

Precision Electroweak Constraints on Near-Conformal Dynamical EWSB

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Early attempts to construct models of Dynamical Electro-Weak Symmetry Breaking (DEWSB), assuming familiar dynamics for the technisector, failed to overcome strict constraints from the oblique correction parameter S . More recently it has been suggested that models involving a technisector with near-conformal dynamics might be more successful in avoiding these constraints. In such theories the reduced strength of chiral symmetry breaking is expected to suppress the splitting between the vector and axial channels, leading to a reduced contribution to S .

We describe an ongoing attempt to measure the contribution to the S parameter from a technisector involving the $SU(2)$ theory with two flavours of adjoint Dirac fermions, which is the main component of the minimal walking-technicolor (MWT) model. The dynamics of this theory have been extensively studied and it has been shown to be consistent with the presence of an infra-red conformal fixed point.

We study the S parameter in this theory using a mixed-action lattice simulation, using Wilson sea fermions and Domain-Wall fermions in the valence sector. We describe the steps involved in the computation, explore important systematic effects, and present current results.

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and from these the parameters S , T and U are defined. Specifically

$$S = 16\pi[\Pi'_{33}(0) - \Pi'_{3Q}(0)]. \quad (1.1)$$

Assuming conservation of iso-spin and parity

$$\Pi_{33} = \frac{1}{4}(\Pi_{VV} + \Pi_{AA}) \quad \Pi_{3Q} = \frac{1}{2}\Pi_{VV} \quad (1.2)$$

which allows us to re-express S as

$$S = -4\pi[\Pi'_{VV}(0) - \Pi'_{AA}(0)] \quad (1.3)$$

It is this expression which allows us to directly calculate S on the lattice. Despite this definition of the S parameter, when discussing experimental bounds on its value, what is conventionally quoted is the difference between the measured value, and the expected theoretical value as predicted in the Standard Model at a certain reference Higgs mass. With this definition, the current bound is $S = 0.03 \pm 0.09$ [5].

There are a number of estimates of the S parameter contribution in different models. The original authors made a number of estimates [3, 4]; to first order in perturbation theory, N_f flavours of fermion in a representation R give a contribution

$$S \simeq \frac{1}{6\pi} \left(\frac{N_f d(R)}{2} \right)$$

where $d(R)$ is the dimension of the representation R . They also attempted a semi-phenomenological estimate evaluating a dispersive integral expression for S using existing QCD data, which led them to numerical estimates which are well approximated by the expression

$$S \simeq 0.3 \frac{N_f d(R)}{2 \cdot 3}$$

where we have inserted the same scaling with the representation of the fermions as in the perturbative expression.

These estimates, however, cannot be simply applied to any gauge sector as they will not account for effects of strongly-coupled physics or deviation of the gauge dynamics from those of QCD. There have been estimates of the S parameter contribution in models of interest for DEWSB, in particular near-conformal models, which suggest that the contribution may be suppressed in such theories [6]. Previous lattice estimates of the S parameter contribution from near-conformal models are restricted to a calculation in the $SU(3)$ theory with a large number of fundamental fermions [7].

2. MWT Calculation

We present a calculation of the S parameter contribution generated when the $SU(2)$ gauge theory with 2 flavours of fermion in the adjoint representation is incorporated in a model of DEWSB. This calculation builds on simulations of the MWT theory used to study the spectroscopy and other aspects of the theory [8]. The ensembles of gauge configurations were generated using the Wilson fermion discretisation at a single gauge coupling $\beta = 2.25$, and we study ensembles with two volumes (32×16^3 and 64×24^3) and several fermion masses approaching the chiral limit.

As we will compute the S parameter in this theory via the difference between the vector and axial channels (1.3), we will require a well conserved chiral symmetry in our lattice calculation. As a result, using the Wilson fermion formulation in both the sea and valence sectors will not suffice. We must conduct a mixed-action computation, using a chiral valence action.

