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Spontaneous supersymmetry breaking in the $2d$ $\mathcal{N}=1$ Wess-Zumino model

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We study the phase diagram of the two-dimensional $\mathcal{N}=1$ Wess-Zumino model using Wilson fermions and the fermion loop formulation. We give a complete non-perturbative determination of the ground state structure in the continuum and infinite volume limit. We also present a determination of the particle spectrum in the supersymmetric phase, in the supersymmetry broken phase and across the supersymmetry breaking phase transition. In the supersymmetry broken phase we observe the emergence of the Goldstino particle.

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1. Motivation and overview

Despite the fact that the latest results from the Large Hadron Collider make it more and more unlikely that supersymmetry, at least in its variety as a minimal extension to the Standard Model, is accommodated in nature, supersymmetric quantum field theories remain to be interesting in their own right. In particular, spontaneous supersymmetry breaking and the corresponding phase transition is an interesting non-perturbative phenonemon which often evades a quantitative description even in simple models such as the $\mathcal{N}=1$ Wess-Zumino model in two dimension on which we focus in these proceedings. Often, a specific model may or may not undergo a supersymmetry breaking phase transition and it is usually not clear how such a transition is realised in detail. While the lattice regularisation provides a convenient setup to perform detailed non-perturbative numerical investigations, for systems which exhibit spontaneous supersymmetry breaking straightforward Monte Carlo simulations are not possible due to a fermion sign problem related to the vanishing of the Witten index [1]. However, it has been shown that the sign problem can be circumvented by using the fermion loop formulation [1, 2, 3] and simulating the system with the open fermion string algorithm [4, 5].

In these proceedings, we present a quantitative non-perturbative investigation of the $2d$ $\mathcal{N}=1$ Wess-Zumino model as follows. First we give a brief definition of the model and then discuss its formulation in terms of fermion loops. After reviewing its vacuum structure and the symmetry breaking pattern we go on to describe quantitatively its mass spectrum in the supersymmetric and the supersymmetry broken phase as well as across the phase transition.

2. The $\mathcal{N}=1$ Wess-Zumino model on the lattice

The $\mathcal{N}=1$ Wess-Zumino model in two dimensions [6] is one of the simplest models which may exhibit spontaneous supersymmetry breaking. Its degrees of freedom consist of one real Majorana fermion field ψ and one real bosonic field ϕ, while its dynamics is described by the Lagrangian density

$$\mathscr{L}=\frac{1}{2}\left(\partial_\mu\phi\right)^2+\frac{1}{2}P'(\phi)^2+\frac{1}{2}\overline{\psi}\left(\partial\!\!\!/+P''(\phi)\right)\psi\,. \tag{2.1}$$

Here, $P(\phi)$ denotes a generic superpotential, and P', P'' its first and second derivative with respect to ϕ. In the following we will concentrate on the specific form

$$P(\phi)=\frac{m^2}{4g}\phi+\frac{1}{3}g\phi^3 \tag{2.2}$$

which leads to a vanishing Witten index $W=0$ and hence allows for spontaneous supersymmetry breaking [7]. The corresponding action enjoys the following two symmetries. First, there is a single supersymmetry given by the transformations

$$\delta\phi=\overline{\varepsilon}\psi\,,\quad \delta\psi=(\partial\!\!\!/\phi-P')\varepsilon\,,\quad \delta\overline{\psi}=0\,, \tag{2.3}$$

and secondly, there is a discrete $\mathbb{Z}(2)$ chiral symmetry given by

$$\phi\to-\phi\,,\quad \psi\to\gamma_5\psi\,,\quad \overline{\psi}\to-\overline{\psi}\gamma_5\,, \tag{2.4}$$

where $\gamma_5 \equiv \sigma_3$ can be chosen to be the third Pauli matrix. The fact that the Witten index is zero for the chosen superpotential can be derived from the transformation properties of the Pfaffian of the Dirac operator under the $\mathbb{Z}(2)$ symmetry $\phi \to -\phi$ [3].

