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Beyond the Standard Model corrections to $K^0 - \bar{K}^0$ mixing

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We calculate the B-parameters for operators arising in theories of new physics beyond the standard model (BSM) using HYP-smeared improved staggered fermions on the MILC asqtad lattices with $N_f = 2+1$ flavors. We use three different lattice spacings ($a \approx 0.045$, 0.06 and 0.09 fm) at a fixed ratio of light to strange quarks, $m_\ell/m_s = 1/5$, to obtain the continuum results. Operator matching is done using perturbative matching at one-loop order, and results are run to 2 or 3 GeV using two-loop running in the $\overline{\text{MS}}$ scheme. For the chiral and continuum extrapolations, we use SU(2) staggered chiral perturbation theory. We present preliminary results with only statistical errors.

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1. Introduction

In the standard model, mixing in the neutral kaon system arises due to the weak interaction. Integrating out the heavy particles, the mixing is described by the matrix element of a $\Delta S = 2$ four-fermion operator ($Q_1^{\rm Cont}$ below), and is parametrized by B_K. B_K is now determined with high precision from lattice QCD [1, 2, 3, 4], and plays an important role in constraining the parameters of the CKM matrix. In BSM theories, additional operators contribute to kaon mixing. If the matrix elements of these operators were known, one could constrain the parameters of these theories in a way that is complementary to direct searches. Here we present a calculation of the new matrix elements using HYP-smeared staggered valence fermions on the MILC asqtad lattices.

We adopt the operator basis used in perturbative calculations of anomalous dimensions [5]

$$Q_1^{\rm Cont} = [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1-\gamma_5) d^b], \tag{1.1}$$
$$Q_2^{\rm Cont} = [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1-\gamma_5) d^b], \tag{1.2}$$
$$Q_3^{\rm Cont} = [\bar{s}^a \sigma_{\mu\nu} (1-\gamma_5) d^a][\bar{s}^b \sigma_{\mu\nu} (1-\gamma_5) d^b], \tag{1.3}$$
$$Q_4^{\rm Cont} = [\bar{s}^a (1-\gamma_5) d^a][\bar{s}^b (1+\gamma_5) d^b], \tag{1.4}$$
$$Q_5^{\rm Cont} = [\bar{s}^a \gamma_\mu (1-\gamma_5) d^a][\bar{s}^b \gamma_\mu (1+\gamma_5) d^b], \tag{1.5}$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$ and a, b are color indices. Q_1 leads to B_K, while Q_{2-5} are the BSM operators. The corresponding BSM B-parameters are defined as

$$B_i = \frac{\langle \overline{K}_0 | Q_i^{\rm Cont} | K_0 \rangle}{N_i \langle \overline{K}_0 | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K_0 \rangle} \qquad (N_2,\ N_3,\ N_4,\ N_5) = (5/3,\ 4,\ -2,\ 4/3)\,. \tag{1.6}$$

We stress that this basis of operators differs slightly from the "SUSY basis" used in other lattice calculations [6, 7]. We prefer the basis of Ref. [5] since we use perturbative matching and running.

2. Methodology and Results

We use the MILC lattices listed in Table 1, setting the scale using $r_1 = 0.3117(6)(^{+12}_{-31})$ fm [9]. For the valence quarks, we use HYP-smeared staggered quarks [10], with parameters chosen to remove $\mathcal{O}(a^2)$ taste-symmetry breaking at tree level. Our valence d and s quarks have masses denoted m_x and m_y, respectively, for which we use 10 different values,

$$am_{x,y} = am_s \times n/10 \qquad \text{with } n = 1, 2, 3, \cdots, 10\,, \tag{2.1}$$

Table 1: MILC lattices used here [8]. a is the nominal value of the lattice spacing. "ens" and "meas" are the number of gauge configurations measurements per configuration, respectively. ID is an identification tag.

