

Two-Color Schrödinger Functional with Six-Flavors of Stout-Smeared Wilson Fermions

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We study the Schrödinger functional running coupling in the $SU(2)$ gauge theory with six-flavors of massless fermions. The aim is to determine whether the above theory has an infrared fixed point (IRFP). We use the standard Wilson gauge action and the stout-smeared Wilson fermion action. Here we present a determination of the critical mass as a function of the bare coupling and a preliminary study of the phase diagram of this lattice action. We also find preliminary indication that this theory has no IRFP. While this conclusion is not yet definite, we also show that with this approach we will be able to take a proper continuum limit and clearly determine the status of this theory with a reasonable amount of computer time.

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1. Introduction

A SM Higgs adequately accounts for all electroweak measurements (for now). However, it has a number of theoretical shortcomings, chiefly among them the hierarchy problem. technicolor is an alternative for electroweak symmetry breaking that avoids the introduction of a fundamental light scalar particle [1]. In particular, a technicolor model based on a walking gauge theory can perhaps account for the standard model fermion masses [2]. Since such a theory is expected to reside just below the conformal window, it is essential to narrow down the extent of this window. The goal of this work is to do that for the $SU(2)$ gauge theories with N_f fermion flavors in the fundamental representation.

The $SU(2)$ gauge theories are particularly interesting in light of the recent discovery of a 125 GeV Higgs-like particle [3, 4]. This newly discovered resonance, while not yet confirmed, is widely expected to have the quantum numbers of a scalar particle. In the framework of strongly coupled theories, light scalars arise as pseudo-Nambu-Goldstone bosons (PNGB), i.e. from explicit breaking of a continuous symmetry which would be broken spontaneously in the absence of explicit breaking. One speculative mechanism is that the breaking of approximate scale invariance, as is present in a walking gauge theory will produce a light dilaton [5]. This gives yet another motivation to search for walking theories. PNGBs are also expected from the breaking of global chiral symmetries. In QCD, this mechanism yields light pseudo-scalars. Two-color theories, due to the pseudo reality of the fundamental representation of $SU(2)$, have an enhanced chiral symmetry and hence novel symmetry breaking patterns [6]. A number of authors use this novel breaking pattern to construct strongly coupled models whose spectrum contain several light scalar PNGBs [7, 8], the lightest of which would be a suitable Higgs candidate.

We take a rather direct route towards narrowing down the extent of the conformal window for two-color theories with N_f flavors of Dirac fermions. We calculate the running coupling in the Schrödinger functional (SF) scheme [9] and directly search for the presence or lack of an infrared fixed point (IRFP) at various N_f . Evidence that $N_f = 10(4)$ is inside (outside) the conformal window is presented by [10]. Additionally Ohki et. al. argue that $N_f = 8$ is inside the conformal window [11]. The case $N_f = 6$ while tackled by many groups [10, 12, 13] remains inconclusive.

In [13], we attempt to determine whether $N_f = 6$ has an IRFP. We used the standard Wilson plaquette gauge action and the Wilson fermion action. Our calculation was inconclusive due to mistuning of the bare fermion mass and an inability to probe sufficiently strong renormalized couplings at computationally feasible lattice volumes due to a lattice artifact driven phase transition. Here we revisit the $N_f = 6$ theory but we move to the stout-smeared [14] Wilson fermion action. This action avoids coupling the fermions to the unphysical fluctuations of the gauge field on the scale of the lattice spacing. This approach allow us to probe a sufficiently large renormalized couplings to either see an IRFP or definitely rule one out. Additionally, we undertake a great deal more careful tuning of the bare fermion mass.