Let us now move on to describe the regularisation of the model on the lattice. For the fermionic fields we use the Wilson lattice discretisation yielding the fermion Lagrangian density

$$\mathscr{L} = \frac{1}{2}\xi^T \mathscr{C}(\gamma_\mu \tilde{\partial}_\mu - \frac{1}{2}\partial^*\partial + P''(\phi))\xi \,,$$

where ξ is a real, 2-component Grassmann field, $\mathscr{C} = -\mathscr{C}^T$ is the charge conjugation matrix and ∂^*, ∂ are the backward and forward lattice derivatives, respectively. In order to guarantee the full supersymmetry in the continuum limit, one needs to introduce the same derivative, in particular the Wilson term, also for the bosonic fields [8]. As a consequence, in addition to the supersymmetry also the $\mathbb{Z}(2)$ chiral symmetry is broken by the lattice regularisation both in the bosonic and the fermionic sector.

Nevertheless we can now use the exact reformulation of the fermionic degrees of freedom in term of closed fermion loops (cf. [1] for further details). Together with the fermion string algorithm [4, 5] this allows simulations with unspecified fermionic boundary conditions which do not suffer from the fermion sign problem [3] and for which critical slowing down is essentially absent even in the presence of a massless fermionic mode such as the Goldstino.

3. Supersymmetry breaking pattern

It is useful to briefly review the (super-)symmetry breaking pattern. The potential for the bosonic field is a standard ϕ^2-theory which may trigger a $\mathbb{Z}(2)$ symmetry breaking phase transition. In particular, for large m/g one expects that the $\mathbb{Z}(2)$ symmetry is broken. In that case, the vacuum expectation value of the boson field $\langle\overline{\phi}\rangle = \pm m/2g$ is expected to select a definite ground state for the system, either bosonic or fermionic. On the other hand, for small m/g one expects the $\mathbb{Z}(2)$ symmetry to be restored with $\langle\overline{\phi}\rangle = 0$ in which case no unique ground state is selected and hence supersymmetry is broken. In fact, the associated tunneling between the two allowed bosonic and fermionic vacua corresponds to the infamous massless Goldstino mode.

In [3] it was indeed demonstrated, using the Witten index

$$W \equiv Z_{\rm pp} = Z_{\mathscr{L}_{00}} - Z_{\mathscr{L}_{10}} - Z_{\mathscr{L}_{01}} - Z_{\mathscr{L}_{11}} \,,$$

as an order parameter, that a supersymmetry breaking phase transition occurs for specific couplings $\hat{g}/\hat{m}$ depending on the lattice spacing set by ag. Here, $Z_{\rm pp}$ denotes the partition function with periodic boundary conditions in both directions while $Z_{\mathscr{L}_{ij}}$ denote partition functions with fixed topological boundary conditions [2]. The expected symmetry breaking pattern and the corresponding vacuum structure follow exactly the expectations described above. In particular, for large m/g one is in a $\mathbb{Z}(2)$ broken phase where supersymmetry is unbroken, while for small m/g the $\mathbb{Z}(2)$ symmetry is restored and the supersymmetry is broken. Note that this situation only holds in the infinite volume limit: at any finite volume the $\mathbb{Z}(2)$ symmetry is always restored (and hence the supersymmetry broken) by soliton solutions which mediate transitions between boson field configurations with $\langle\overline{\phi}\rangle = \pm m/2g$ [9]. We have now further confirmed this scenario using the Ward identity $\langle P'\rangle$.