a (fm)	am_l/am_s	size	ens $\times$ meas	ID
0.12	0.01/0.05	$20^3 \times 64$	671×9	C3
0.09	0.0062/0.031	$28^3 \times 96$	995×9	F1
0.06	0.0036/0.018	$48^3 \times 144$	749×9	S1
0.045	0.0028/0.014	$64^3 \times 192$	747×1	U1

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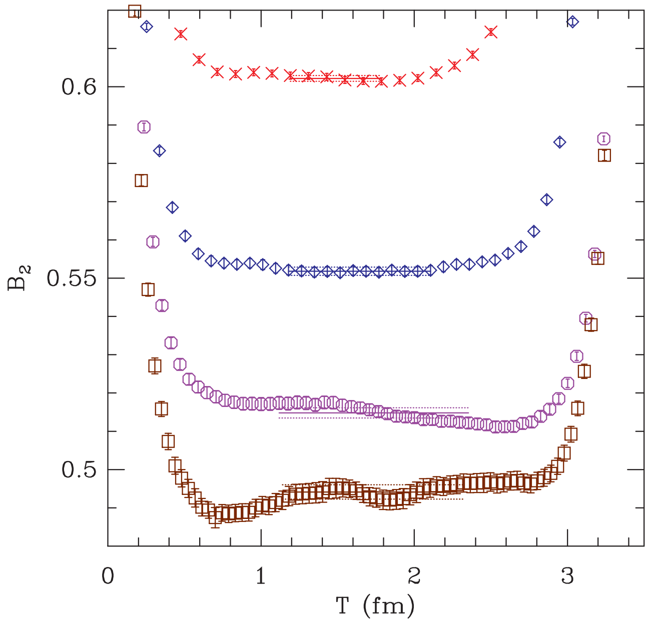


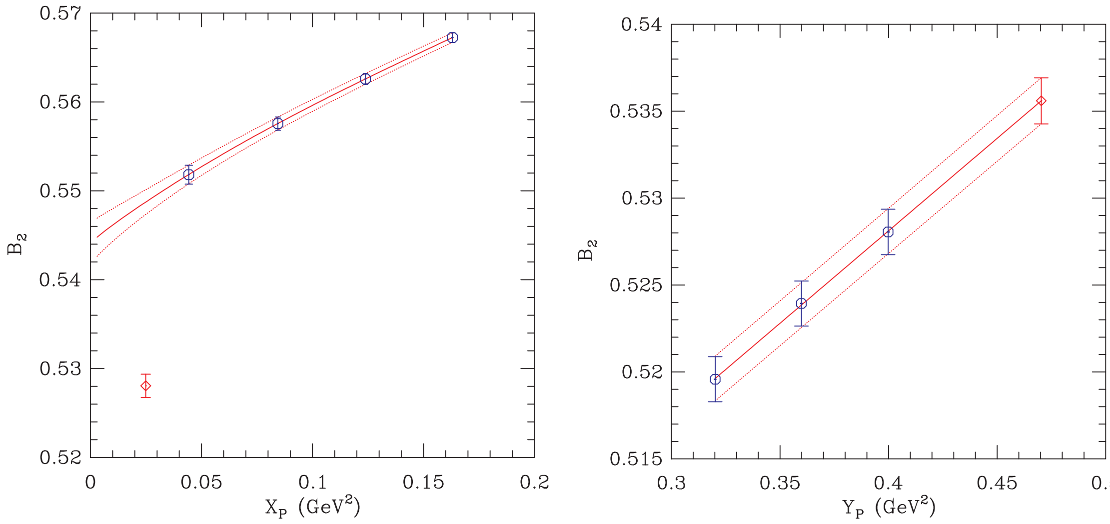
Figure 1: $B_2(\mu = 1/a)$ as a function of $T = t - t_1$. (Red) crosses are from the coarse ensemble C3, with $(am_x, am_y) = (0.005, 0.05)$; (blue) diamonds are from the fine ensemble F1, with $(am_x, am_y) = (0.003, 0.03)$; (purple) octagons are from the superfine ensemble S1, with $(am_x, am_y) = (0.0018, 0.018)$; and (brown) squares are from the ultrafine ensemble U1, with $(am_x, am_y) = (0.0014, 0.014)$.

with m_s is the nominal sea strange quark mass given in Table 1.

