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Mapping the Conformal Window: SU(2) with 4, 6 and 10 flavors of fermions

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We present studies of the SU(2) gauge theory with 4, 6 and 10 fermion flavors. These models are expected to lie on both sides of the edge of the conformal window, where the theory has an infrared fixed point. We observe that the coupling grows with the length scale at four flavors, implying QCD-like behavior. At ten flavors the results are compatible with a Bank-Zaks type fixed point. The results at six flavors remain inconclusive: the running is slow towards the infrared but the range and accuracy of the study are insufficient for determining the existence of a fixed point.
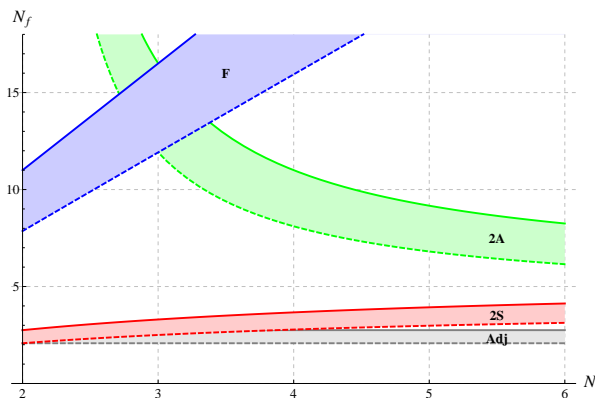
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Figure 1: The phase diagram of SU(N) gauge theory as a function of the number of colors, flavors and fermion representations (F = Fundamental, 2A = 2-index antisymmetric, 2S = 2-index symmetric, Adj = Adjoint). The shaded bands indicate the estimated conformal windows.

1. Introduction

Within the possible phase space of gauge theories there is a group of models with a nontrivial infrared fixed point. In these conformal models, under renormalization group evolution, the coupling runs to smaller values at small distances, exhibiting asymptotic freedom, but runs to a constant at large distances. They have applications in phenomenological model building, such as for technicolor theories [1, 2, 3], where the Higgs sector is replaced with a strongly interacting sector with chiral symmetry breaking. From purely theoretical point of view, mapping the phase diagram of gauge theories in the number of colors N and fermion flavors N_f is interesting for understanding their nonperturbative dynamics from first principles. Many lattice studies of the conformal window have already appeared in the literature: for example SU(2) with fundamental representation fermions [4], SU(2) with adjoint fermions [5, 6, **?**, 8, 9, 10] and SU(3) with fermions in the fundamental [11, 12, 13] or in the two-index symmetric [14], i.e. the sextet, representation.

In figure 1 we sketch a phase diagram for SU(N) gauge theories as a function of N and N_f for model with fermions interacting with the fundamental, two-index (anti)symmetric and adjoint representations of the gauge field. The upper boundary corresponds to the loss of asymptotic freedom, when the first coefficient of the perturbative expansion of the beta function is zero: $\beta_0 = 11/3N - 4/3N_f T(R) = 0$, where $T(R)$ is the group theory factor for the fermion representation R. Just below the upper bound the value of the fixed point is expected to be small and perturbation theory applicable. When the number of flavors is lowered, however, the fixed point is expected to move to higher coupling. Finally, as one comes to the lower limit, the critical coupling for the chiral symmetry breaking becomes smaller than the expected fixed point and the model becomes chirally broken. The lower bound is therefore and inexact approximation in a region where perturbation theory may not be applicable, and needs to be checked using nonperturbative methods. This provides and interesting challenge for the lattice community.

In this study we investigate the phase diagram of SU(2) gauge theory with $N_f = 10, 6$ and 4. The results have been published in reference [15]. The models with 10 and 4 fermions flavors are expected to lie well within and below the conformal window respectively. The model with 6

fermions should be close to the lower boundary of the conformal window and is therefore the most challenging of these models.

Since large discretization errors have been observed in previous studies with unimproved Wilson fermions, we measure the coupling using the Schrödinger functional method with perturbatively improved Wilson fermions. The models with 4 and 10 fermion flavors behave as expected and are in the confining and conformal phases respectively. Unfortunately, in the 6 flavor case we are unable to resolve whether a fixed point exists, but the possible locations of the fixed point is are at a much higher value of the coupling than suggested by previous unimproved results.