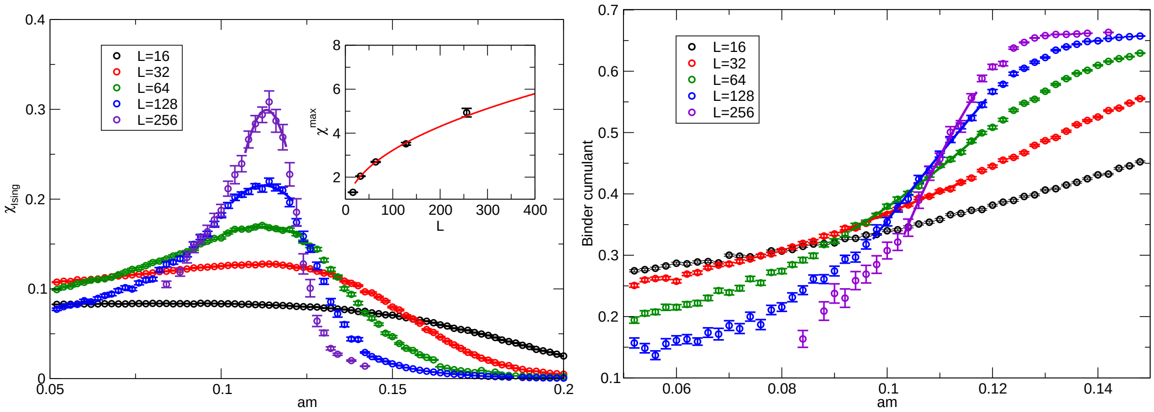

Figure 1: Susceptibility of the volume averaged Ising projected boson field (left plot) and the Binder cumulant of the boson field (right plot) for several volumes at $ag = 0.0625$.

3.1 $\mathbb{Z}(2)$ phase transition

In order to further quantify the phase transition we investigated in detail several order parameters sensitive to the $\mathbb{Z}(2)$ phase transition. It should be noted that there exists no real order parameter for the $\mathbb{Z}(2)$ transition, since the Wilson lattice discretisation breaks not just the supersymmetry, but also the $\mathbb{Z}(2)$ chiral symmetry in the bosonic sector. However, it turns out that at the lattice spacings $ag \leq 0.25$ which we simulated, the system behaves sufficiently continuum-like, so that accurate determinations of the phase transition are possible without problems. This is exemplified in figure 1. In the left plot we show the susceptibility $\chi_{\rm Ising}$ of the volume averaged Ising projected boson field $\overline{\phi}_{\rm Ising} = 1/V \sum_x \mathrm{sign}[\phi_x]$. The susceptibility shows a nice finite volume scaling and the scaling of the susceptibility peak indicates a second order phase transition, presumably in the universality class of the 2d Ising model. The right plot of figure 1 shows the Binder cumulant of the boson field for various volumes, all at fixed lattice spacing $ag = 0.0625$. From the position of the susceptibility peak and the crossing of the Binder cumulant one can infer the critical bare mass am_c at which the phase transition occurs.

In general, different order parameters consistently indicate a phase transition only in the thermodynamic limit when the finite volume pseudo-phase transition becomes a true one. In the left plot of figure 2 we show the critical bare mass am_c as a function of the inverse volume expressed in units of g, as obtained from the two (pseudo-)order parameters discussed above. We find that the determination from the Binder cumulant shows rather large finite size effects, in contrast to the one from the susceptibility. However, in the thermodynamic limit they both agree and this is sustained for all lattice spacings (right plot). The inset finally shows the continuum extrapolation of the critical coupling $f_c = g/m_c$ using the bare mass am_c and the one renormalised using 1-loop continuum perturbation theory, $am_c^{\rm R}$. The renormalised critical coupling in the continuum can now be compared to the one obtained in [10] using a different discretisation and algorithm.