For this we choose the Domain Wall Fermion (DWF) action for the valence sector. To do this we must tune the bare DWF fermion mass m_f to reproduce the physics as measured in the unitary Wilson calculation. We do this primarily via the effective pseudoscalar mass $m_{PS}(t)$. An example of this tuning is shown in Fig. 1a. The bare fermion mass m_f at which the tuning is achieved is found to be largely independent of the volume in question, and so we are able to perform the tuning on the smaller volume and use the value obtained in computations on the larger volume.

We monitor the degree to which the chiral symmetry is explicitly broken in our calculation by measuring the residual mass m_{res} [9], defined through the pseudoscalar currents on the boundary J_5 and at the center J_{5q} of the fifth dimension, which has extent L_5 . The residual mass is expected to decay as L_5 is increased, and the vanishing of m_{res} in the limit of infinite L_5 restores the explicit chiral symmetry.

At present all data in this study are computed at $L_5 = 16$. To achieve a reasonable degree of chiral symmetry preservation in our calculation we would like maintain a ratio $\frac{m_{res}}{m} \ll 1$. In Fig. 1b we see that the ratio of m_{res} to the quark mass m grows in the chiral limit, meaning the effect of the chiral symmetry breaking on the physics of interest will be enhanced as we reduce the quark mass, keeping L_5 fixed. This is a systematic effect which we have not quantified at present, and so should be borne in mind when interpreting our current results. It will be necessary to analyse the dependence of our results on the level of chiral symmetry preservation by varying L_5 but at present this has not been performed.

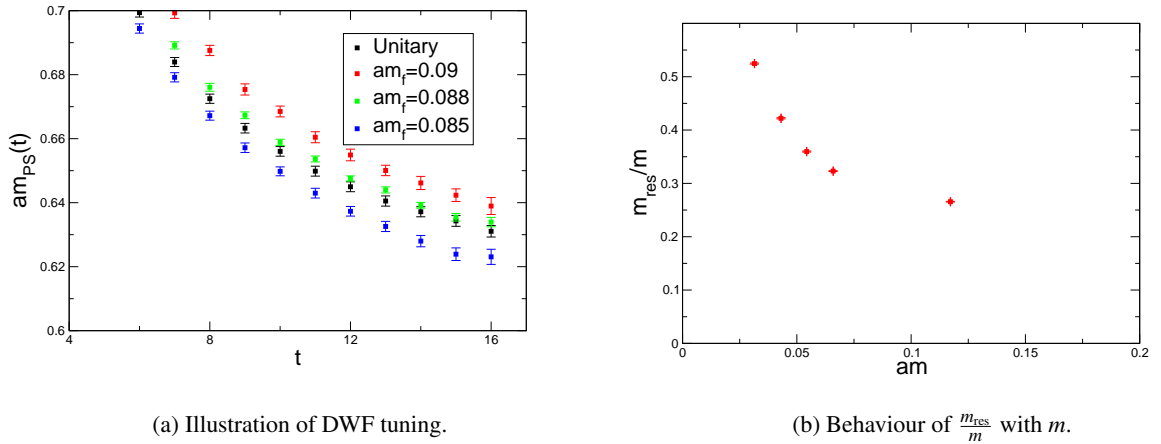


Figure 1: Diagnostics on our simulation.

3. Correlators

We will compute the S parameter contribution via the $V - A$ expression (1.3) for which we

need to compute the VV and AA vacuum polarisation functions. We will analyse the vacuum polarisations $\Pi^{\mathcal{J}}$ and Π^J which represent the conserved-local and local-local vector-vector and axial-vector-axial-vector correlators

$$\begin{aligned}\Pi_{\mu\nu}^{\mathcal{J}}(\bar{q}) &= Z_J \sum_{\mathbf{x}} e^{iq \cdot x} \langle \mathcal{J}(\mathbf{x}) J(0) \rangle \\ \Pi_{\mu\nu}^J(\bar{q}) &= Z_J^2 \sum_{\mathbf{x}} e^{iq \cdot x} \langle J(\mathbf{x}) J(0) \rangle\end{aligned}$$

where $\mathcal{J}(x)$ represents the conserved DWF vector or axial current and $J(x)$ the local DWF current. Z_J here is a renormalisation factor. In our calculation we rely on the equality $Z_V = Z_A$ which holds up to terms proportional to m_{res} . We calculate Z_V from the ratio of the conserved-local and local-local vector two-point functions at zero momentum. From the $\Pi_{\mu\nu}$ we form the conserved-local and local-local V – A vacuum polarisation tensors

$$\begin{aligned}\Pi_{\mu\nu}^{\mathcal{V}-\mathcal{A}} &= \Pi_{\mu\nu}^{\mathcal{V}} - \Pi_{\mu\nu}^{\mathcal{A}} \\ \Pi_{\mu\nu}^{V-A} &= \Pi_{\mu\nu}^V - \Pi_{\mu\nu}^A\end{aligned}$$

