perturbation can have large non-Gaussianities in some cases. We consider how strongly such non-Gaussianities will be constrained by future CMB experiments. As a theoretical model of the extra radiation, we focus on the scenario that it is produced by decay of inflaton or the other scalar field or both. We derive the formulas for $N_{\rm eff}$, S_{DR} and non-Gaussianities and consider the implication of observational constraints for that model.

2. The model and the formalism

We consider a cosmological model with two scalar fields: the inflaton ϕ and the “curvaton" σ, which is light during inflation and decays later than inflaton. They obtain large-scale quantum fluctuations and may generate isocurvature perturbations as well as adiabatic ones.

All the components of the current Universe, photon (denoted by γ), neutrino (denoted by ν), dark matter, baryon and extra radiation (denoted by X) arise from both ϕ and σ. The temporal change of the components of the Universe is summarized in Table 1. Here, we write the Standard Model radiation (plus dark matter) which exists prior to the neutrino decoupling as r. This model can be characterized by the three parameters: the branching ratio of inflaton decay into r r_ϕ, that of curvaton r_σ and the ratio of the curvaton energy density to the total energy density at its decay R_σ.

[1]The standard value of $N_{\rm eff}$ is not 3 but $\simeq 3.046$, due to partial heating of the neutrinos at $e^\pm$ annihilation.

Table 1: Energy components of the Universe at each epoch except for CDM and baryon. Here $\Gamma_\phi\,(\Gamma_\sigma)$ is the decay rate of the inflaton (curvaton), and Γ_ν denotes the neutrino interaction rate at the neutrino freezeout. $\Gamma_{e^\pm}$ denotes the Hubble parameter at the $e^\pm$ annihilation. r_e denotes the plasma consisting of γ and $e^\pm$. The superscripts (ϕ) and (σ) represent the origin of the fluid.

epoch	component	energy transfer
$\Gamma_\phi < H$	ϕ, σ	$\phi \to X^{(\phi)} + r^{(\phi)}$
$\Gamma_\sigma < H < \Gamma_\phi$	$X^{(\phi)}, r^{(\phi)}, \sigma$	$\sigma \to X^{(\sigma)} + r^{(\sigma)}$
$\Gamma_\nu < H < \Gamma_\sigma$	X, r	$r \to \nu + r_e$
$\Gamma_{e^\pm} < H < \Gamma_\nu$	X, ν, r_e (DR $= X + \nu$)	$e^\pm \to \gamma$
$H < \Gamma_{e^\pm}$	X, ν, γ (DR $= X + \nu$)	

We define the adiabatic perturbation ζ as the curvature perturbation on the uniform density slice. The isocurvature perturbation of DR is defined as

$$S_{DR} \equiv 3(\zeta_{DR} - \zeta). \tag{2.1}$$

Here the curvature perturbation of DR, ζ_{DR} is defined as

$$\rho_{DR}(\vec{x}) = \bar{\rho}_i e^{4(\zeta_{DR}-\zeta)}, \tag{2.2}$$

where $\rho_{DR}(\vec{x})$ is evaluated on the uniform density slice. ζ and S_{DR} are determined by the quantum fluctuation of scalar fields, $\delta\phi$ and $\delta\sigma$. Up to the second order of them, ζ and S_{DR} are expanded as

$$\zeta = \sum_i N_{\phi_i}\delta\phi_i + \sum_{i,j}\frac{1}{2}N_{\phi_i\phi_j}\delta\phi_i\delta\phi_j,\ S_{DR} = \sum_i S_{\phi_i}\delta\phi_i + \sum_{i,j}\frac{1}{2}S_{\phi_i\phi_j}\delta\phi_i\delta\phi_j, \tag{2.3}$$

where $\delta\phi_i$ is either $\delta\phi$ or $\delta\sigma$. Up to the leading term, the power spectra of ζ and S_{DR} are given by

$$\langle A_1(\vec{k}_1)A_2(\vec{k}_2)\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_{A_1A_2}(k_1), \tag{2.4}$$

where A_i denotes either ζ or S_{DR} and

$$P_{\zeta\zeta}(k) = [N_\phi^2 + N_\sigma^2]P_{\delta\phi}(k),\ P_{\zeta S_{\rm DR}}(k) = [N_\phi S_\phi + N_\sigma S_\sigma]P_{\delta\phi}(k),\ P_{S_{\rm DR}S_{\rm DR}}(k) = [S_\phi^2 + S_\sigma^2]P_{\delta\phi}(k), \tag{2.5}$$

