PROCEEDINGS OF SCIENCE

Screening in two-dimensional gauge theories

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We analyze the problem of screening in 1+1 dimensional gauge theories. Using QED_2 as a warm-up for the non-abelian models we show the mechanism of the string breaking, in particular the vanishing overlap of the Wilson loops to the broken-string ground state that has been conjectured in higher-dimensional analyses. We attempt to extend our analysis to non-integer charges in the quenched and unquenched cases, in pursuit of the numerical check of a renowned result for the string tension between arbitrarily-charged fermions in the massive Schwinger model.

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1. Introduction

Two-dimensional $U(1)$ gauge theory with fermions (QED_2) has long been a test-bed for concepts relating to four-dimensional gauge theories. Being extremely simple compared to QCD it captures some of its crucial non-perturbative features such as confinement and string breaking. Although over the years there has been a plethora of numerical studies of QED_2 using various methods such as discrete light cone quantization, hamiltonian lattice field theory, euclidean LFT (see Refs. [1, 2, 3] and references therein), this system still attracts a lot of attention, see e.g. Refs. [2, 4].

As far as the pure gauge theory (Quantum Maxwell Dynamics, QMD_2) is concerned, when formulated in $\mathbb{R}^2$ it turns out to be trivial and results in a linear confining potential for the probe charges [5]. On a torus the situation is altered – due to the fact that one cannot gauge away the fields along closed contours there exists a single space-independent quantum degree of freedom and the spectrum of the theory yields the Manton's model of QMD_2 on a circle [6] in the continuum limit [7].

The massless case for $N_f = 1$ was solved by Schwinger [8] using bosonisation trick, that however is not easily generalized to the massive or multiflavour case. The solution shows that all real values of external charge Q_{ext} are screened by the vacuum polarization[1]. Subsequently, a perturbative addition of a small fermion mass was introduced by Coleman et al. in Ref. [9] – it was shown that with $m \neq 0$ only the integer probe charges are screened by fermion-antifermion pairs, in a mechanism reminiscent of the string breaking in QCD. The non-integer charges are expected to have a non-vanishing string tension in accordance with

$$\sigma \sim m\,(1-\cos(2\pi Q_{\text{ext}}))\,. \tag{1.1}$$

Non-abelian two-dimensional models, which are the ultimate aim of our investigation [10] are even more interesting. There are theoretical predictions for the spectrum of QCD_2 with fundamental fermions [11] and the large-N limit of the theory was analytically solved by 't Hooft [12]. The practicality of this solution is however limited by the fact that the fundamental matter is quenched in the large-N limit, thus even more interesting are the large-N limits of theories with two-index representation matter where the fermion dynamics plays an important role.

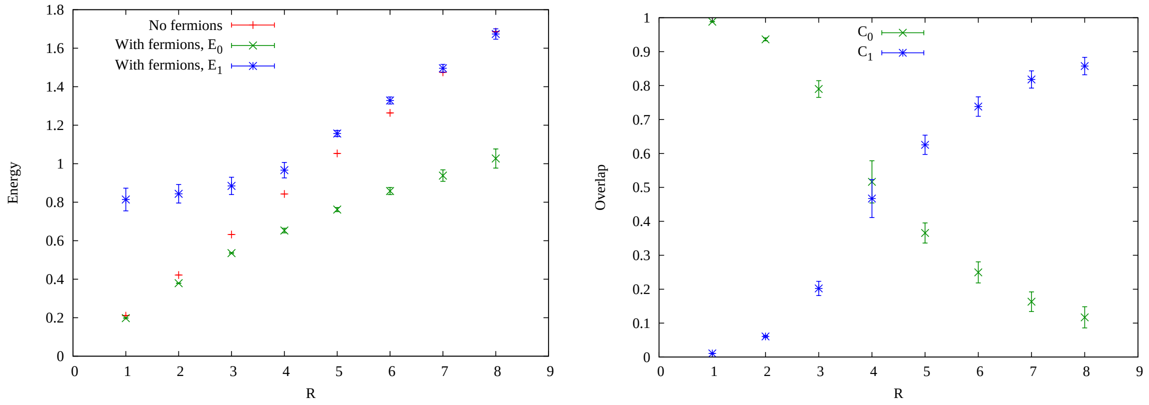
For example, the adjoint fermions in 1+1 dimensions were analyzed both theoretically, giving encouraging results [13], and numerically by DLCQ techniques [14]. However, no lattice analysis has been performed until recently.

In this letter we report the results of our study of QED_2, treated as a warm-up for the non-abelian models. In Section 2 we study the string breaking in QED_2 using Wilson loops and in Section 3 we sum up our attempts to define operators carrying fractional charges and to calculate the string tension with them, which would lead to a lattice verification of Eq. (1.1).

