Beyond the Standard Model corrections to $K^0 - \bar{K}^0$ mixing

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We calculate the B-parameters for operators arising in theories of new physics beyond the standard model (BSM) using HYP-smeared improved staggered fermions on the MILC asqtad lattices with $N_f = 2 + 1$ flavors. We use three different lattice spacings ($a \approx 0.045, 0.06$ and $0.09$ fm) at a fixed ratio of light to strange quarks, $m_\ell/m_s = 1/5$, to obtain the continuum results. Operator matching is done using perturbative matching at one-loop order, and results are run to 2 or 3 GeV using two-loop running in the $\overline{\text{MS}}$ scheme. For the chiral and continuum extrapolations, we use SU(2) staggered chiral perturbation theory. We present preliminary results with only statistical errors.
1. Introduction

In the standard model, mixing in the neutral kaon system arises due to the weak interaction. Integrating out the heavy particles, the mixing is described by the matrix element of a $\Delta S = 2$ four-fermion operator ($Q^\text{Cont}_{i}$ below), and is parametrized by $B_K$. $B_K$ is now determined with high precision from lattice QCD $[1, 2, 3, 4]$, and plays an important role in constraining the parameters of the CKM matrix. In BSM theories, additional operators contribute to kaon mixing. If the matrix elements of these operators were known, one could constrain the parameters of these theories in a way that is complementary to direct searches. Here we present a calculation of the new matrix elements using HYP-smeared staggered valence fermions on the MILC asqtad lattices.

We adopt the operator basis used in perturbative calculations of anomalous dimensions $[5]$

\[
\begin{align*}
Q^\text{Cont} & = [\bar{s} \gamma_{\mu} (1 - \gamma_5) d^\mu] [\bar{d} \gamma_{\nu} (1 - \gamma_5) b^\nu], \\
Q^\text{Cont} & = [\bar{s} (1 - \gamma_5) d^\mu] [\bar{d} (1 - \gamma_5) b^\mu], \\
Q^\text{Cont} & = [\bar{s} \sigma_{\mu\nu} (1 - \gamma_5) d^\mu] [\bar{d} \sigma_{\mu\nu} (1 - \gamma_5) b^\mu], \\
Q^\text{Cont} & = [\bar{s} (1 - \gamma_5) d^\mu] [\bar{d} (1 + \gamma_5) b^\mu], \\
Q^\text{Cont} & = [\bar{s} \gamma_{\mu} (1 - \gamma_5) d^\mu] [\bar{d} \gamma_{\nu} (1 - \gamma_5) b^\nu],
\end{align*}
\]

where $\sigma_{\mu\nu} = [\gamma_{\mu}, \gamma_{\nu}] / 2$ and $a, b$ are color indices. $Q_i$ leads to $B_K$, while $Q_{2-5}$ are the BSM operators. The corresponding BSM $B$-parameters are defined as

\[
B_i = \frac{\langle K_0 | Q^\text{Cont}_i | K_0 \rangle}{N_i \langle \bar{K}_0 | \bar{s} \gamma_5 d | 0 \rangle \langle 0 | \bar{s} \gamma_5 d | K_0 \rangle},
\]

We stress that this basis of operators differs slightly from the “SUSY basis” used in other lattice calculations $[6, 7]$. We prefer the basis of Ref. $[5]$ since we use perturbative matching and running.

2. Methodology and Results

We use the MILC lattices listed in Table 1, setting the scale using $r_l = 0.3117(6)(^{+0.12}_{-0.31})$ fm $[9]$. For the valence quarks, we use HYP-smeared staggered quarks $[10]$, with parameters chosen to remove $O(a^2)$ taste-symmetry breaking at tree level. Our valence $d$ and $s$ quarks have masses denoted $m_x$ and $m_y$, respectively, for which we use 10 different values,

\[am_{x,y} = am_x \times n / 10 \quad \text{with } n = 1, 2, 3, \cdots, 10, \quad (2.1)\]

Table 1: MILC lattices used here $[8]$. $a$ is the nominal value of the lattice spacing. “ens” and “meas” are the number of gauge configurations measurements per configuration, respectively. ID is an identification tag.