These tensors can be decomposed in their transverse and longitudinal components as

$$\Pi_{\mu\nu}(\bar{q}) = (\bar{q}^2 \delta_{\mu\nu} - \bar{q}_\mu \bar{q}_\nu) \Pi_T(\bar{q}^2) + \bar{q}_\mu \bar{q}_\nu \Pi_L(\bar{q}^2)$$

where \bar{q} signifies the lattice momentum

$$\bar{q}_\mu = \frac{2}{a} \sin\left(\frac{aq_\mu}{2}\right) = \frac{2}{a} \sin\left(\frac{\pi n_\mu}{L_\mu}\right) \quad (3.1)$$

where n_μ is a 4-vector of integers representing the discrete lattice momenta and L_μ is the 4-vector of integers representing the extent of the lattice in each of the space-time directions. The conserved-local vector vacuum polarisation function obeys an exact Ward Identity

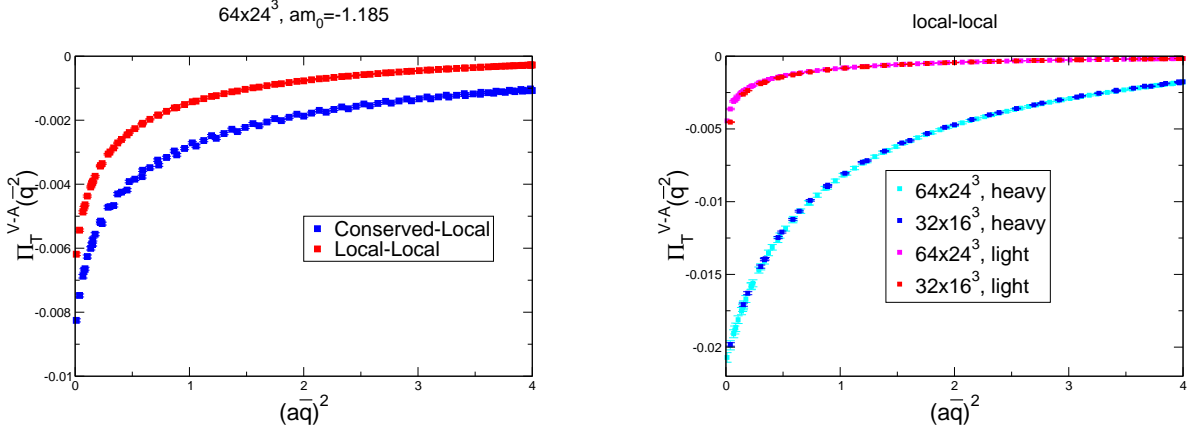
$\bar{q}_\mu e^{\frac{iq_\mu a}{2}} \Pi_{\mu\nu}^{\mathcal{V}} = 0$ which is equivalent to the vanishing of the longitudinal component of this polarisation tensor. In Figure 2a we show examples of the transverse component of the conserved-local and local-local Π^{V-A} . In Fig. 2b we show how this function varies on ensembles with different masses and volumes.

4. Results

In order to extract the S parameter contribution from our data, we must compute

$$S = 4\pi \frac{d}{d(\bar{q})^2} \Pi_T^{V-A}(\bar{q}^2) = 4\pi \frac{d}{d(a\bar{q})^2} a^2 \Pi_T^{V-A}(\bar{q}^2)$$

where the last equality reassures us that we need not concern ourselves with fixing the scale in our simulation in order to compute the result we seek at on this single value of the coupling. We must also subtract from our result the alteration of the theoretical prediction for the S parameter made by removing the Standard Model Higgs in building the complete model of DEWSB. To estimate this quantity we follow [7], finding that due to the fact that the mass splitting between the PS and



(a) Examples of the transverse component of the $V - A$ vacuum polarisation function. (b) Further examples of the transverse vacuum polarisation from ensembles at different volumes and quark masses.