2. String breaking in QED_2

We analyze lattice QED_2 with $N_f = 1$ (the massive Schwinger model) by means of euclidean

[1] In this letter we always write Q_{ext} as a dimensionless quantity, i.e. the multiplicity of the fundamental electric charge.



(a) Extracted energies E_i (b) Extracted overlaps C_i

Figure 1: Energies and overlaps to the Wilson loop of the ground state and first excited state, $V = 24 \times 24$, $g^{-2} = 1.5$, $\kappa = 0.245$. The red points of in Fig. (a), inserted for comparison, are the numerical calculation of the (analytically known) pure gauge result.

Monte Carlo simulations. We use Wilson fermions and generate the configurations using the Rational Hybrid Monte Carlo algorithm. As a test of the possibilities of the simulation we analyze the string breaking using Wilson loops.

It is well known that QED_2 resembles QCD_4 in this respect. It exhibits a confining linear potential in the pure gauge case (as does pure gauge lattice QCD_4 [15]) and charge screening at large distances, interpreted as string breaking by quark-antiquark pair [9, 16]. The string breaking is very hard to observe in the lattice QCD using Wilson loops alone – a poor overlap of the Wilson loop to the broken-string ground state was postulated and a larger set of operators had to be used to deliver a firm observation [17].

As QED_2 is computationally much less demanding than QCD_4 we are able to use a different approach. By using very high statistics (tens of millions configurations) we are able to perform two-exponent fits to the unsmeared Wilson loop data

$$W(R,T) \cong C_0 e^{-E_0(R)T} + C_1 e^{-E_1(R)T}, \tag{2.1}$$

extracting the ground state and the first excited state from the Wilson loop together with the corresponding overlaps. Sample results are presented in Fig. 1. It is natural to interpret Fig. 1(a) as a confined state and a broken-string state, where the latter becomes the ground state around $R \simeq 4-5$, with quantum repulsion (i.e. mixing) of states observed. Perhaps the most instructive result of this exercise it Fig. 1(b) where one can see how the Wilson loop clearly prefers the confined state, explaining the difficulty of observing the string breaking using solely this operator.

3. Non-integer charges

In this section we analyze the possibility of introducing operators carrying arbitrary real charge Q_{ext} to verify the equation (1.1) (from now on, to be concise, we refer to Q_{ext} simply as Q). This is a relatively unexplored topic on the lattice. One notable exception is Ref. [18] presenting a hamiltonian lattice analysis where real charges were introduced by means of constant electric field.

In this work we have approached a different method. We define the “charged Wilson loop” over a contour Γ as

$$W_Q(\Gamma) = \prod_{j\in\Gamma} (U_j)^Q. \tag{3.1}$$

Note that one can formulate an alternative definition as

$$W_Q^{(\mathrm{alt})}(\Gamma) = \Big(\prod_{j\in\Gamma} U_j\Big)^Q. \tag{3.2}$$

While the two coincide for integer charges, they can give vastly different results for the non-integer case when the complex logarithms used to define the non-integer powers fall on different branches. The numerical results presented in this work are obtained using the first definition (3.1). We will briefly discuss the different results given by the second method later in this section.

To find the string tension dependence on Q in the massive Schwinger model we have performed similar analysis as for the “ordinary” Wilson loops in the previous section. For every real Q we found that up to additional perimeter terms, that do not influence the string tension, the Wilson loops with charge Q behave in a very similar manner to the ones with the integer charge closest to Q – thus for every charge analyzed the data is consistent with $\sigma_Q = 0$.

To understand this result we went to an even simpler theory i.e. the pure gauge model (QMD$_2$). There we know the exact results for the string tension of integer charges both on infinite [5] and finite lattices [7]. In the continuum limit one might also expect the well-known continuum result $\sigma_Q \sim Q^2/2$ for all real charges.

One might thus expect that σ_Q for non-integer charges smoothly interpolate between the analytically known results for integer values. However, the values of the string tension obtained from the single-exponent fits (see Fig. 2(a)) show that the string tension is projected to the nearest integer-charged value[2]. A similar behaviour is observed in the pure gauge case of Manton’s model of continuum QED$_2$ on a spatial circle [6]. In this model one can set the gauge so that $A_x(x,t)$ is independent of x and that $A_x \in [0,1)$ with both ends identified – the periodicity of the physical space results in the periodicity of the field space. One then obtains a quantum-mechanical system with a hamiltonian

$$H = \frac{\pi}{e^2}\dot{A}_x^2 \tag{3.3}$$

and the Hilbert space consisting of wave functions satisfying $\psi(A_x = 0) = \psi(A_x = 1)$. Only integer-charged states satisfy periodicity. A state created by an operator with arbitrary charge Q is a result of projection onto the physical Hilbert space. To see this let us define spatial Polyakov loop in this model as $P(A_x,\tau) = e^{2\pi i A_x(\tau)}$. The “charged” Polyakov loop is then simply:

