B → π form factor with 2 flavours of $O(a)$ improved Wilson quarks

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The determinations of $|V_{ub}|$ from the exclusive branching ratios of $B \to \tau \nu$ and $B \to \pi l \nu$ tend to show a tension at the level of 3σ [1]. On the theoretical side they depend on the lattice computation of the hadronic matrix elements f_B and the $B \to \pi$ form factor $f_+(q^2)$. To understand the tension, improved precision and a careful analysis of the systematics involved are necessary. Working towards this goal, we present preliminary lattice results of the ALPHA collaboration for the $B \to \pi$ form factor $f_+(q^2)$ with $N_f = 2$ flavours of $O(a)$-improved Wilson fermions. Our computation uses HQET in the static limit, pion masses ranging down to ~ 250 MeV, large volumes with $m_\pi L > 4$, three lattice spacings, and non-perturbative renormalization. We describe the techniques adopted to reduce the statistical noise (stochastic all-to-all with full time dilution) and the contamination from excited states (smearing for the B and the pion). We estimate the size of the chiral and continuum extrapolations. We discuss the impact our result could have to clarify the above mentioned discrepancy in the determination of $|V_{ub}|$.

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1. Introduction

The precise determination of the CKM matrix elements is a key for testing the Standard Model. Violations of CKM unitary or discrepancies between independent determinations of the same matrix element can provide hints for New Physics. At the time when we started our work, a tension at the level of 3σ between two exclusive determinations of $|V_{ub}|$ existed, as reported e.g. in the PDG of 2012. These determinations use the branching ratio (BR) for the processes $B → \pi l \nu$ and $B → \tau \nu$ from experiment combined with the form factor $f_+(q^2)$ and the B decay constant f_B, respectively, from the lattice. Also an inclusive determination, based on a perturbative expansion in α_s and an expansion in $1/m_b$, is possible. The results reported by the PDG 2012, computed before ICHEP 2012, can be summarized as follows [1]:

$$
\begin{aligned}
|V_{ub}| &= 0.00323(31) \qquad B → \pi l \nu \\
|V_{ub}| &= 0.00510(47) \qquad B → \tau \nu \\
|V_{ub}| &= 0.00441(34) \qquad \text{incl.}
\end{aligned}
\tag{1.1}
$$

At ICHEP 2012 the Belle collaboration reported a new result for $\mathrm{BR}(B → \tau \nu)$ [8] based on a new set of data, obtained with a more sophisticated tagging of the B. This result, taken alone, would yield a value for $|V_{ub}|$ that is consistent with the exclusive determination from $B → \pi$. However, more data and a careful inspection of the systematics involved are needed to draw more definitive conclusions.

While the experimental precision in the differential decay rate for $B → \pi l \nu$ has by now reached good precision, $B → \tau \nu$ events are more difficult to reconstruct and an error of the order of 20% on the total branching ratio has to be expected. The situation on the theoretical side is the opposite: the lattice computation of a form factor is more challenging than a decay constant.

The results for the determination of f_B by the ALPHA collaboration have been presented in a poster at this conference [5]. The aim of the work described here is the determination of $f_+(q^2)$ in the same setup, which uses the same CLS configurations. In particular in our final results we will use fully non-perturbative renormalization and matching. At this conference we have presented the progress reached so far, putting some emphasis on the techniques employed to deal with the sizable finite-T effects and with the signal-to-noise ratio problems.

2. General Setup

Neglecting the lepton masses, the SM prediction for the $B → \pi l \nu$ decay rate is:

$$
\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} |p_\pi|^3 |f_+(q^2)|^2
\tag{2.1}
$$

where $q^\mu = p_B^\mu - p_\pi^\mu$ and the form factor $f_+(q^2)$ is defined through the Lorentz decomposition of the matrix element:

Figure 1: Quark line diagram for the three point function.

$$
\langle \pi(p_\pi)|V^\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu .
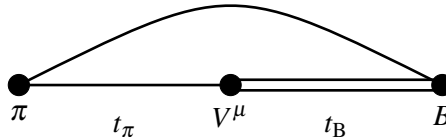\tag{2.2}
$$