2. The Method and Results

The model is defined by the lattice action $S = S_G + S_F$, where S_G is the standard Wilson plaquette action and S_F is the clover improved fermion action

$$S_F = a^4 \sum_{\alpha=1}^{N_f} \sum_x \left[\bar{\psi}_\alpha(x)(iD + m_0)\psi_\alpha(x) + ac_{\text{sw}} \bar{\psi}_\alpha(x)\frac{i}{4}\sigma_{\mu\nu}F_{\mu\nu}(x)\psi_\alpha(x) \right], \tag{2.1}$$

where D is the standard Wilson-Dirac operator. We set the improvement coefficient c_{sw} to the perturbative value [17] $c_{\text{sw}} = 1 + 0.1551(1)g_0^2 + O(g_0^4)$. We have performed short measurements with $N_f = 6$ and 10 that suggest that this is close to the nonperturbative value at large coupling. This is not the case with $N_f = 2$ [18], where c_{sw} seems to diverge when g^2 is increased. We have also included perturbative improvement at the Schrödinger functional boundaries,

$$\delta S_{c_t} = \frac{\beta_L}{4} \sum_{p_T} (c_t - 1)\text{tr}(1 - U(p))$$

$$\delta S_{\tilde{c}_t} = a^4 \sum_x (\tilde{c}_t - 1)\frac{1}{a} \bar{\psi}(x)\psi(x)(\delta(x_0 - a) + \delta(x_0 - (L - a)))$$

as described in [18].

We measure the running coupling using the Schrödinger functional method [19, 20, 21, 22]. We consider a lattice of volume $V = (Na)^4$. The spatial links at the timelike boundaries of the lattice are fixed to the values

$$U_\mu(\bar{x}, t = 0) = e^{-i\eta\sigma_3 a/L}, \ U_\mu(\bar{x}, t = L) = e^{-i(\pi-\eta)\sigma_3 a/L} \tag{2.2}$$

where σ_3 is the third Pauli matrix. The spatial boundary conditions are periodic for the gauge field. The fermion fields are set to vanish at the the $t = 0$ and $t = L$ boundaries and twisted periodic boundary conditions $\psi(x + L\hat{i}) = \exp(i\pi/5)\psi(x)$ are set at the spatial boundaries. At the classical level the boundaries generate a constant chromoelectric field and the response of the field to the boundaries can be easily calculated,

$$\frac{\partial S^{\text{cl.}}}{\partial \eta} = \frac{k}{g_0^2}, \tag{2.3}$$

where the constant k is a function of $N = L/a$ and η [20]. At full quantum level we define the running coupling g^2 trough