<table>
<thead>
<tr>
<th>$a$ (fm)</th>
<th>$am_x/am_y$</th>
<th>size</th>
<th>ens × meas</th>
<th>ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.01/0.05</td>
<td>$20^3 \times 64$</td>
<td>671 × 9</td>
<td>C3</td>
</tr>
<tr>
<td>0.09</td>
<td>0.0062/0.031</td>
<td>$28^3 \times 96$</td>
<td>995 × 9</td>
<td>F1</td>
</tr>
<tr>
<td>0.06</td>
<td>0.0036/0.018</td>
<td>$48^3 \times 144$</td>
<td>749 × 9</td>
<td>S1</td>
</tr>
<tr>
<td>0.045</td>
<td>0.0028/0.014</td>
<td>$64^3 \times 192$</td>
<td>747 × 1</td>
<td>U1</td>
</tr>
</tbody>
</table>
Figure 1: $B_2(\mu = 1/a)$ as a function of $T = t - t_1$. (Red) crosses are from the coarse ensemble C3, with $(a m_x, a m_y) = (0.005, 0.05)$; (blue) diamonds are from the fine ensemble F1, with $(a m_x, a m_y) = (0.003, 0.03)$; (purple) octagons are from the superfine ensemble S1, with $(a m_x, a m_y) = (0.0018, 0.018)$; and (brown) squares are from the ultrafine ensemble U1, with $(a m_x, a m_y) = (0.0014, 0.014)$.

with $m_s$ is the nominal sea strange quark mass given in Table 1.

The methodology of the calculation for the BSM B-parameters is very similar to that used for $B_K$ [11]. Many details of the lattice operators and the perturbative matching are given in Refs. [13], although some additional subtleties related to the use of the new operator basis have led to small changes [14]. These, together with the renormalization group running, will be explained in Ref. [15]. The kaon and anti-kaon are produced using U(1)-noise wall-sources placed at timeslices $t_1$ and $t_2 > t_1$, while the four-quark operators (and bilinears needed for the B-parameters) are placed at an intermediate time $t$. The resulting B-parameters should be independent of $t$ when $t$ is far enough from the wall-sources, so that contamination from excited states is small. Hence we fit the data to a constant in the plateau region. The fitting range is determined using the two-point correlator from the wall-sources to the taste-$\xi_5$ axial current. In Fig. 1, we show results for $B_2$ as a function of $T = t - t_1$ with our most physical kaon. When fitting, we ignore the correlations between timeslices (diagonal approximation for the covariance matrix) to avoid an instability of the fit due to small eigenvalues of the covariance matrix. The fitting errors are estimated using the jackknife method.

To increase statistics, we perform multiple measurements with randomly chosen $t_1$ on each gauge configuration (see Table 1). We find considerable autocorrelation for the BSM B-parameters on the fine, superfine, and ultrafine ensembles. Hence we bin the data on these ensembles, using a bin size of 5.

After calculating the BSM B-parameters for 55 valence quark mass combinations, we perform the chiral extrapolation to the physical down and strange quark masses. We first extrapolate $m_x$ to
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Figure 2: (a) $B_2(\mu = 1/a)$ from the NNNLO Bayesian X-fit vs. $X_P$, on F1, for $am_y = 0.03$. The red diamond represents the physical point. (b) $B_2(\mu = 1/a)$ from the Y-fit vs. $Y_P$, on F1. The red diamond corresponds to the physical strange quark mass.

$m_d^{\text{phys}}$ at fixed $m_y$ (“X-fit”), and then linearly extrapolate $m_y$ to $m_\ell^{\text{phys}}$ (“Y-fit”). In the X-fit, we fit to the form from SU(2) staggered chiral perturbation theory (SChPT), which requires $m_\ell \ll m_y$. Hence we take lightest four quark masses for $m_\ell$ (e.g. $m_\ell = \{0.005, 0.01, 0.015, 0.02\}$ on the coarse ensemble) and the heaviest three quark masses for the $m_y$ (e.g. $m_y = \{0.04, 0.045, 0.05\}$ on the coarse ensemble).

For the X-fit we use the next-to-leading order (NLO) SChPT result for the $B_j$ from Ref. [13], extended to higher order:

$$B_j(\text{X-fit, NNNLO}) = c_1 F_0(j) + c_2 X + c_3 X^2 + c_4 X^2 (\ln(X))^2 + c_5 X^2 \ln(X) + c_6 X^3.$$  \hspace{1cm} (2.2)

Here $X = X_P/\Lambda^2$, with $X_P$ the squared mass (in physical units) of the taste-$\xi_5$ pion composed of two light quarks, $X_P = M^2_{\text{cc, p}}$. For the chiral renormalization scale we take $\Lambda = 1$ GeV. $F_0(j)$ contains the leading order and NLO chiral logarithms, and is completely known in terms of $f_\pi$ and measured lattice pion masses [13]. The $c_2$ term is the NLO analytic term. We include three “generic” NNLO terms: the $c_3$ term whis is representative of NNLO analytic terms, and the $c_4$ and $c_5$ terms, which are representative of NNLO chiral logarithms in continuum ChPT. We also include one NNNLO term, with coefficient $c_6$. We fit using the Bayesian method [16] with parameters $c_{4-6}$ constrained to be of order unity, which is the expectation from chiral power-counting. Specifically, we constrain them to be $c_{4-6} = 0 \pm 1$. The full correlation matrix is included in the X-fit.