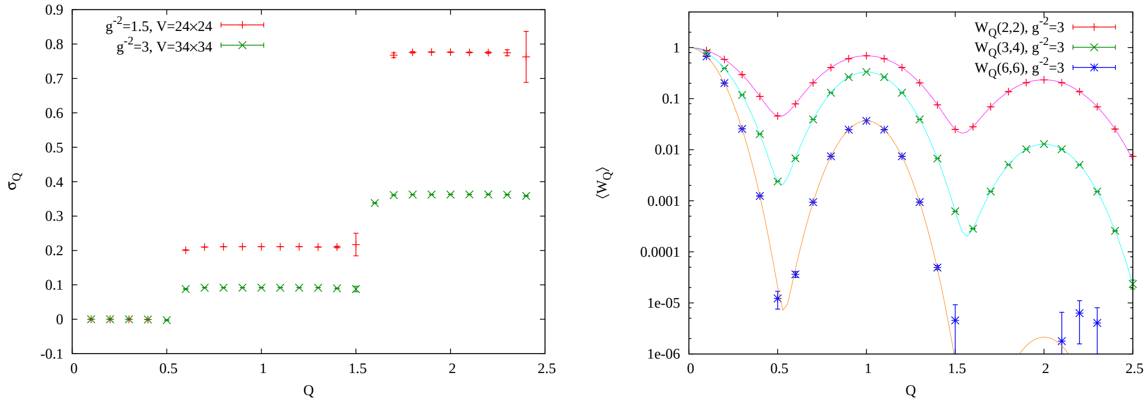
$$P_Q(A_x,\tau) = e^{2\pi i Q A_x(\tau)}. \tag{3.4}$$

The correlation function of the Polyakov loops is then a sum over integer-charged states with proper overlaps:

$$\langle P_Q^\dagger(A_x,\tau)P_Q(A_x,0)\rangle = \sum_{n=0}^{\infty} e^{-n^2e^2\pi\tau}\left(\frac{\sin(\pi(Q-n))}{\pi(Q-n)}\right)^2. \tag{3.5}$$

Driven by this analogy, we formulated an ansatz that very well describes the data (see Fig. 2(b)):

[2]This result shows no sign of volume dependence, up to the largest analysed lattice of volume 256×256.



(a) String tension as a function of Q (b) Ansatz vs. data (not a fit)

Figure 2: Charged Wilson loops in QMD_2.

$$W_Q(R,T) = \sum_{n=0}^{\infty} \left(\frac{I_n(\beta)}{I_0(\beta)}\right)^{RT} \left(\frac{\sin(\pi(Q-n))}{\pi(Q-n)}\right)^{2R+2T}. \tag{3.6}$$

The ansatz assumes that the finite size effects for the integer charges are negligible. In fact, using the results of Ref. [7] one can include them, leading to an exact result [19].

The results obtained using the second definition, Eq. (3.2) lead to similar conclusions with a similarly successful ansatz of the form

$$W_Q^{(\mathrm{alt})}(R,T) = \sum_{n=0}^{\infty} \left(\frac{I_n(\beta)}{I_0(\beta)}\right)^{RT} \left(\frac{\sin(\pi(Q-n))}{\pi(Q-n)}\right), \tag{3.7}$$

the only difference being the exponent of the sine part[3].

The periodicity of the configuration space in the compact formulation implies that the operator with non-integer charge, which does not satisfy periodicity

$$e^{iQ(A+2\pi)} \neq e^{iQA}, \tag{3.8}$$

creates a state which is a combination of states belonging to the Hilbert space, i.e. the integer-charged states. We conjecture that the same phenomenon occurs for dynamical fermions and is the reason of the incompatibility of our string tension calculations and Eq. (1.1). When only the integer part of the probe charge has a physical meaning then the effects of fractional charges are projected out, which explains the vanishing of the string tension for all real Q.

4. Conclusions

In this work we have studied the phenomena of confinement and screening in two-dimensional lattice gauge theories. Using high precision data from Wilson loops we investigated the string breaking in QED_2 and found a rapidly decreasing overlap of Wilson loops on the broken-string

[3]Also calculations using Polyakov loops defined with both methods lead to the same conclusions and differ from the Wilson loops only by the exponents over the sine parts.

state, which accounts for the difficulties in observing the string breaking in lattice QCD simulations which use solely this operator.

Then we proposed two generalizations of Wilson loops for arbitrary real charges Q. We found that for both of them the string tension vanishes for any Q, which would be in disagreement with the prediction of Coleman et al. [9]. Based on the experience from the pure gauge theory, we suggested an explanation by pointing out that in the compact formulation the effects of fractional charges are projected out of the Hilbert space and only the integer charges have a physical meaning. The value for the string tension between integer charges predicted by Eq. (1.1) is indeed zero for all values of the fermion mass.

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