Figure 2: Examples of the transverse component of $V - A$.

V channels is heavily suppressed in this theory [8], the result is small relative to the scale and precision of our current results.

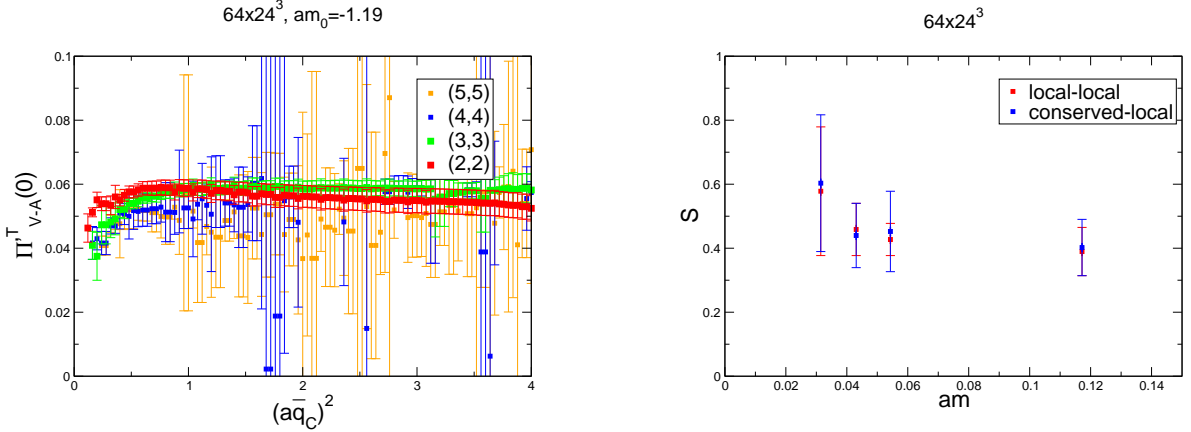
Our results for S will depend on the slope of the transverse $V - A$ polarisation function, and so we must interpolate a continuous form for this function from our data. To do this we fit to the functional form using a series of Pade approximants, taking direction from similar techniques used to fit the vector polarisation function in order to extract the leading order hadronic contribution to the anomalous magnetic moment of the muon in QCD.

In order to analyse the stability of fits obtained using a given fit-form we will vary the data points included in the fit by fitting in the range $[0, (a\bar{q}_C)^2]$ and varying $(a\bar{q}_C)^2$. We estimate the systematic uncertainty owing to the uncertainty over how to perform this fit by choosing a range of results which are compatible with the results given by a number of different fit forms over a certain range of $(a\bar{q}_C)^2$. In Fig. 3a we show how the results typically depend on the fit ansatz used. We choose a range in q_C^2 where the results are reasonably extracted from a number of different ansatze, and chose a final result for the slope at the origin which is compatible with all results obtained in this range.

Using this prescription, the results we obtain for the S parameter are shown as a function of quark mass in Fig. 3b. These results are provisional and come with caveats, primarily that the chiral symmetry breaking is increasing in our calculation as we move to lighter quark masses, which might be expected to have a significant effect on our results. This effect will be explored in the near future. However at this point no suppression of the S parameter in this theory is identified, nor do we identify any trend in the chiral limit.

5. Conclusions

We have obtained early provisional results for the S parameter contribution from a MWT model of DEWSB. We will improve these results by resolving the dependence of the results on the



(a) Example of the varying results obtained from different Pade ansatze. (b) Provisional results obtained for the S parameter in MWT.

Figure 3: Results for S obtained using our fitting procedure.

level of chiral symmetry breaking, and by improving the systematics of the study with regard to the tuning of the valence action, the renormalisation factors used, and the interpolation of the vacuum polarisation to zero momentum. In addition we will explore the consequences of alternating the order in which we take the zero momentum and chiral limits, as suggested in [10].

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