

The sphaleron rate at the electroweak crossover with 125 GeV Higgs mass

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We measure the sphaleron rate with the physical parameters of the Standard Model, including the recently found Higgs mass of $m_H = 125$ GeV. The sphaleron rate tells us about the efficiency of baryon number violation through sphaleron transitions. These occur above the electroweak scale ~ 100 GeV and get exponentially suppressed at temperatures substantially below the electroweak crossover. The sphaleron rate enters computations of Baryogenesis via Leptogenesis, where non-zero lepton number is converted into non-zero baryon number. We simulate the effective electroweak theory on the lattice with multicanonical and real-time methods to calculate the sphaleron rate as a function of temperature through the electroweak crossover.

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1. Introduction

Baryon and lepton number are not exactly conserved quantities in the Standard Model because of the axial anomaly, which connects them to the Chern-Simons number of the weak gauge field. Vacua in the electroweak theory are labeled by an integer-valued Chern-Simons number

$$N_{CS} = \int d^3x J_{CS}^0 = -\frac{g^2}{64\pi} \int d^3x \varepsilon^{ijk} \text{Tr} \left(A_i F_{jk} + i\frac{g}{3} A_i A_j A_k \right). \quad (1.1)$$

In the early Universe, when $T > 100$ GeV, it was possible to move from one vacuum to another surmounting the potential barrier between vacua through sphaleron transitions, thus changing Chern-Simons number, B and L according to

$$\frac{1}{n_G} [B(t) - B(0)] = L_i(t) - L_i(0) = N_{CS}(t) - N_{CS}(0), \quad (1.2)$$

with n_G the number of generations of fermions. At zero temperature, where the potential barrier is high, the sphaleron rate is exponentially small and, when the Higgs field expectation value $v \gg T$, the rate is negligible. In electroweak Baryogenesis scenarios [1] the baryon number of the Universe is generated during the electroweak phase transition. However, this scenario does not work in the Standard Model: it requires stronger CP violation than observed and a first order phase transition, whereas the Standard Model has a smooth crossover [2].

Nevertheless, the sphaleron rate during the electroweak crossover in the Standard Model is relevant for Baryogenesis via Leptogenesis: indeed the sphaleron rate converts a lepton asymmetry into a baryon asymmetry. If the lepton asymmetry is generated just before or during the electroweak phase transition, how the sphaleron rate shuts off has an effect on the generated baryon number.

The sphaleron rate has been studied in the broken phase before, but either with unphysical Higgs masses [3, 4, 5] or not very deeply in the broken phase [3]. Both perturbative calculations [6, 7] and lattice simulations [3, 8, 9, 10] have been used.

In this paper we follow the procedure of our previous works [11, 12] in determining the sphaleron rate at the energy range of the electroweak crossover, with the crucial difference that now we are able to perform the simulations with the recently found Higgs mass, $m_H = 125$ GeV. Our results are compared to analytical estimates both in the broken and symmetric phases [6].

2. Theory on the lattice

The thermodynamics of the 4-dimensional electroweak theory is studied in 3 dimensions by means of dimensional reduction [13], a perturbative technique that gives the correspondence between 4D and 3D parameters. The result is a SU(2) effective theory with the Higgs field ϕ and gauge field A_μ (F_{ij})

$$L = \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + m_3^2 \phi^\dagger \phi + \lambda_3 (\phi^\dagger \phi)^2, \quad (2.1)$$

and 3D effective parameters g_3^2 , λ_3 and m_3^2 .

Bödeker showed [14] that at leading order in $\log(1/g)$ the time evolution of this effective SU(2) Higgs model is governed by Langevin dynamics. The latter, though, is very slow to simulate on

the lattice and can be substituted by any other dissipative procedure, e. g. heat bath. One heat-bath sweep through the lattice corresponds to the real-time step [4]

$$\Delta t = \frac{a^2 \sigma_{el}}{4}, \quad (2.2)$$

with

$$\sigma_{el}^{-1} = \frac{3}{m_D^2} \gamma, \quad \gamma = \frac{Ng^2T}{4\pi} \left[\ln \frac{m_D}{\gamma} + 3.041 \right] \quad (2.3)$$

where σ_{el} is the non-abelian color conductivity, which quantifies the current response to infrared external fields, N is the dimension of the SU(N) gauge group, and m_D is the Debye mass, determining the length scale $l_D \sim 1/m_D \sim 1/gT$. We made use of a 32^3 lattice, with $\beta_G \equiv \frac{4}{g_3^2 a} = 9$, where g_3 is the 3D gauge coupling and a the lattice spacing. In real-time simulations, for each temperature we computed 4 trajectories for every 1000 initial configurations.