2. Preliminary Study of the Lattice Action

For this work we use the standard Wilson plaquette gauge action and the stout-smeared Wilson fermion action. The Wilson fermion action contains an additional irrelevant operator that lifts

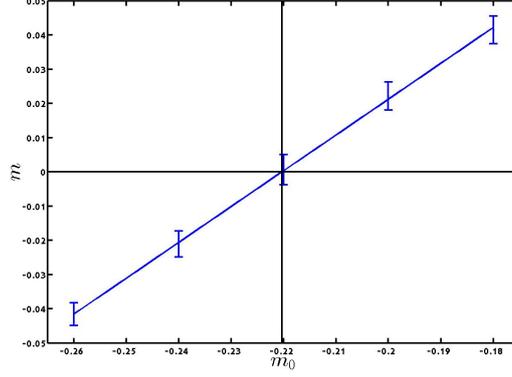


Figure 1: PCAC quark mass at $g_0^2 = 1.4$ and $L/a = 12$ plotted versus the bare quark mass m_0 . A linear fit to these points is included as well. The root of this linear function determines the critical mass $m_c(g_0^2 = 1.2, \frac{L}{a} = 12)$, the value of the bare mass which results in restored chiral symmetry signaled by a zero PCAC mass. The root is highlighted in this figure by a vertical line. This figure is representative of all critical mass determinations at different bare couplings and lattice volumes.

the mass of the fermion doublers to the cutoff scale so they decouple from the calculation. This additional term explicitly breaks chiral symmetry and as a result the fermion mass is additively renormalized. As a result the bare mass must be carefully tuned in order to restore chiral symmetry. This critical value of the bare mass (as a function of the bare coupling) $m_c(g_0^2)$ is defined as the bare mass value, m_0 , that results in a zero PCAC quark mass [15]. In practice, m_c is determined, at fixed bare gauge coupling g_0^2 and lattice volume $(L/a)^3 \times 2L/a$, as the root of a fitted linear function to measurements of the PCAC quark mass versus the bare quark mass. One example of this procedure is shown below in figure 1. This is done for a range of bare couplings and lattice volumes and fit to a polynomial given by

$$m_c^{\text{fit}}\left(g_0^2, \frac{a}{L}\right) = \sum_{i=1}^n g_0^{2i} \left[a_i + b_i \left(\frac{a}{L} \right) \right]. \quad (2.1)$$

Finally, $m_c^{\text{fit}}(g_0^2, 0)$ will be used in all following running coupling calculations. All data used to fit m_c^{fit} and $m_c^{\text{fit}}(g_0^2, 0)$ show below in figure 2.

In order to guarantee that we can take a continuum limit, we need to ensure that we obtain data from the weak-coupling side of any spurious lattice phase transition. With this in mind, we scanned through the bare parameter space and located peaks in the plaquette susceptibility on a $L/a = 10^3 \times 11$ lattice. This search indicates a line in the $m_0 - g_0^2$ plane of first order phase transition that ends at a critical point at around $g_0^2 \approx 2.2$. For $g_0^2 \lesssim 2.3$, we see crossover behavior. In Figure 3, we show the above transition line plotted along with $m_c^{\text{fit}}(g_0^2, 0)$. Figure 3 indicates that the six flavor massless fundamental Wilson fermion action has a sensible continuum limit only for $g_0^2 \lesssim 2.175$. Therefore, we will only examine the running coupling on lattices with a bare coupling $g_0^2 < 2.175$.

The final choices required to specify our action are detailing the gauge field smearing operator used in the fermion action. We use only one level of stout-smearing with a smearing parameter $\rho = 0.25$. These properties of the smearing operator are described in [14]. As all calculations

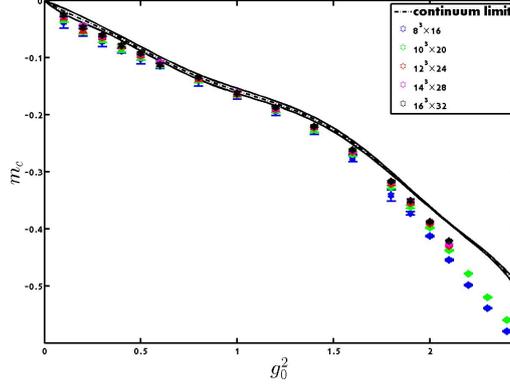


Figure 2: All critical masses determined by similar procedure to that which is shown in figure 1. All m_c determinations are fit to the polynomial $m_c^{\text{fit}}(g_0^2, \frac{a}{L})$ given in eq. (2.1). We show an extrapolation to the continuum critical mass which is given by $m_c^{\text{fit}}(g_0^2, 0)$.