To extract this matrix element from lattice simulations, we consider the ratio:

$$R(t_\pi, t_B) \equiv \frac{\sum_{\vec{x}_\pi, \vec{x}_B} e^{-i\vec{p}\cdot\vec{x}_\pi} \langle P_{ll}(t_\pi + t_B, \vec{x}_\pi) V^\mu(t_B, \vec{x}_B) P_{hl}(0) \rangle}{\sqrt{\sum_{\vec{x}_\pi} e^{-i\vec{p}\cdot\vec{x}_\pi} \langle P_{ll}(x_\pi) P_{ll}(0) \rangle \sum_{\vec{x}_B} \langle P_{hl}(x_B) P_{hl}(0) \rangle}} \,, \tag{2.3}$$

$$\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle = \lim_{T, t_B, t_\pi \to \infty} R(t_\pi, t_B) e^{E_\pi t_\pi/2} e^{m_B t_B/2} \tag{2.4}$$

where P_{ll} and P_{hl} are interpolating operators for the π and the B meson, respectively. Our conventions for t_B and t_π are illustrated in the quark line diagram, in Fig. 1.

We treat the b quark in HQET, where a different Lorentz decomposition of $\langle \pi(p_\pi) | V^\mu | B(p_B) \rangle$ in the B rest frame is more convenient:

$$\langle \pi | V^0 | B \rangle = \sqrt{2m_B} f_\parallel \,,$$
$$\langle \pi | V^k | B \rangle = \sqrt{2m_B} p_\pi^k f_\perp \,.$$

In the static limit f_+ is proportional to $f_\perp$:

$$f_+ = \frac{\sqrt{m_B}}{\sqrt{2}} f_\perp \,. \tag{2.5}$$

We have computed the ratio eq. (2.3) on some of the CLS ensembles, as listed in Table 1, with different pion masses and three lattice spacings [9]. These simulations use two $O(a)$-improved Wilson quarks in the sea and relatively fine lattice spacings. Boundary conditions are periodic and the time extent is $T = 2L$. Finite volume effects are expected to contribute less than 1% to the global systematic error, since $m_\pi L > 4$ on all ensembles.

As the b quark cannot be treated relativistically with present lattice spacings, we em-

id	L/a	a [fm]	m_π[MeV]	$m_\pi L$
A2	32	0.0755	630	7.7
E4	32	0.0658	580	6.2
E5	32		420	4.7
F6	48		310	5.0
O7	64	0.0486	270	4.2

Table 1: CLS ensembles used in this exploratory study.

ploy HQET. This is an expansion of QCD in powers of $1/m_b$ and can be used to describe processes for which the relevant momentum scale in the rest frame of the B-meson, in our case $|\vec{p}_\pi|$, satisfies $|\vec{p}_\pi| \ll m_b$. The main advantage of this approach is that HQET is renormalizable, so that the continuum limit can be taken and errors coming from this extrapolation can be estimated. Power divergences have to be removed non-perturbatively, and of course also the renormalization of the vector current should be done with non-perturbative precision. We here use the results of [6], where first the renormalization group invariant (RGI) renormalization factor was computed non-perturbatively. Then the matching factors only contain the physical m_b-dependence through the ratio $M_b/\Lambda_{\overline{MS}}$ where M_b is the RGI b-quark mass. The necessary matching factor, C_V has been computed at 3-loops in continuum perturbation theory [4]. For the operator, which enters in the

computation of the form factor $f_+(q^2)$ in the static limit, the renormalization and matching reads:

$$V_k^{QCD} = C_V(M_b/\Lambda_{\overline{MS}})Z_{A,RGI}^{stat}(g_0)V_k^{stat} + O(1/M_b). \tag{2.6}$$

However, the accuracy of the perturbative expansion for B-physics has been questioned in [12]. We use this perturbative (3-loop) matching only in the present exploratory work, but will employ non-perturbative matching similar to [2] in our future analysis. We will then also include $O(a)$-improvement for the vector current operator, while in the present setup we expect discretisation effects of $O(a\alpha_s)$.