The power spectrum of $\delta\phi_i$ ($\delta\phi$ or $\delta\sigma$) is defined as

$$\langle \delta\phi_i(\vec{k}_1)\delta\phi_j(\vec{k}_2)\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2)P_{\delta\phi}(k_1)\delta_{ij}, \tag{2.6}$$

$$P_{\delta\phi}(k) = \frac{H_{\rm inf}^2}{2k^3}\left(\frac{k}{k_0}\right)^{n_s-1}, \tag{2.7}$$

where $H_{\rm inf}$ is the Hubble parameter during inflation, n_s is the scalar spectral index [2] and k_0 is the pivot scale chosen as $k_0 = 0.002{\rm Mpc}^{-1}$. The amplitude of the isocurvature perturbation and the correlation between it and the adiabatic one are parametrized by α and γ, which are defined as

$$\begin{pmatrix} \mathscr{P}_{\zeta\zeta}(k) & \mathscr{P}_{\zeta S_{\rm DR}}(k) \\ \mathscr{P}_{S_{\rm DR}\zeta}(k) & \mathscr{P}_{S_{\rm DR}S_{\rm DR}}(k) \end{pmatrix} = A_s\left(\frac{k}{k_0}\right)^{n_s-1}\begin{pmatrix} 1-\alpha & \gamma\sqrt{\alpha(1-\alpha)} \\ \gamma\sqrt{\alpha(1-\alpha)} & \alpha \end{pmatrix}, \tag{2.8}$$

[2] The scalar spectral indices for ϕ and σ do not coincide in general. In the following we assume they are the same just for simplicity.

where $\mathscr{P}_{AB}(k) \equiv (k^3/2\pi^2)P_{AB}(k)$ and $A_s \approx 2\times 10^{-9}$ is the amplitude of primordial power spectra. $\gamma = 0$, $\gamma = +1$ and $\gamma = -1$ mean that the adiabatic perturbation and the DR isocurvature perturbation are uncorrelated, totally correlated and totally anti-correlated, respectively. The bispectra of ζ and S_{DR} are defined as

$$\langle A_1(\vec{k}_1)A_2(\vec{k}_2)A_3(\vec{k}_3)\rangle \equiv (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)B_{A_1A_2A_3}(k_1,k_2,k_3). \tag{2.9}$$

We define the non-Gaussianity parameters $f_{NL}^{A_1,A_2A_3}$, which represent magnitudes of non-Gaussianities, as

$$B^{A_1A_2A_3}(k_1,k_2,k_3) = f_{NL}^{A_1,A_2A_3}(k_1,k_2,k_3)P_{\zeta\zeta}(k_2)P_{\zeta\zeta}(k_3) + (2 \text{ cyclics of } \{123\}), \tag{2.10}$$

They are written in terms of the coefficients in (2.3) as

$$\begin{aligned}
f_{\rm NL}^{\zeta,\zeta\zeta} \equiv f_{NL}^{(1)} = \frac{\sum_{i,j} N_{\phi_i}N_{\phi_j}N_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},\ f_{\rm NL}^{S_{\rm DR},\zeta\zeta} \equiv f_{NL}^{(2)} = \frac{\sum_{i,j} S_{\phi_i}N_{\phi_j}N_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},\\
f_{\rm NL}^{\zeta,S_{\rm DR}\zeta} = f_{\rm NL}^{\zeta,\zeta S_{\rm DR}} \equiv f_{NL}^{(3)} = \frac{\sum_{i,j} N_{\phi_i}N_{\phi_j}S_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},\ f_{\rm NL}^{\zeta,S_{\rm DR}S_{\rm DR}} \equiv f_{NL}^{(4)} = \frac{\sum_{i,j} N_{\phi_i}S_{\phi_j}S_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},\\
f_{\rm NL}^{S_{\rm DR},\zeta S_{\rm DR}} = f_{\rm NL}^{S_{\rm DR},S_{\rm DR}\zeta} \equiv f_{NL}^{(5)} = \frac{\sum_{i,j} S_{\phi_i}S_{\phi_j}N_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},\ f_{\rm NL}^{S_{\rm DR},S_{\rm DR}S_{\rm DR}} \equiv f_{NL}^{(6)} = \frac{\sum_{i,j} S_{\phi_i}S_{\phi_j}S_{\phi_i\phi_j}}{(\sum_i N_{\phi_i}^2)^2},
\end{aligned} \tag{2.11}$$

up to the leading order [3] .