$$\left\langle \frac{\partial S}{\partial \eta} \right\rangle = \frac{k}{g^2}.$$
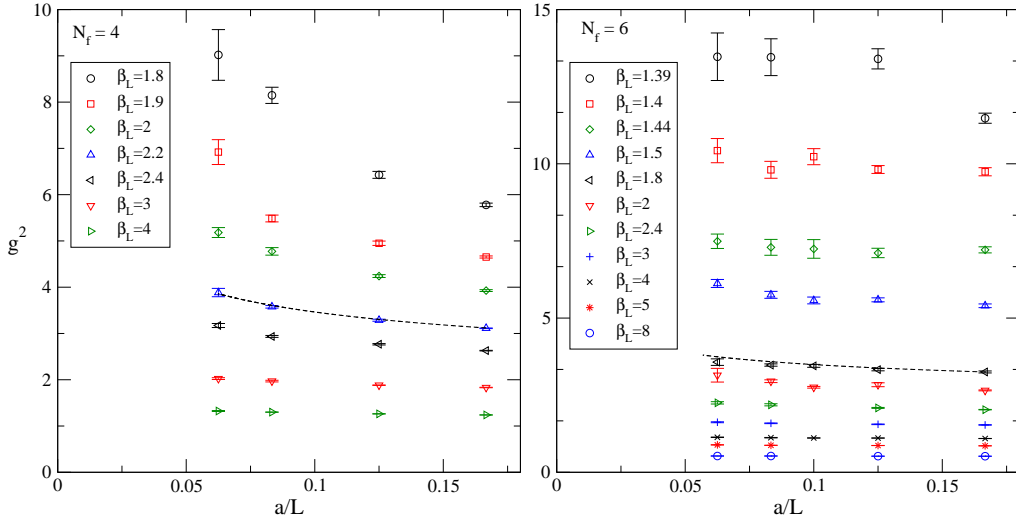
Figure 2: The measured values of $g^2(g_0^2, L/a)$ against the inverse lattice size a/L with 4 and 6 flavors of fermions. The Black dashed lines give an example of the running in perturbation theory to 2-loop order at a modest coupling, normalized to match the measurement at $L/a = 6$.
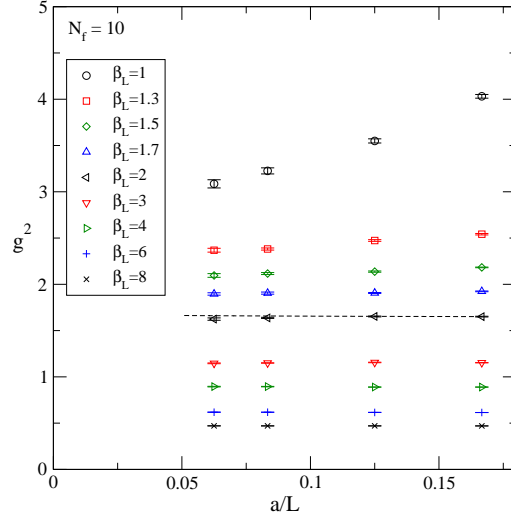

Figure 3: The measured values of $g^2(g_0^2, L/a)$ against the inverse lattice size a/L with 10 flavors of fermions. The Black dashed lines give an example of the running in perturbation theory to 2-loop order at a modest coupling, normalized to match the measurement at $L/a = 6$.

The measured values of g^2 for the models with $N_f = 4$ and 6 are given in figure 2 and for the model with $N_f = 10$ in figure 3. The figures show that in the four fermion model the running of the coupling with the energy scale stays negative and increases in magnitude with the coupling. In the 10 fermion model the running is slow at small coupling and changes sign between $g^2 = 1$ or $g^2 = 2$. In the six fermion model the running remains slow to very high coupling but does not seem to change sign, although the within the errors it is impossible to make any conclusion above $g^2 = 10$.
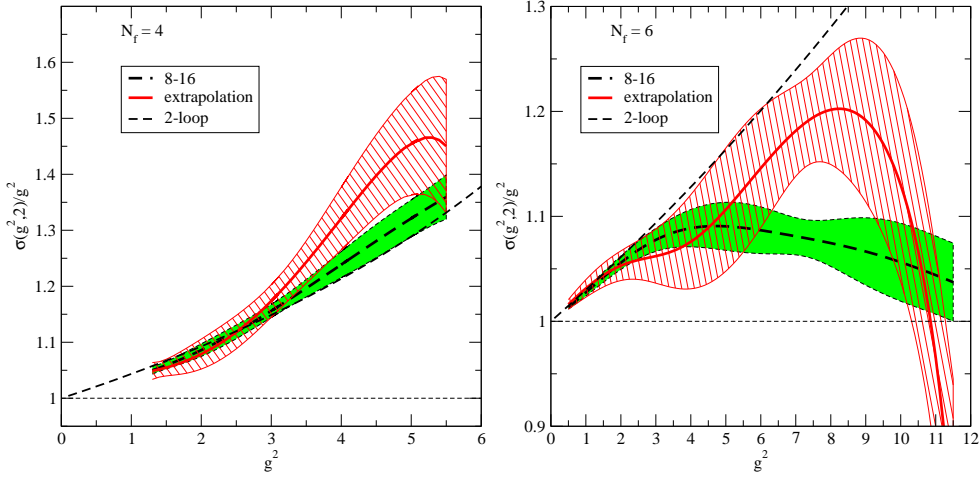
Figure 4: (color online) The scaled step scaling function $\sigma(g^2,2)/g^2$ with 4 and 6 fermions. The thick red line corresponds to the continuum extrapolation, and the hashed band to the statistical errors of the extrapolation. The thick dashed line with the shaded error band is the largest volume step scaling function without extrapolation. The thin dashed line is the 2-loop perturbative value of $\sigma(g^2,2)/g^2$.