3. Techniques to reduce contamination(s) from excited states and finite T

Significant contributions to the 3-point function in eq. (2.3) may arise from a pion which propagates "around the world" in the T-periodic time direction of the lattice. Apart from a free pion state, also a π-B state, which we denote by $[\pi B]$, contributes to these finite-T effects. This can be understood by considering the transfer matrix representation for eq. (2.3) at suitable time separations such that the contamination from other excited states is negligible. This is equivalent to the requirements

$$\Delta_\pi t_\pi \gg 1 , \quad \Delta_\pi(T - t_\pi) \gg 1 , \quad \Delta_B t_B \gg 1 , \tag{3.1}$$

where Δ_π and Δ_B are the energy shifts between the ground state and the first excited state in the π- and in the B-channel, respectively. We then obtain:

Figure 2: t_π dependence for the considered ratio eq. (2.3), for E5 lattice (see Table 1). In these curves the interpolating operator for the B meson is always smeared. Circles and triangles correspond respectively to a local and a smeared one for the pion. The red curve is obtained by fitting the finite T effects with eq. (4.1).

$$R(t_\pi, t_B)e^{E_\pi t_\pi/2}e^{M_B t_B/2} \to \frac{\langle\pi|V^\mu|B\rangle + \langle 0|V^\mu|[\pi B]\rangle e^{-E_\pi(T-2t_\pi)-\Delta_{[\pi B]}t_B}}{\sqrt{1 + e^{-E_\pi(T-2t_\pi)}}} \tag{3.2}$$

where:

$$\Delta_{[\pi B]} = E_{[\pi B]} - E_\pi - E_B . \tag{3.3}$$

The finite-T effect is hence proportional to the matrix element $\langle 0|V^\mu|[\pi B]\rangle$. If we assume (also based on numerical evidence) that $\langle 0|V^\mu|[\pi B]\rangle$ and $\langle\pi|V^\mu|B\rangle$ have comparable magnitude and that $\Delta_{[\pi B]} \approx 0$, we conclude that finite-T effects dominate for $t_\pi > T/2$. The region where one expects to observe an approximate plateau is limited to $t_\pi < T/2 - k/E_\pi$ where k depends on $\langle 0|V^\mu|[\pi B]\rangle/\langle\pi|V^\mu|B\rangle$. It would be optimal to consider lattices with $T > 2L$ or with other boundary conditions to compute these 3-point functions [10]. Here, we just employ gaussian smeared wave functions both for the B and the π, to allow the extraction of the matrix element $\langle\pi|V^\mu|B\rangle$ at
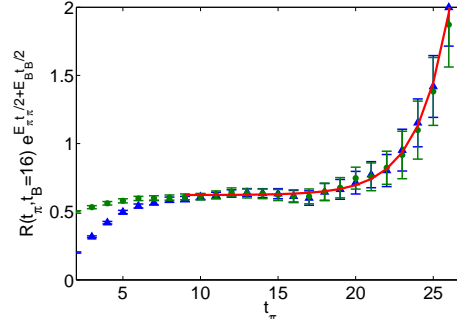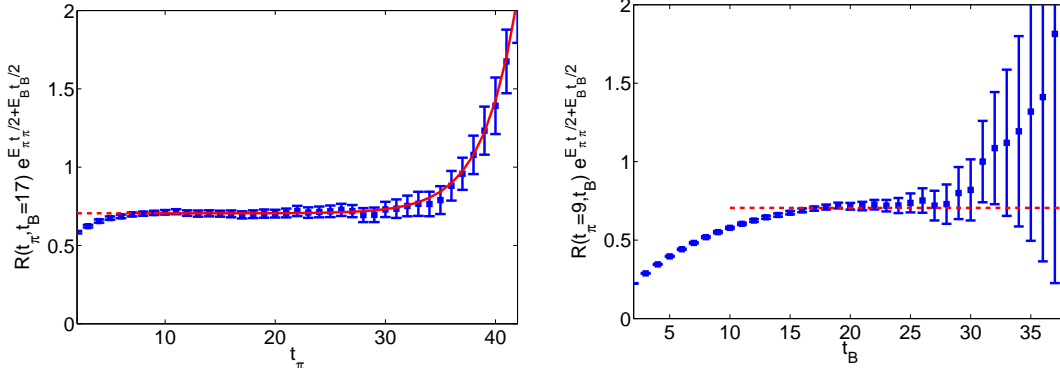
Figure 3: Left (Right): t_π (t_B) dependence of the ratio in eq. (2.3) at fixed t_B (t_π) for ensemble F6 (see Table 1 for detailed parameters) and smeared wave functions for both the B and the π. The red curve is the expected finite-T behaviour fitted according to eq. (4.1).

relatively small time separations t_π and t_B.

Using smeared pion wave functions also helps to improve the signal-to-noise ratio which is of the order $e^{-(E_\pi - m_\pi)t_\pi}$ for the pion propagation. The effect of the smearing is shown in Fig. 2. In the future we will also consider the possibility to use non-spherical smearing for non-zero momentum pion wave functions [7].