3. Measurement of the sphaleron rate

In the symmetric phase we make use of canonical Monte Carlo simulations and approach the broken phase. At very low temperatures, the rate is highly suppressed and canonical methods do not work anymore. We need multicanonical methods, which calculate a weight function that compensates the high potential barrier between the vacua, thus allowing transitions. The exact value of the sphaleron rate

$$\Gamma \equiv \lim_{t \rightarrow \infty} \frac{\langle (N_{CS}(t) - N_{CS}(0))^2 \rangle}{V t} \quad (3.1)$$

is obtained, in the broken phase, through a method similar to the one used in [3, 4].

1. Once done the multicanonical simulations, we obtain the canonical (physical) probability distribution of the Chern-Simons number $p_{\text{phys.}}(N_{CS})$.
2. We choose a narrow interval $1/2 - \varepsilon/2 \leq N_{CS} \leq 1/2 + \varepsilon/2$ around the point that separates vacuum $N_{CS} = 0$ from the vacuum $N_{CS} = 1$. The relative probability of finding a configuration here is

$$P(|N_{CS} - 1/2| < \varepsilon/2) = \int_{1/2 - \varepsilon/2}^{1/2 + \varepsilon/2} dN p_{\text{phys.}}(N). \quad (3.2)$$

This is where we need multicanonical methods, as the probability of being on top of the barrier is extremely small, and to get a reliable estimate would take an impractically long time with canonical sampling.

3. Let us now take a random configuration from the canonical distribution but with the constraint $1/2 - \varepsilon/2 < N_{CS} < 1/2 + \varepsilon/2$; i.e. near the top of the potential barrier. Starting from this configuration, we now generate two real-time trajectories using heat-bath dynamics. The trajectories are evolved until the Chern-Simons number falls near a vacuum value. Interpreting one of the trajectories as evolving backwards in time, we can glue the trajectories together at the starting point and obtain a vacuum-to-vacuum trajectory. The trajectory can either return to the starting vacuum or be a genuine tunneling trajectory. Only the latter-type trajectories contribute to the sphaleron rate.

4. We can obtain the tunneling rate by measuring $|\Delta N_{\text{CS}}/\Delta t|$ from the trajectories at the moment they cross the value $N_{\text{CS}} = 1/2$. Here Δt is the time interval between successive measurements, and ΔN_{CS} the change in Chern-Simons number. This characterizes the probability flux through the top of the barrier. We obtain the physical time difference from the relation between the heat-bath “time” and physical time, equation (2.2).
5. If the tunneling trajectories would go straight across the top, the ingredients above would be sufficient to calculate the total rate. However, typically the trajectories “random walk” near the top of the barrier and can cross the value $N_{\text{CS}} = 1/2$ several times. Because the trajectories were chosen starting from a set of configurations near the top of the barrier, this leads to overcounting: the evolution could be started at any point the $N_{\text{CS}} = 1/2$ limit is crossed. This can be compensated by calculating a dynamical prefactor

$$d = \frac{1}{N_{\text{traj}}} \sum_{\text{traj}} \frac{\delta_{\text{tunnel}}}{\# \text{ crossings}}, \quad (3.3)$$

where the sum goes over the ensemble of trajectories, N_{traj} is the number of trajectories, δ_{tunnel} is 0 if the trajectory does not lead to a change of the vacuum and 1 if it does, and ($\# \text{ crossings}$) is the number of times the trajectory crosses $N_{\text{CS}} = 1/2$.

With these ingredients, the sphaleron rate now becomes

$$\Gamma = \frac{P(|N_{\text{CS}} - 1/2| < \varepsilon/2)}{\varepsilon} \left\langle \left| \frac{\Delta N_{\text{CS}}}{\Delta t} \right| \right\rangle d. \quad (3.4)$$

We note that the result is independent of ε as long as $\varepsilon \ll 1$. It is also independent of the frequency Δt with which the Chern-Simons number is measured: if we decrease the measurement interval, the trajectories become more jagged due to the random-walk nature of the heat-bath updates. This will increase the number of the crossings of the value $N_{\text{CS}} = 1/2$ and hence decrease d . However, the latter is completely compensated by a corresponding increase in $\langle |\Delta N_{\text{CS}}/\Delta t| \rangle$. If the measurement interval Δt is small enough, random walk arguments imply $d \propto (\Delta t)^{1/2}$ and $\langle |\Delta N_{\text{CS}}/\Delta t| \rangle \propto (\Delta t)^{-1/2}$. This is corroborated by the numerical data. Thus, equation (3.4) has a well-defined continuum limit.