To quantify the running and facilitate taking the continuum limit, we use the step scaling function $\Sigma(u,s,L/a)$ introduced in [19]:

$$\Sigma(u,s,L/a) = g^2(g_0^2,sL/a)\big|_{g^2(g_0^2,L/a)=u} \tag{2.4}$$

$$\sigma(u,s) = \lim_{a/L\to 0}\Sigma(u,s,L/a). \tag{2.5}$$

We choose $s=2$ and calculate $\Sigma(u,s,L/a)$ at $L=6$ and 8. Since we expect most $\mathcal{O}(a)$ effects to be absent in the improved model, we obtain a continuum limit using quadratic extrapolation.

For the continuum extrapolation we need to calculate $\Sigma(u,s,L/a)$ at the same measured coupling $u = g^2$ on both lattice sizes. We use an interpolating function to define the measured coupling $g^2(g_0^2,L/a)$ in a continuous range of g_0^2. At each volume L/a we fit the data to the function

$$\frac{1}{g^2(g_0^2,L/a)} = \frac{1}{g_0^2}\left[\frac{1+\sum_{i=1}^n a_i g_0^{2i}}{1+\sum_{i=1}^m b_i g_0^{2i}}\right].$$

For the models with 4 and 6 fermion flavors we find the best fit using the parameters $m=2, n=2$ and for the model with 10 flavors $m=1, n=2$.

In figures 4 and 5 we show the step scaling function for SU(2) with 4, 6 and 10 fermion flavors. Both the models with 4 and 10 fermions behave as expected, with the renormalized step scaling function $\sigma(g^2)/g^2$ increasing with the coupling in the case with 4 fermions and clearly crossing $\sigma(g^2)/g^2 = 1$ in the case with 10 fermions. In both models the continuum extrapolation starts to deviate from the lattice result at large coupling, implying the presence of discretization effects. The results for the model with 6 fermions are inconclusive: the running remains slow but the discretization errors start to dominate before there is possibility of a fixed point.
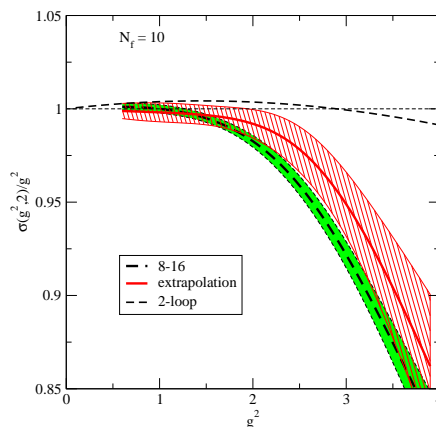
Figure 5: As in figure 4 but with 10 fermion flavors.

To solve the problem we have investigated using smeared actions in combination with $\mathscr{O}(a)$ improvement to reduce the systematic errors and allow simulations with larger lattice sizes. A hypercubic smearing procedure was used to study the running coupling in [10] and [23]. It was demonstrated that smearing can reduce the systematic errors and stabilize the simulation. We are currently investigating SU(2) with two adjoint fermions using a hypercubic stout smearing similar to the one used in [24, 25], using a smeared fermion action and a partially smeared gauge action.

Acknowledgments

References

[1] S. Weinberg, *Implications Of Dynamical Symmetry Breaking: An Addendum*, Phys. Rev. D **19**, 1277 (1979); L. Susskind, *Dynamics Of Spontaneous Symmetry Breaking In The Weinberg-Salam Theory*, Phys. Rev. D **20**, 2619 (1979).