The methodology of the calculation for the BSM B-parameters is very similar to that used for B_K [11]. Many details of the lattice operators and the perturbative matching are given in Refs. [13], although some additional subtleties related to the use of the new operator basis have led to small changes [14]. These, together with the renormalization group running, will be explained in Ref. [15]. The kaon and anti-kaon are produced using U(1)-noise wall-sources placed at timeslices t_1 and $t_2 > t_1$, while the four-quark operators (and bilinears needed for the B-parameters) are placed at an intermediate time t. The resulting B-parameters should be independent of t when t is far enough from the wall-sources, so that contamination from excited states is small. Hence we fit the data to a constant in the plateau region. The fitting range is determined using the two-point correlator from the wall-sources to the taste-ξ_5 axial current. In Fig. 1, we show results for B_2 as a function of $T = t - t_1$ with our most physical kaon. When fitting, we ignore the correlations between timeslices (diagonal approximation for the covariance matrix) to avoid an instability of the fit due to small eigenvalues of the covariance matrix. The fitting errors are estimated using the jackknife method.

To increase statistics, we perform multiple measurements with randomly chosen t_1 on each gauge configuration (see Table 1). We find considerable autocorrelation for the BSM B-parameters on the fine, superfine, and ultrafine ensembles. Hence we bin the data on these ensembles, using a bin size of 5.

After calculating the BSM B-parameters for 55 valence quark mass combinations, we perform the chiral extrapolation to the physical down and strange quark masses. We first extrapolate m_x to



(a) X-fit (b) Y-fit

Figure 2: (a) $B_2(\mu = 1/a)$ from the NNNLO Bayesian X-fit vs. X_P, on F1, for $am_y = 0.03$. The red diamond represents the physical point. (b) $B_2(\mu = 1/a)$ from the Y-fit vs. Y_P, on F1. The red diamond corresponds to the physical strange quark mass.

m_d^{phys} at fixed m_y ("X-fit"), and then linearly extrapolate m_y to m_s^{phys} ("Y-fit"). In the X-fit, we fit to the form from SU(2) staggered chiral perturbation theory (SChPT), which requires $m_x \ll m_y$. Hence we take lightest four quark masses for m_x (e.g. $m_x = \{0.005,\ 0.01,\ 0.015,\ 0.02\}$ on the coarse ensemble) and the heaviest three quark masses for the m_y (e.g. $m_y = \{0.04,\ 0.045,\ 0.05\}$ on the coarse ensemble).

For the X-fit we use the next-to-leading order (NLO) SChPT result for the B_j from Ref. [13], extended to higher order:

$$B_j(\text{X-fit, NNNLO}) = c_1 F_0(j) + c_2 X + c_3 X^2 + c_4 X^2 \big(\ln(X)\big)^2 + c_5 X^2 \ln(X) + c_6 X^3\,. \tag{2.2}$$

Here $X = X_P/\Lambda_\chi^2$, with X_P the squared mass (in physical units) of the taste-ξ_5 pion composed of two light quarks, $X_P = M_{xx:\text{P}}^2$. For the chiral renormalization scale we take $\Lambda_\chi = 1\,\text{GeV}$. $F_0(j)$ contains the leading order and NLO chiral logarithms, and is completely known in terms of f_π and measured lattice pion masses [13]. The c_2 term is the NLO analytic term. We include three "generic" NNLO terms: the c_3 term whis is representative of NNLO analytic terms, and the c_4 and c_5 terms, which are representative of NNLO chiral logarithms in continuum ChPT. We also include one NNNLO term, with coefficient c_6.

We fit using the Bayesian method [16] with parameters c_{4-6} constrained to be of order unity, which is the expectation from chiral power-counting. Specifically, we constrain them to be $c_{4-6} = 0 \pm 1$. The full correlation matrix is included in the X-fit.