4. Results

We obtain the sphaleron rate Γ/T^4 and the expectation value for the Higgs field $\langle \phi^2 \rangle$ for Higgs mass $m_H = 125 \text{ GeV}$. Figure 3 shows the Higgs field expectation value behavior as a function of temperature. We start from the “symmetric phase” with canonical Monte Carlo simulations and lower the temperature to reach the “broken phase”, where we switch to multicanonical simulations. We can see the Higgs field assuming a non-zero value when approaching the broken phase, and the transition from the two methods occurring smoothly.

The sphaleron rate as a function of temperature is shown in Figure 4. Here again we perform the simulations with canonical Monte Carlo at high temperatures and continue with multicanonical methods when reaching the cold broken phase. The sphaleron rate changes from its asymptotic

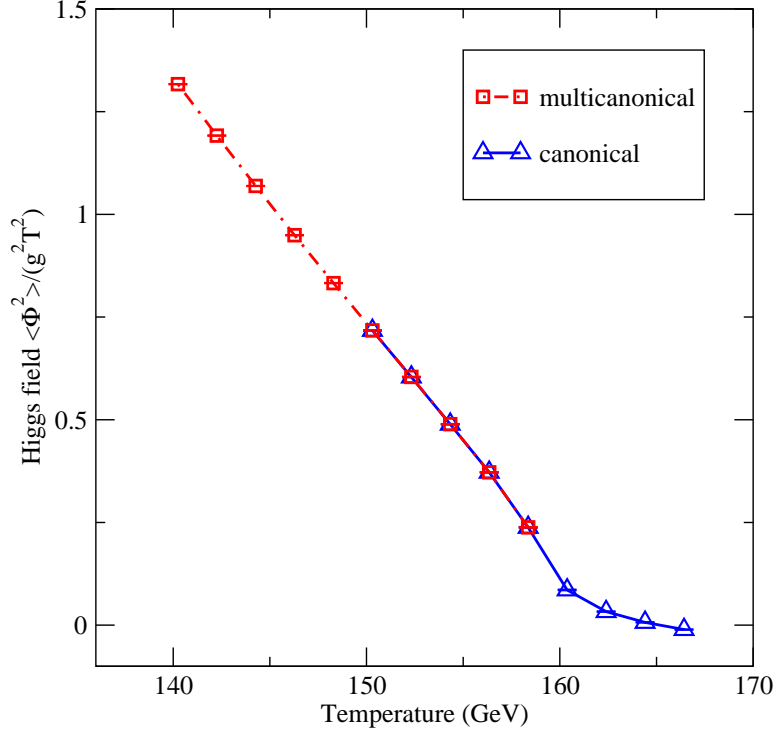


Figure 1: The Higgs expectation value $\langle \phi^2 \rangle$ for Higgs mass of 125 GeV as a function of temperature. The high-temperature canonical and low-temperature multicanonical results match beautifully in the transition region.

value to become exponentially suppressed at very low temperatures. The canonical and multicanonical methods are in good agreement. The theoretical curves, used to compare our results, were obtained separately for the broken and symmetric phases, through perturbative calculations in [6].

5. Conclusion

We improved the previous estimates for the sphaleron rate and determined its behavior from the symmetric to the broken phase, through the electroweak crossover. Our results are in agreement with previous estimates in the symmetric phase, and in the broken phase the slope of our curve is the same as in the analytic calculation [6].

Even though the Standard Model has a too weak source of CP-violation in the quark sector, Baryogenesis might still be viable through lepton number violating processes. The sphaleron rate plays an important role in Leptogenesis, as the conversion of lepton to baryon number depends on it, and it is therefore important to know its size rather accurately.