[2] C. T. Hill and E. H. Simmons, Phys. Rept. **381**, 235 (2003) [Erratum-ibid. **390**, 553 (2004)] [arXiv:hep-ph/0203079].

[3] F. Sannino, arXiv:0804.0182 [hep-ph].

[4] F. Bursa, L. Del Debbio, D. Henty, E. Kerrane, B. Lucini, A. Patella, C. Pica and T. Pickup *et al.*, Phys. Rev. D **84**, 034506 (2011) [arXiv:1104.4301 [hep-lat]].

[5] S. Catterall and F. Sannino, Phys. Rev. D **76**, 034504 (2007) [arXiv:0705.1664 [hep-lat]].

[6] A. J. Hietanen, J. Rantaharju, K. Rummukainen and K. Tuominen, JHEP **0905**, 025 (2009) [arXiv:0812.1467 [hep-lat]].

[7] L. Del Debbio, A. Patella and C. Pica, Phys. Rev. D **81**, 094503 (2010) [arXiv:0805.2058 [hep-lat]].

[8] S. Catterall, J. Giedt, F. Sannino and J. Schneible, JHEP **0811**, 009 (2008) [arXiv:0807.0792 [hep-lat]].

[9] A. J. Hietanen, K. Rummukainen and K. Tuominen, Phys. Rev. D **80**, 094504 (2009) [arXiv:0904.0864 [hep-lat]].

[10] T. DeGrand, Y. Shamir and B. Svetitsky, Phys. Rev. D **83**, 074507 (2011) [arXiv:1102.2843 [hep-lat]].

[11] T. Appelquist, G. T. Fleming and E. T. Neil, Phys. Rev. Lett. **100**, 171607 (2008) [Erratum-ibid. **102**, 149902 (2009)] [arXiv:0712.0609 [hep-ph]].

[12] Z. Fodor, K. Holland, J. Kuti, D. Nogradi and C. Schroeder, Phys. Lett. B **681**, 353 (2009) [arXiv:0907.4562 [hep-lat]].

[13] A. Deuzeman, M. P. Lombardo and E. Pallante, Phys. Lett. B **670**, 41 (2008) [arXiv:0804.2905 [hep-lat]].

[14] Y. Shamir, B. Svetitsky and T. DeGrand, Phys. Rev. D **78**, 031502 (2008) [arXiv:0803.1707 [hep-lat]].

[15] T. Karavirta, J. Rantaharju, K. Rummukainen and K. Tuominen, JHEP **1205**, 003 (2012) [arXiv:1111.4104 [hep-lat]].

[16] T. W. Appelquist, D. Karabali and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986).

[17] M. Luscher and P. Weisz, Nucl. Phys. B **479**, 429 (1996) [arXiv:hep-lat/9606016].

[18] T. Karavirta, A. Mykkanen, J. Rantaharju, K. Rummukainen and K. Tuominen, JHEP **1106**, 061 (2011) [arXiv:1101.0154 [hep-lat]].

[19] M. Luscher, R. Narayanan, P. Weisz and U. Wolff, Nucl. Phys. B **384**, 168 (1992) [arXiv:hep-lat/9207009].

[20] M. Luscher, R. Narayanan, R. Sommer, U. Wolff and P. Weisz, Nucl. Phys. Proc. Suppl. **30**, 139 (1993).

[21] M. Luscher, R. Sommer, P. Weisz and U. Wolff, Nucl. Phys. B **413**, 481 (1994) [arXiv:hep-lat/9309005].

[22] M. Della Morte *et al.* [ALPHA Collaboration], Nucl. Phys. B **713**, 378 (2005) [hep-lat/0411025].

[23] T. DeGrand, Y. Shamir and B. Svetitsky, arXiv:1201.0935 [hep-lat].

[24] S. Durr *et al.*, JHEP **1108**, 148 (2011) [arXiv:1011.2711 [hep-lat]].

[25] A. Hasenfratz, R. Hoffmann and S. Schaefer, JHEP **0705**, 029 (2007) [arXiv:hep-lat/0702028].