4. Mass spectrum

We determine the mass spectrum from the temporal behaviour of correlators projected to zero spatial momentum, $C(t) \sim \langle \mathcal{O}(0)\mathcal{O}^{(T)}(t)\rangle$. For the boson masses we use the $\mathbb{Z}(2)$-odd and -even

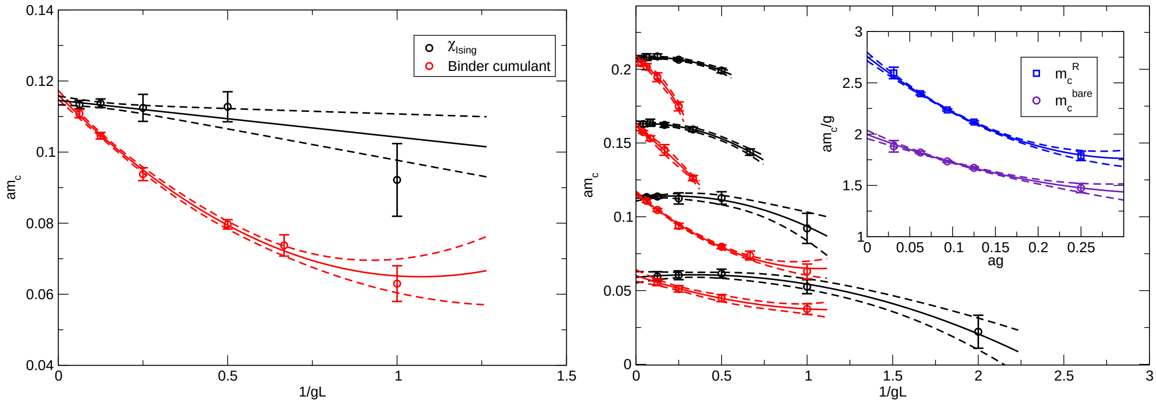

Figure 2: Thermodynamic limit of the critical mass am_c from the Binder cumulant and the peak of the susceptibility at $ag = 0.0625$ (left plot) and for a range of other couplings $ag = 0.25 - 0.03125$ (right plot). The inset shows the continuum limit of the bare and the renormalised critical coupling $f_c = g/m_c^{\text{bare,R}}$.

operators $\mathscr{O} = \phi$ and ϕ^2, respectively, while for the fermion masses we use $\mathscr{O} = \xi$ and $\xi\phi$. We note that in the supersymmetric/$\mathbb{Z}(2)$-broken phase the vacuum can not distinguish between the even and odd states and hence we extract the same mass using the two operators, while in the supersymmetry (SUSY) broken/$\mathbb{Z}(2)$ restored phase, the vacuum respects the $\mathbb{Z}(2)$ symmetry and distinguishes between states with different $\mathbb{Z}(2)$ quantum numbers. Furthermore, in the SUSY broken phase we can measure excitations both in the bosonic vacuum, i.e. in $Z_{\mathscr{L}_{00}}$, and in the fermionic one, i.e. $Z_{\mathscr{L}_{10}} + Z_{\mathscr{L}_{01}} + Z_{\mathscr{L}_{11}}$. We emphasise that simulations in the SUSY broken phase are only feasible due to the fact that the fermion loop algorithm essentially eliminates critical slowing down [4, 5], despite the emergence of the (would-be) Goldstino.

In figure 3 we show examples of boson mass extractions in the SUSY broken/$\mathbb{Z}(2)$-symmetric (left plot) and in the supersymmetric/$\mathbb{Z}(2)$ broken phase (right plot), both in the bosonic vacuum. The top panel shows the full correlator, the middle one the connected part and the lowest one the corresponding effective masses. In the SUSY broken phase we can fit double exponentials (plus a small shift due to the residual $\mathbb{Z}(2)$ breaking), while in the $\mathbb{Z}(2)$ broken phase only one exponential can be fitted, since the signal is quickly dominated by the fluctuations stemming from the large disconnected contribution.

In figure 4 we show examples of fermion mass extractions in both phases. In the left plot (SUSY broken phase) the top panel shows the correlator of the $\mathbb{Z}(2)$-even state which can be well fitted with a double exponential with the lowest mass corresponding to the Goldstino mass. The middle panel shows the $\mathbb{Z}(2)$-odd state fitted with a single exponential. The right plot shows the fermion correlator in the supersymmetric phase (top panel), on a log scale (middle panel) and the corresponding effective masses (bottom panel). It is remarkable that the signal of the fermion correlator can be followed over more than six orders of magnitude. Of course this just reflects the efficiency of the employed fermion loop algorithm [4].