In the model considered in this paper, we can calculate the coefficients in (2.3) using δN-formalism[4, 5] as[2],

$$\begin{aligned}
N_\phi = \tfrac{1}{M_{\rm P}^2}\tfrac{V}{V_\phi},\ N_{\phi\phi} = \tfrac{1}{M_{\rm P}^2}\left(1-\frac{VV_{\phi\phi}}{V_\phi^2}\right),\ N_\sigma = \tfrac{3+R}{6\sigma_i}\left(\tfrac{\hat{R}_rR_r^{(\sigma)}}{R_r}+\tfrac{\hat{R}_XR_X^{(\sigma)}}{R_X}\right),\\
N_{\sigma\sigma} = \tfrac{2}{9\sigma_i^2}\tfrac{3+R}{4}\left(\tfrac{\hat{R}_rR_r^{(\sigma)}}{R_r}+\tfrac{\hat{R}_XR_X^{(\sigma)}}{R_X}\right)\left[3+4R-2R^2-2(3+R)\left(\tfrac{\hat{R}_rR_r^{(\sigma)}}{R_r}+\tfrac{\hat{R}_XR_X^{(\sigma)}}{R_X}\right)\right],
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
S_\phi = S_{\phi\phi} = 0,\ S_\sigma = -\tfrac{3+R}{2\sigma_i}\tfrac{\hat{R}_r\hat{R}_X}{\hat{R}_{\rm DR}}\left(\tfrac{R_r^{(\sigma)}}{R_r}-\tfrac{R_X^{(\sigma)}}{R_X}\right)(1-\hat{c}_\nu),\\
S_{\sigma\sigma} = \tfrac{3+R}{2\sigma_i^2}\tfrac{\hat{R}_r\hat{R}_X}{\hat{R}_{\rm DR}}\left(\tfrac{R_r^{(\sigma)}}{R_r}-\tfrac{R_X^{(\sigma)}}{R_X}\right)\tfrac{(1-\hat{c}_\nu)}{3}\left[2R^2-4R-3+\tfrac{3+R}{\hat{R}_{\rm DR}}\left(\tfrac{\hat{R}_rR_r^{(\sigma)}(\hat{c}_\nu+\hat{R}_{\rm DR})}{R_r}+\tfrac{\hat{R}_XR_X^{(\sigma)}(1+\hat{R}_{\rm DR})}{R_X}\right)\right].
\end{aligned} \tag{2.13}$$

The meanings of the symbols are as follows: M_P is the reduced Planck mass. V is the potential of ϕ and V_ϕ and $V_{\phi\phi}$ are the first and second derivatives of V, respectively, evaluated when observable scales exit the horizon. $R \equiv 3R_\sigma/(4-R_\sigma)$. R_i is the ratio of the energy density of a fluid i to the total energy density at the decay of σ, and $\hat{R}_i$ is that at the $e^\pm$ annihilation. $R_i^{(\sigma)}$ is the ratio of energy density of the fluid i generated by σ decay at that time. σ_i is the amplitude of the oscillation of σ when it starts to oscillate. $\hat{c}_\nu \simeq 0.405$ is the ratio of the energy density of neutrino to that of standard model relativistic particles (photons and neutrinos) after the electron-positron annihilation. The excess of $N_{\rm eff}$ to the standard value in this scenario is

$$\Delta N_{\rm eff} = \frac{3\hat{R}_X}{\hat{c}_\nu\hat{R}_r}. \tag{2.14}$$

[3]There are also terms which arises from the product of three quadratic terms of $\delta\phi_i$ in ζ or $S_{\rm DR}$, as shown in [3].

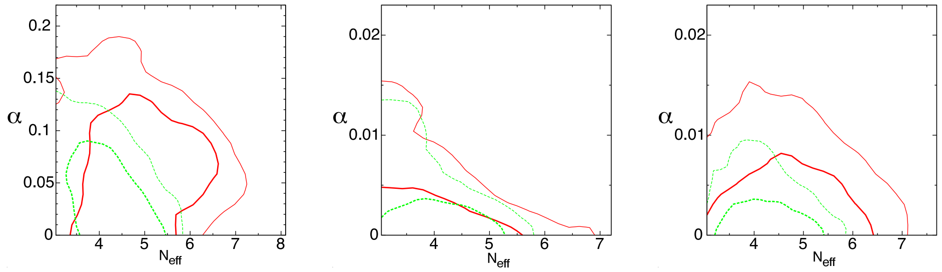


Figure 1: 1σ and 2σ C.L. constraints in $N_{\rm eff}$-α plane from the CMB (red solid) and ALL (green dashed) datasets. From left to right, shown are the constraints for the uncorrelated ($\gamma = 0$), totally correlated ($\gamma = 1$) and totally anti-correlated ($\gamma = -1$) cases. Note that the scales are not same among three panels.