Having determined the parameters $c_{1-6}$, we can simultaneously extrapolate the results to the physical point $m_\ell = m_\ell^{\text{phys}}$ and remove lattice artifacts due to taste-breaking in pion masses in the chiral logarithms $F_0(j)$, as explained in Ref. [11]. We also set $m_\ell \neq m_\ell^{\text{phys}}$ in the logarithms. In Fig. 2(a), we show the X-fit for $B_2$ on ensemble F1.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure2a.png}
\includegraphics[width=0.4\textwidth]{figure2b.png}
\caption{(a) $B_2(\mu = 1/a)$ from the NNNLO Bayesian X-fit vs. $X_P$, on F1, for $am_y = 0.03$. The red diamond represents the physical point. (b) $B_2(\mu = 1/a)$ from the Y-fit vs. $Y_P$, on F1. The red diamond corresponds to the physical strange quark mass.}
\end{figure}
After the X-fit, we perform the Y-fit, in which we extrapolate \( m_s \) to the physical strange quark mass \( m_s^{\text{phys}} \). We expect that the \( B_j \) are smooth, analytic functions of \( Y_P \). It turns out that a linear form describes the data well:

\[
B_j(Y\text{-fit}) = b_1 + b_2 Y_P ,
\]

where \( Y_P = M_{yy,P}^2 \) is the squared mass of the valence pion with composition \( yy\bar{y} \) and taste \( \xi_5 \). Fig. 2(b) shows results of the Y-fit for \( B_2 \) on F1. At this stage, we use uncorrelated fitting for the Y-fit.

After the chiral extrapolations, we know the BSM B-parameters evaluated at a fixed lattice spacing and matched to the \( \overline{\text{MS}} \) scheme at a scale \( \mu = 1/a \). In order to extrapolate to the continuum limit \((a = 0)\), we need to first run the results to a common scale \( \mu \). In the RG running, operator mixing arises in pairs: \((Q_{\text{Cont}}^2, Q_{\text{Cont}}^3)\) and \((Q_{\text{Cont}}^4, Q_{\text{Cont}}^5)\).

The anomalous dimension matrix for the BSM \( \Delta S = 2 \) operators in the basis of Eqs. (1.2)-(1.5) is calculated up to two-loop order in Ref. [5]. This is in the \( \overline{\text{MS}} \) scheme with naive-dimensional regularization of \( \gamma_5 \) and with the choice of evanescent operators made by Ref. [5].\(^1\) Hence we calculate the RG evolution matrix for the BSM B-parameters at that order. In the case of RG running for \( B_4,5 \), there is a removable singularity in the standard two-loop approximate analytic solution. To resolve this, we use the analytic continuation method introduced in Ref. [17]. We have checked the results by numerical evolution of the RG equations. The resulting BSM B-parameters evaluated at \( \mu = 2 \text{ GeV} \) and 3 GeV are given in the Tables 2 and 3. We note that statistical errors in the BSM B-parameters are smaller than those in \( B_K \).

The final step is the continuum extrapolation of the results. We know that the leading \( a \) and \( \alpha_s \) dependence to be [14]

\[
B_j = d_1 + d_2(a \Lambda)^2 + d_3(a \Lambda)^2 \alpha_s + d_4 \alpha_s^2 + d_5(a \Lambda)^4 + \cdots,
\]

where \( \alpha_s = \alpha_s^{\overline{\text{MS}}}(1/a) \). We do a Bayesian fit to this form, taking the QCD scale determining the magnitude of discretization errors to be \( \Lambda = 300 \text{ MeV} \), and constraining \( d_{2-5} = 0 \pm 2 \). As for \( B_K \), we find that fits to all four lattice spacings are very poor, with \( \chi^2_{\text{aug}}/\text{dof} = 6.6 \sim 30 \) for \( B_{2-5} \). Thus we drop the results from the coarse lattice and fit to the finest three spacings (F1, S1 and U1).

\[\text{Table 2: Preliminary results for BSM B-parameters and } B_K \text{ at } \mu = 2 \text{ GeV. Continuum values are obtained using linear extrapolation. Only statistical errors are shown.}\]