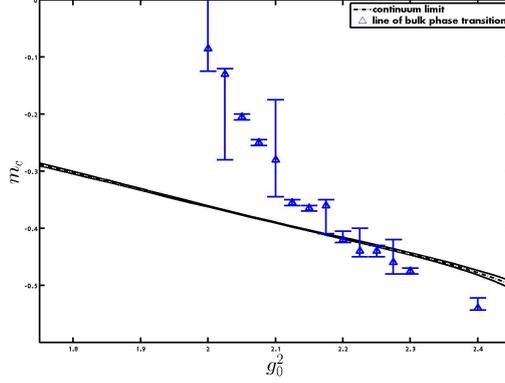


Figure 3: $m_c^{\text{fit}}(g_0^2, 0)$ plotted along with the peak in the plaquette susceptibility. We collect all running coupling data along the critical mass line on the weak coupling side of the phase transition line.

in this work are done with Dirichlet boundary conditions in the time directions, there is some ambiguity in how to implement the smearing of the gauge field near this boundary. It is clearly important to not smear boundary links with bulk links. Even with this constraint there is still some ambiguity in how we define our smearing operator. Specifically we can choose whether to allow the bulk links near the boundary to be smeared with boundary links.

The chief observable of interest is the SF running coupling, roughly it is the derivative of the action with respect to the SF boundary conditions [16]. If we smear bulk links with boundary links, then this observable becomes significantly more complicated and difficult to calculate. Hence we would prefer to define our smearing operator to avoid this. We did not fully appreciate this issue until after completing the preliminary study of the phase diagram of our action. However we recomputed the critical mass and the plaquette susceptibility on a limited sample with the preferable definition of the smearing operator. We show a comparison between the critical mass calculated

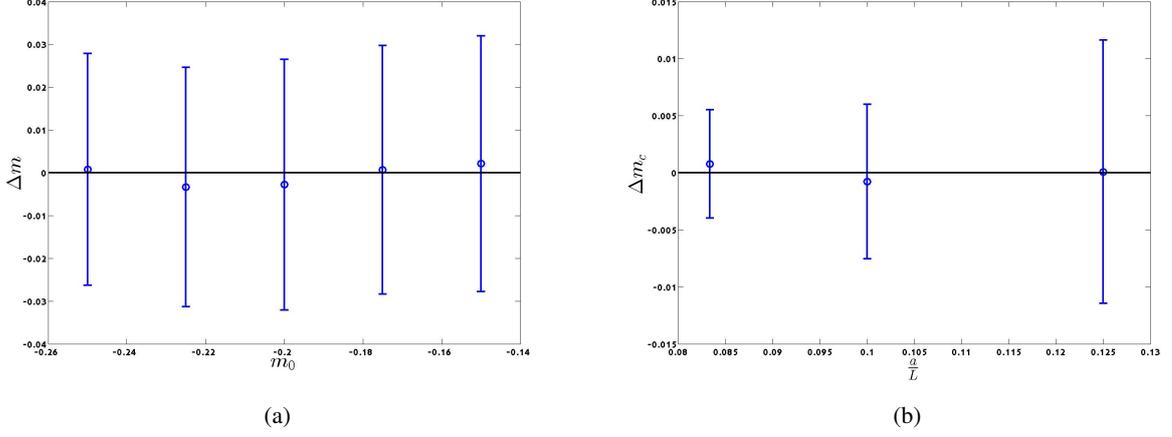


Figure 4: A representative sampling of differences, when calculated using two different definitions of the smearing operator near the boundaries in the time directions, in two quantities relevant in the analysis of the critical mass. All data shown in (a) and (b) obtained at $g_0^2 = 1.2$. In (a) we show the difference in the PCAC quark mass m at $L/a = 8$ versus several bare quark masses m_0 near the critical mass. In (b), the difference in the critical mass m_c ($g_0^2 = 1.2, L/a$) is shown versus L/a . Both (a) and (b) show no statistically significant difference.

with the two different smearing procedures in figure 4. Additionally, no significant shift in the phase boundary shown in figure 3 was found. A comparison of the peak susceptibility (used in determining the phase boundary) on a sample of the data is shown in figure 5. We conclude that any shift in quantities used to determine the phase diagram, due to modifying the smearing near the boundaries, is well below all other sources of errors and we proceed to use the preferable smearing definition for the running coupling while using the determined calculated critical mass and phase boundary.