We refer to Ref. [2] for details and derivations of these quantities and here only comment that R's in the above equations are written in terms of r_ϕ, r_σ and R_σ and so are (2.12)-(2.14).

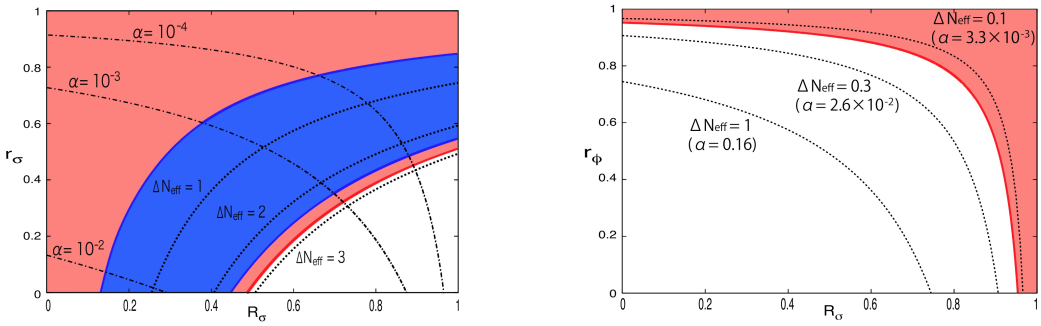
3. Constraints from current observations and implications to the model

In this section we derive constraints on the extra radiation with isocurvature perturbations from current cosmological observations. The energy density of the extra radiation changes the Hubble expansion rate and affect the CMB power spectrum. The free-streaming of DR also affects the CMB perturbation. The isocurvature perturbation of DR, which is equivalent to the neutrino isocurvature perturbation, induce the different type of the CMB spectrum than that induced by the adiabatic perturbation. Due to these effects, we can constrain $N_{\rm eff}$ and S_{DR} using the CMB data.

We adopt CMB data of WMAP 7-year result [6, 7, 8] and ACT [9, 10, 11] at small angular scales. We also include data from the baryon acoustic oscillation (BAO) in the power spectrum of SDSS galaxies [12] and the direct measurement of the Hubble constant (H0) [13]. Although these observations do not constrain $N_{\rm eff}$ or S_{DR} directly, including them leads to breaking of the parameter degeneracies. Hereafter, we will refer to sets of combined datasets of WMAP+ACT and WMAP+ACT+BAO+H0 as “CMB" and “ALL", respectively.

Using a modified version of the publicly available CosmoMC code[14], we derive the constraints on parameters by the MCMC analysis[2]. Figure 1 shows the constraint in $N_{\rm eff}$-α plane when γ is fixed to 0 or ± 1. We see that the data of BAO and H0 actually tighten the constraints. $N_{\rm eff} = 3.046$ is out of the 1σ allowed region in the uncorrelated and totally anti-correlated cases, but consistent with the 2σ C.L. level. As is well known, the current CMB data does not show any signatures of the isocurvature perturbation and $\alpha = 0$ is consistent in all the cases.

Let us apply the above result to some limiting cases in our cosmological model, which are motivated by particle physics. First, we consider the “uncorrelated" case, where the inflaton does not decay into X ($r_\phi = 1$) and the adiabatic perturbation is dominantly produced by the inflaton ($N_\phi \gg N_\sigma$). Then, the perturbation of X is uncorrelated with ζ. Such a situation can arise in the KSVZ axion model[15, 16], where saxion decays mainly into axion, identifying σ with saxion and ϕ with another scalar field. Figure 2(a) shows the constraint in the R_σ-r_σ plane, in addition to contours of α and $\Delta N_{\rm eff}$. Here, $2R\delta\sigma/\sigma_i$ is set to $0.1 \times \zeta_\phi \simeq 5 \times 10^{-6}$.



(a) The constraint in R_σ-r_σ plane for the uncorrelated case. In this figure $2R\delta\sigma/\sigma_i \simeq 5\times10^{-6}$ is fixed.

(b) The constraint in R_σ-r_ϕ plane for the totally anti-correlated case. There is no 1σ allowed region.