Finally, in figure 5 we show the full boson and fermion mass spectrum in the left and right plot, respectively, as a function of the bare mass am across the supersymmetry breaking phase transition occurring at around $am_c \sim 0.042$. We see how the mass spectrum in the SUSY broken/$\mathbb{Z}(2)$-symmetric phase fans out into the $\mathbb{Z}(2)$-even and -odd states, with bosonic and fermionic masses non-degenerate, while in the supersymmetric/$\mathbb{Z}(2)$-broken phase the states collapse onto a degen-

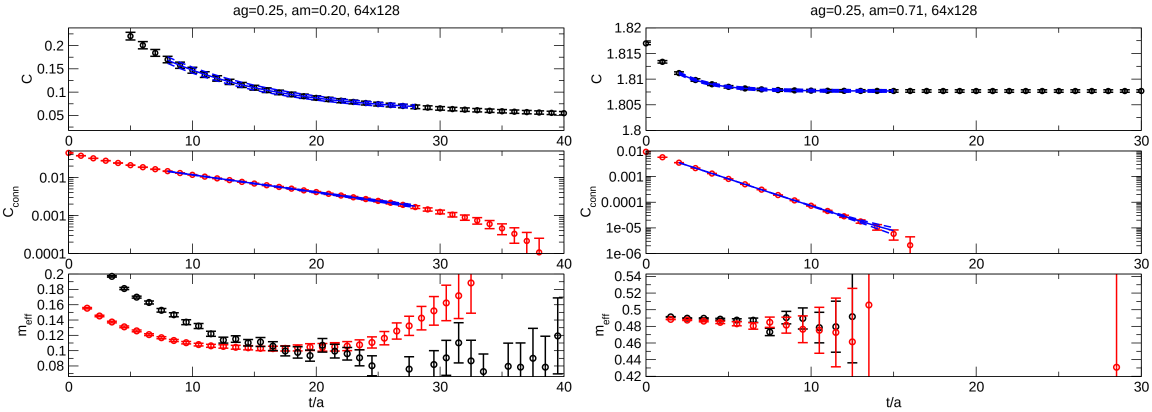

Figure 3: Boson mass extraction in the SUSY broken/$\mathbb{Z}(2)$-symmetric (left plot) and in the supersymmetric/$\mathbb{Z}(2)$ broken phase (right plot).

erate mass, in addition to the boson and fermion masses being equal. In the SUSY broken phase we can crosscheck the mass determination in the bosonic sector with the one in the fermionic sector and we find very convincing consistency. This agreement in the SUSY broken phase and the degeneracy of the boson and fermion masses in the supersymmetric phase is rather surprising, given the fact that the simulations are at finite and rather coarse lattice spacing $ag = 0.25$. Moreover, it should be kept in mind, that in the SUSY broken phase it is rather difficult to keep the systematic effects from mixing with higher excited states under control.

A first preliminary investigation of the effects of the finite volume on the spectrum reveals that they are essentially negligible for the volume $L/a = 64$ that we are using here. This is not quite the case for the boson mass spectrum in the SUSY broken phase. In fact, the investigation in [11] suggests a distinct finite volume scaling of the boson masses with the lowest boson mass vanishing towards the thermodynamic limit.