3. Running Coupling and Conclusions

We calculate the SF running coupling, as defined here [16], at several fixed bare coupling and a variety of lattice volumes. Results are shown in figure 6. No attempt is made to take a continuum limit hence no rigorous conclusions about the IR nature of this theory can be made. However this plot does give a preliminary indication that the two-color six-flavor gauge theory is confined and chirally broken in the IR. This can be inferred from the lack of evidence of an IRFP up to renormalized couplings well beyond the strength that would be required to break chiral symmetry dynamically [17]. Moreover the renormalized coupling crosses this threshold on lattice volumes as low as $L/a = 10$ in the range of bare couplings we are free to probe. This implies that an accurate determination of the IR status of this theory is computationally feasible with our choice of action. The qualitative behavior of figure 6 stands in contrast to that of figure 1 of [19]. There a theory, which is widely believed to be inside the conformal window and whose renormalized couplings, when plotted in the manner of figure 6 here, show behavior that is consistent with the existence of an IRFP.

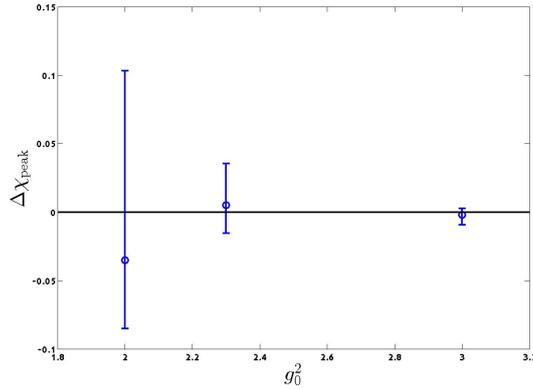


Figure 5: Difference in determination of the peak plaquette susceptibility, when calculated using two different definitions of the smearing operator near the boundaries in the time directions, versus g_0^2 . All points calculated on lattice volumes $L/a = 10^3 \times 11$. No statistically significant differences found.

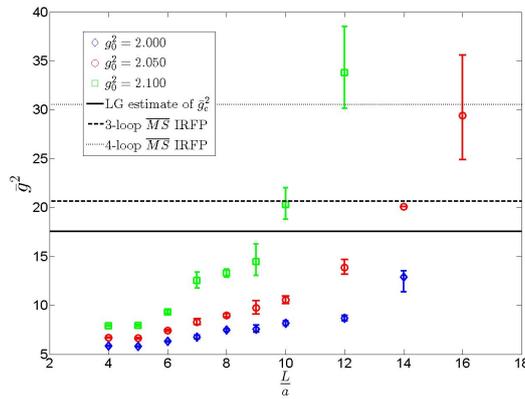


Figure 6: The SF running coupling evaluated, at fixed $g_0^2 = 2.0, 2.05, \text{ and } 2.1$, versus L/a . The ladder gap estimate of the critical renormalized coupling strength required to break chiral symmetry is shown along with the 3 and 4 loop \overline{MS} scheme IRFP strengths. The 2 loop IRFP strength is $\bar{g}_*^2 \approx 144$.

Of course an IRFP can appear once we take a continuum limit. To obtain such a limit, we would need to produce a series of curves like those in figure 6 but at many more values of the bare coupling. The finally step would be to do a step scaling analysis as in [18, 13]. This is currently in progress.

We have presented on the current status of our SF running coupling calculation of the two-color six-flavor gauge theory. A thorough study of the parameter space of our action has been undertaken and we are confident that we have sufficiently tuned the bare fermion mass. The phase diagram has also been studied to ensure that all results are inferred from the weak side of any spurious lattice phase transition. Study of the phase diagram in conjunction with our preliminary running coupling calculation imply that the definite determination status of this theory is computationally feasible. Moreover, we see a preliminary indication that this theory has no IRFP,

but a definite conclusion will have to wait.

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