Having determined the parameters c_{1-6}, we can simultaneously extrapolate the results to the physical point $m_x = m_d^{\text{phys}}$ and remove lattice artifacts due to taste-breaking in pion masses in the chiral logarithms $F_0(j)$, as explained in Ref. [11]. We also set $m_\ell \neq m_\ell^{\text{phys}}$ in the logarithms. In Fig. 2(a), we show the X-fit for B_2 on ensemble F1.

After the X-fit, we perform the Y-fit, in which we extrapolate m_y to the physical strange quark mass $m_s^{\rm phys}$. We expect that the B_j are smooth, analytic functions of Y_P. It turns out that a linear form describes the data well:

$$B_j(\text{Y-fit}) = b_1 + b_2 Y_P \,, \tag{2.3}$$

where $Y_P = M^2_{yy,P}$ is the squared mass of the valence pion with composition $y\bar{y}$ and taste ξ_5. Fig. 2(b) shows results of the Y-fit for B_2 on F1. At this stage, we use uncorrelated fitting for the Y-fit.

After the chiral extrapolations, we know the BSM B-parameters evaluated at a fixed lattice spacing and matched to the $\overline{\rm MS}$ scheme at a scale $\mu = 1/a$. In order to extrapolate to the continuum limit ($a = 0$), we need to first run the results to a common scale μ. In the RG running, operator mixing arises in pairs: $(Q_2^{\rm Cont}, Q_3^{\rm Cont})$ and $(Q_4^{\rm Cont}, Q_5^{\rm Cont})$.

The anomalous dimension matrix for the BSM $\Delta S = 2$ operators in the basis of Eqs. (1.2)-(1.5) is calculated up to two-loop order in Ref. [5]. This is in the $\overline{\rm MS}$ scheme with naive-dimensional regularization of γ_5 and with the choice of evanescent operators made by Ref. [5].[1] Hence we calculate the RG evolution matrix for the BSM B-parameters at that order. In the case of RG running for $B_{4,5}$, there is a removable singularity in the standard two-loop approximate analytic solution. To resolve this, we use the analytic continuation method introduced in Ref. [17]. We have checked the results by numerical evolution of the RG equations. The resulting BSM B-parameters evaluated at $\mu = 2\,$GeV and $3\,$GeV are given in the Tables 2 and 3. We note that statistical errors in the BSM B-parameters are smaller than those in B_K.

The final step is the continuum extrapolation of the results. We know that the leading a and α_s dependence to be [14]

$$B_j = d_1 + d_2(a\Lambda)^2 + d_3(a\Lambda)^2\alpha_s + d_4\alpha_s^2 + d_5(a\Lambda)^4 + \cdots \,, \tag{2.4}$$

where $\alpha_s = \alpha_s^{\overline{\rm MS}}(1/a)$. We do a Bayesian fit to this form, taking the QCD scale determining the magnitude of discretization errors to be $\Lambda = 300$MeV, and constraining $d_{2-5} = 0 \pm 2$. As for B_K, we find that fits to all four lattice spacings are very poor, with $\chi^2_{\rm aug}/{\rm dof} = 6.6 \sim 30$ for B_{2-5}. Thus we drop the results from the coarse lattice and fit to the finest three spacings (F1, S1 and U1). In

Table 2: Preliminary results for BSM B-parameters and B_K at $\mu = 2\,$GeV. Continuum values are obtained using linear extrapolation. Only statistical errors are shown.

B_j \ Lat	C3	F1	S1	U1	Continuum
B_K	0.5672(52)	0.5295(43)	0.5362(38)	0.5318(70)	0.5383(66)
B_2	0.5404(09)	0.5646(14)	0.5967(19)	0.6058(31)	0.6245(30)
B_3	0.3689(06)	0.4148(10)	0.4594(14)	0.4805(24)	0.5032(22)
B_4	1.0965(23)	1.1260(28)	1.0911(37)	1.0942(57)	1.0698(56)
B_5	0.9278(20)	0.9381(25)	0.8875(31)	0.8720(49)	0.8432(48)

[1] A different choice of evanescent operators was made in the one-loop matching calculation of Ref. [12]. We have now extended this calculation to the scheme of Ref. [5]. Results will be reported in Ref. [15].