<table>
<thead>
<tr>
<th>( B_j )</th>
<th>( \text{C3} )</th>
<th>( \text{F1} )</th>
<th>( \text{S1} )</th>
<th>( \text{U1} )</th>
<th>( \text{Continuum} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_K )</td>
<td>0.5672(52)</td>
<td>0.5295(43)</td>
<td>0.5362(38)</td>
<td>0.5318(70)</td>
<td>0.5383(66)</td>
</tr>
<tr>
<td>( B_2 )</td>
<td>0.5404(09)</td>
<td>0.5646(14)</td>
<td>0.5967(19)</td>
<td>0.6058(31)</td>
<td>0.6245(30)</td>
</tr>
<tr>
<td>( B_3 )</td>
<td>0.3689(06)</td>
<td>0.4148(10)</td>
<td>0.4594(14)</td>
<td>0.4805(24)</td>
<td>0.5032(22)</td>
</tr>
<tr>
<td>( B_4 )</td>
<td>1.0965(23)</td>
<td>1.1260(28)</td>
<td>1.0911(37)</td>
<td>1.0942(57)</td>
<td>1.0698(56)</td>
</tr>
<tr>
<td>( B_5 )</td>
<td>0.9278(20)</td>
<td>0.9381(25)</td>
<td>0.8875(31)</td>
<td>0.8720(49)</td>
<td>0.8432(48)</td>
</tr>
</tbody>
</table>

\(^1\)A different choice of evanescent operators was made in the one-loop matching calculation of Ref. [12]. We have now extended this calculation to the scheme of Ref. [5]. Results will be reported in Ref. [15].
Table 3: Preliminary results for BSM B-parameters and $B_K$ at $\mu = 3$ GeV. Notation as in Table 2.

<table>
<thead>
<tr>
<th>$B_j$ \ Lat</th>
<th>C3</th>
<th>F1</th>
<th>S1</th>
<th>U1</th>
<th>Continuum</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_K$</td>
<td>0.5478(50)</td>
<td>0.5114(42)</td>
<td>0.5179(37)</td>
<td>0.5137(67)</td>
<td>0.5199(64)</td>
</tr>
<tr>
<td>$B_2$</td>
<td>0.4779(08)</td>
<td>0.4993(12)</td>
<td>0.5277(17)</td>
<td>0.5358(28)</td>
<td>0.5524(26)</td>
</tr>
<tr>
<td>$B_3$</td>
<td>0.3152(05)</td>
<td>0.3496(08)</td>
<td>0.3840(12)</td>
<td>0.3997(20)</td>
<td>0.4174(19)</td>
</tr>
<tr>
<td>$B_4$</td>
<td>1.0462(22)</td>
<td>1.0750(26)</td>
<td>1.0421(36)</td>
<td>1.0452(55)</td>
<td>1.0222(54)</td>
</tr>
<tr>
<td>$B_5$</td>
<td>0.9132(19)</td>
<td>0.9272(24)</td>
<td>0.8824(31)</td>
<td>0.8714(48)</td>
<td>0.8450(47)</td>
</tr>
</tbody>
</table>

Figure 3: Continuum extrapolation of $B_2$ at 2GeV. (Red) diamond is the result from the linear fitting function; and (blue) circle is the result from the Bayesian constrained fitting with the fitting function given in Eq. (2.4).

In this case, both the linear fitting (keeping only $d_1$ and $d_2$) and the constrained fitting (with $d_{1-5}$) work well. In Fig. 3, we show an example of the continuum extrapolation for $B_2$. In Tables 2 and 3, we quote the results from the linear extrapolation. Clearly the systematic errors associated with the choice of continuum extrapolation are significantly larger than the statistical errors.

3. Outlook

The next stage in our calculation is to quantify all sources of systematic error and so draw up a complete error budget. This requires results at other values of the light sea-quark masses to estimate residual $m_\ell$ dependence, and at other volumes to estimate finite volume effects. The latter can also be estimated using SChPT, and are expected to be small. We also plan to investigate whether the use of ratios which cancel chiral logarithms reduces errors in the analysis, and to compare the results to those from an analysis using SU(3) SChPT. We expect that, as for $B_K$, our dominant errors will come from the use of one-loop matching and the continuum extrapolation.
Although our results are preliminary, it is interesting to compare them to those found in Refs. [6, 7]. Changing to the SUSY basis, $B_2$, $B_4$ and $B_5$ are unchanged, while $B_3^{\text{SUSY}} = (5B_2 - 3B_3)/2$. Thus our preliminary results in the tables translate into $B_3^{\text{SUSY}} = 0.81$ and $0.75$ at $\mu = 2$ and $3$ GeV, respectively. There are some disagreements between our results and those of Refs. [6, 7] at the 25% level. Determining whether these are significant will require our full error budget.

Acknowledgments

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References