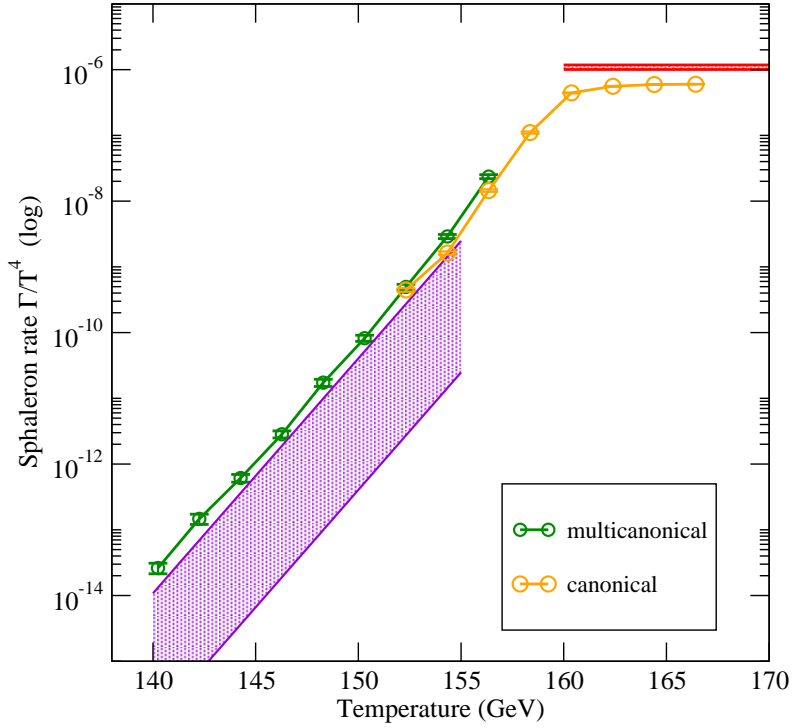


Figure 2: The sphaleron rate for a Higgs mass of 125 GeV. The high-temperature canonical and low-temperature multicanonical results again match very well in the transition region. Also shown are previous high-temperature estimates $\sim 5 \times 10^{-7} T^4$, (top, horizontal line) and perturbative calculations in the low-temperature phase (bottom, wide band), both from [6].

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References

- [1] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, *Phys. Lett. B* **155** (1985) 36.
- [2] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, *Phys. Rev. Lett.* **77** (1996) 2887 [arXiv:hep-ph/9605288].
- [3] G. D. Moore, *Phys. Rev. D* **59**, 014503 (1999) [arXiv:hep-ph/9805264].
- [4] G. D. Moore and K. Rummukainen, *Phys. Rev.* **D63** (2001) 045002. [hep-ph/0009132].

- [5] W. H. Tang and J. Smit, Nucl. Phys. B **482** (1996) 265 [arXiv:hep-lat/9605016].
- [6] Y. Burnier, M. Laine and M. Shaposhnikov, JCAP **0602** (2006) 007 [arXiv:hep-ph/0511246].
- [7] O. Philipsen, Phys. Lett. B **358**, 210 (1995) [arXiv:hep-ph/9506478].
- [8] J. Ambjorn, T. Askgaard, H. Porter and M. E. Shaposhnikov, Nucl. Phys. B **353** (1991) 346.
J. Ambjorn and A. Krasnitz, Phys. Lett. B **362**, 97 (1995) [arXiv:hep-ph/9508202].
D. Bodeker, G. D. Moore and K. Rummukainen, Phys. Rev. D **61**, 056003 (2000) [arXiv:hep-ph/9907545].
G. D. Moore, C. r. Hu and B. Muller, Phys. Rev. D **58** (1998) 045001 [arXiv:hep-ph/9710436].
G. D. Moore and K. Rummukainen, Phys. Rev. D **61** (2000) 105008 [arXiv:hep-ph/9906259].
G. D. Moore, Phys. Rev. D **62**, 085011 (2000) [arXiv:hep-ph/0001216].
- [9] A. Krasnitz and R. Potting, Phys. Lett. B **318** (1993) 492 [arXiv:hep-ph/9308307].
- [10] H. P. Shanahan and A. C. Davis, Phys. Lett. B **431** (1998) 135 [arXiv:hep-ph/9804203].
- [11] M. D'Onofrio, K. Rummukainen and A. Tranberg, PoS LATTICE **2010** (2010) 048 [arXiv:1011.2074 [hep-lat]].
- [12] M. D'Onofrio, K. Rummukainen and A. Tranberg, JHEP **1208** (2012) 123 [arXiv:1207.0685 [hep-ph]].
- [13] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. **B458** (1996) 90-136. [hep-ph/9508379].
- [14] D. Bodeker, Phys. Lett. B **426** (1998) 351 [arXiv:hep-ph/9801430].