Table 3: Preliminary results for BSM B-parameters and B_K at $\mu = 3\,\text{GeV}$. Notation as in Table 2.

B_j \ Lat	C3	F1	S1	U1	Continuum
B_K	0.5478(50)	0.5114(42)	0.5179(37)	0.5137(67)	0.5199(64)
B_2	0.4779(08)	0.4993(12)	0.5277(17)	0.5358(28)	0.5524(26)
B_3	0.3152(05)	0.3496(08)	0.3840(12)	0.3997(20)	0.4174(19)
B_4	1.0462(22)	1.0750(26)	1.0421(36)	1.0452(55)	1.0222(54)
B_5	0.9132(19)	0.9272(24)	0.8824(31)	0.8714(48)	0.8450(47)

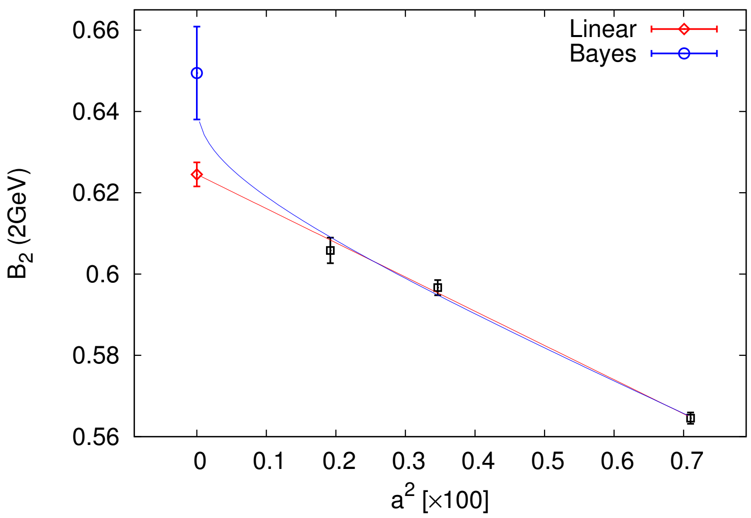


Figure 3: Continuum extrapolation of B_2 at 2GeV. (Red) diamond is the result from the linear fitting function; and (blue) circle is the result from the Bayesian constrained fitting with the fitting function given in Eq. (2.4).

this case, both the linear fitting (keeping only d_1 and d_2) and the constrained fitting (with d_{1-5}) work well. In Fig. 3, we show an example of the continuum extrapolation for B_2. In Tables 2 and 3. we quote the results from the linear extrapolation. Clearly the systematic errors associated with the choice of continuum extrapolation are significantly larger than the statistical errors.

3. Outlook

The next stage in our calculation is to quantify all sources of systematic error and so draw up a complete error budget. This requires results at other values of the light sea-quark masses to estimate residual m_ℓ dependence, and at other volumes to estimate finite volume effects. The latter can also be estimated using SChPT, and are expected to be small. We also plan to investigate whether the use of ratios which cancel chiral logarithms reduces errors in the analysis, and to compare the results to those from an analysis using SU(3) SChPT. We expect that, as for B_K, our dominant errors will come from to the use of one-loop matching and the continuum extrapolation.

Although our results are preliminary, it is interesting to compare them to those found in Refs. [6, 7]. Changing to the SUSY basis, B_2, B_4 and B_5 are unchanged, while $B_3^{\rm SUSY} = (5B_2 - 3B_3)/2$. Thus our preliminary results in the tables translate into $B_3^{\rm SUSY} = 0.81$ and 0.75 at $\mu = 2$ and 3 GeV, respectively. There are some disagreements between our results and those of Refs. [6, 7] at the 25% level. Determining whether these are significant will require our full error budget.

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