Figure 2: The constraints on the model parameters in the uncorrelated case and the totally anti-correlated case. 1σ and 2σ allowed regions are shown by blue and orange, respectively. Contours of α and $\Delta N_{\rm eff}$ are also shown. There is the one-to-one correspondence between α and $N_{\rm eff}$.

Next, we consider the "totally anti-correlated" case, where σ does not decay into X ($r_\sigma = 1$) and it dominantly produces the curvature perturbation ($N_\phi \ll N_\sigma$). σ is the curvaton in the original meaning. In this case, X produced by inflaton decay has isocurvature perturbation anti-correlated with the adiabatic one. Such a situation arises in the DFSZ axion model[17, 18], where saxion decay mainly into Higgs bosons, and eventually lighter SM particles. Figure 2(b) shows the constraints on R_σ-r_ϕ plane in this case. There is not 1σ allowed region and only the 2σ allowed region is shown. Besides, we need $R \gtrsim 0.01$, in order not to have too large non-Gaussianity.

4. Forecast for CMB constraints on the non-Gaussianity parameters

Although the primordial curvature perturbation is almost Gaussian, current CMB data imply that it might have non-Gaussianity at 1σ level[1] and future observations might confirm it. In our cosmological scenario the primordial perturbations have quadratic terms of $\delta\phi_i$, which means that they non-Gaussian, like the curvaton scenario. Then bispectra of them might be large and this might lead to the bispectra of the CMB fluctuation detectable in the future. Note that the isocurvature perturbation can be dominant source of non-Gaussianity of the CMB, although it is smaller than the adiabatic one as the source of the power spectra. Like the power spectra, the CMB bispectra induced by ζ and those by S_{DR} have different feature. Therefore, the CMB bispectra, if detectable, provide us the novel constraints on the extra radiation with the isocurvature perturbation.

The magnitude of each type of bispectra is parametrized by the non-Gaussianity parameters $f_{NL}^{(i)}$. We can perform the Fisher matrix analysis[19, 20, 21] in order to find how strongly they will be constrained in future observations, choosing the fiducial parameter that all f_{NL}'s are 0[3] . The resultant uncertainties in Planck survey[22] and the cosmic variance limited survey (CVL), in which the instrumental noise is so small that the uncertainties are determined only by the cosmic variance, are summarized in Table 2 for $N_{\rm eff} = 3.04$ and in Table 3 $N_{\rm eff} = 4$. If the non-Gaussianity parameters are larger than the values in Table 2 and Table 3, we will detect the non-Gaussianity in the CMB anisotropy and obtain some information on the extra radiation.

Table 2: Expected uncertainties for non-Gaussianity parameters for the case with $N_{\rm eff} = 3.04$. “P" and “C" means Planck and CVL survey respectively.

	$f_{\rm NL}^{(1)}$	$f_{\rm NL}^{(2)}$	$f_{\rm NL}^{(3)}$	$f_{\rm NL}^{(4)}$	$f_{\rm NL}^{(5)}$	$f_{\rm NL}^{(6)}$
P	21	126	27	187	257	339
C	3.5	18.3	5.0	27.2	26.4	39.3

Table 3: Same as in Table 2 but for the case with $N_{\rm eff} = 4$.

	$f_{\rm NL}^{(1)}$	$f_{\rm NL}^{(2)}$	$f_{\rm NL}^{(3)}$	$f_{\rm NL}^{(4)}$	$f_{\rm NL}^{(5)}$	$f_{\rm NL}^{(6)}$
P	22	101	21	116	163	164
C	3.5	14.0	3.7	15.9	15.4	17.3

5. Summary

Motivated by recent observations, in this paper we considered the situation that there are some non-interacting relativistic particles different from neutrinos, dubbed the extra radiation. We focused on the possibility that the dark radiation (DR), the mixture of neutrino and the extra radiation, has the isocurvature perturbation S_{DR}, which is motivated by the scenario that the extra radiation is produced by particle decay. We considered the general framework in which there are inflaton ϕ and the other light scalar field (curvaton) σ and all components in the Universe are produced by their decay. We derived the formulas for the primordial perturbations, the non-Gaussianity parameter for them and the effective neutrino number in terms of the model parameters, the branching ratio of ϕ and σ decay and the energy density ratio of σ. Then, we found the constraints on the effective neutrino number $N_{\rm eff}$ and S_{DR} from current cosmological observations, using the MCMC method, and applied them to some limiting cases in our cosmological model. Finally, we forecast the constraints on non-Gaussianity parameters from future CMB observations using the Fisher analysis.

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