An interesting feature of the spectrum of a theory with spontaneously broken supersymmetry is of course the occurrence of the massless Goldstino. Since in our regularisation the supersymmetry is broken explicitely at any finite lattice spacing, the Goldstino is only approximately massless as

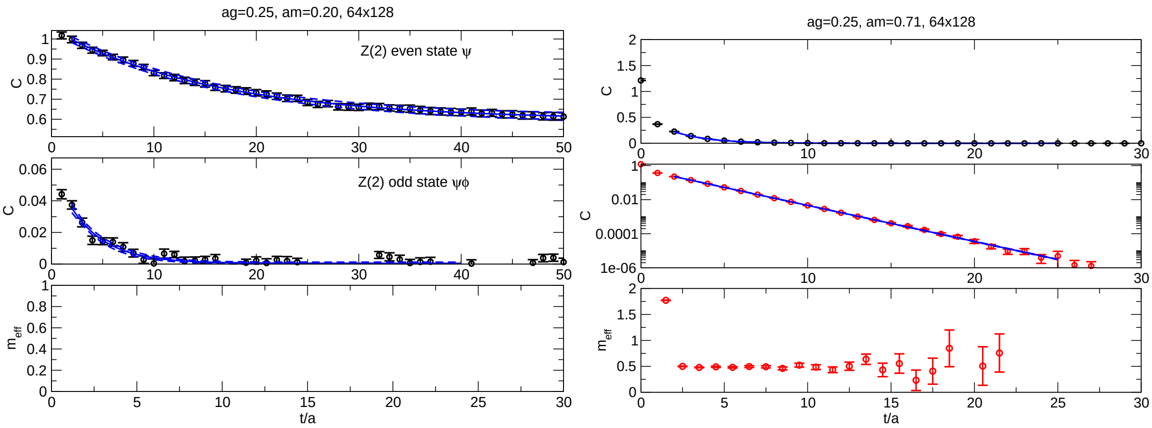

Figure 4: Fermion mass extraction in the SUSY broken/$\mathbb{Z}(2)$-symmetric (left plot) and in the supersymmetric/$\mathbb{Z}(2)$ broken phase (right plot).

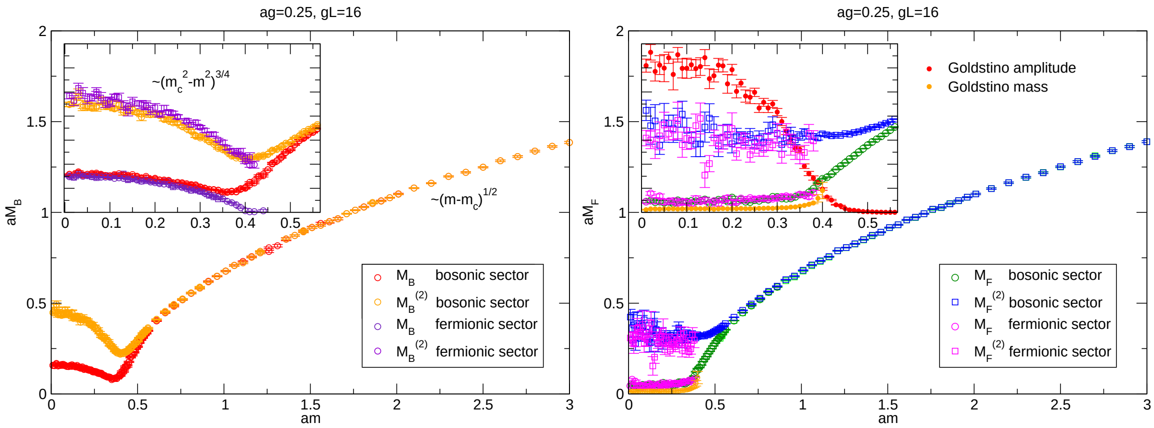

Figure 5: Mass spectrum for bosonic (left plot) and fermionic excitations (right plot). The superscript $^{(2)}$ denotes the excited state.

can be seen in figure 5. To corroborate the identification of this low mass state as the Goldstino, we plot in the inset of the right plot also the contribution (amplitude) of that state to the full fermion correlator. It turns out that the amplitude decreases as we increase the bare mass and vanishes at the transition to the supersymmetric phase, i.e. the Goldstino decouples from the system at the supersymmetry restoring phase transition.

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