4. Results

To extract the form factor on each ensemble we have tried two different methods. The first is a linear two-parameter fit at a fixed t_B

$$\frac{A + B_{t_B} e^{-E_\pi(T-2t_\pi)}}{\sqrt{1 + e^{-E_\pi(T-2t_\pi)}}} \tag{4.1}$$

The value of t_B needs to be chosen small enough that the signal-to-noise ratio is satisfactory but large enough that the excited states effects can be neglected. The good description of the data (see red curve in Fig. (2)) which we achieve with this fit ansatz indicates that the finite-T effects are properly taken into account.

The second is a weighted average over the plateau region, which has a relatively small extent due to the combined presence of excited states and large finite-T effects. The quality of our plateaux is shown in Fig. 3, where the t_π (t_B) dependence at fixed t_B (t_π) is plotted for one of our lattices (F6, see Table 1). We note that the smearing in the pion sector is very efficient and a plateau sets in at early t_π, while in the B sector excited state effects are quite large and the plateau only starts when the noise is already quite sizable. This is quite different from the computation of the B-meson energy and decay constant from two-point functions [5].

Both our methods to extract the form factors give consistent results. All results presented in this work were obtained with the two-parameter fit method. We have also checked that results obtained at different time slices t_B (inside the plateau region), or with a different fit range in t_π are compatible. The errors were computed using the methods described in [11].
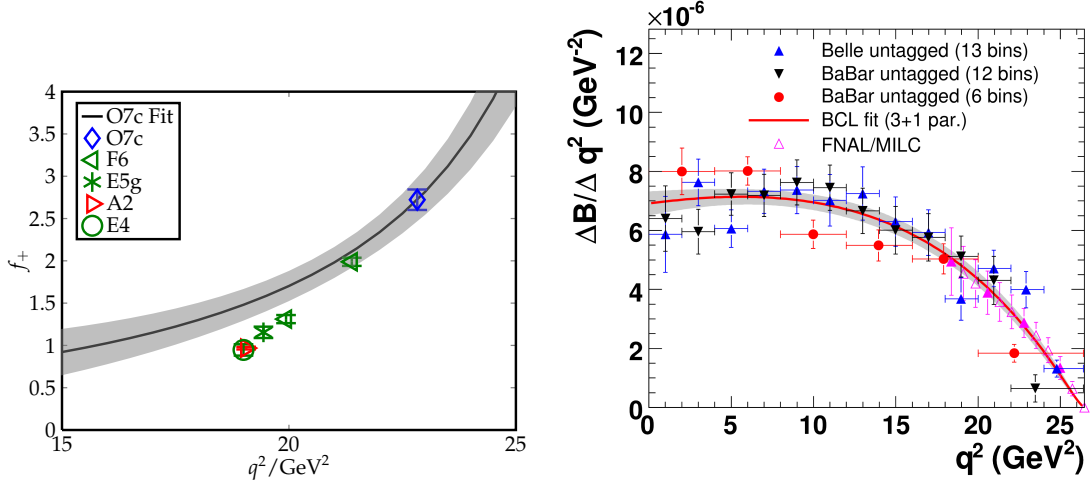
Figure 4: Fit of the q^2-dependence of the form factor using the parametrization proposed in [3]. The left panel shows a fit using only the computation from O7 and the experimental data available. The figure from [1] in the right panel illustrates the experimental situation. Note that points at lower q^2 are more affected by $O(a)$ and $1/m_b$ effects.

5. Conclusions and Outlook

The results collected so far do not yet allow an extrapolation to the continuum limit or to the physical pion mass. Moreover, the vector current is not yet $O(a)$-improved beyond tree-level of perturbation theory and we work in the static approximation of HQET. Therefore, cutoff effects of order $aE_\pi \alpha_s$ and truncation errors of order $O(E_\pi/m_B)$ are present. We are currently working to include in our computation all $O(a)$ and $1/m_b$ terms and to perform the matching with QCD at non-perturbative accuracy.

The encouraging outcome of our present study is that we are able to extract a signal for the form factor on our ensembles with a precision of around $5-10\%$. Using a parametrization of the q^2-dependence of the form factor [3], we have fitted the available experimental data together with our determination of $f_+(q^2)$ at the smallest available p_π (largest q^2) from just one of our ensembles (O7 which is closest to the physical point), This parametrization is based on the so called z-expansion and very general principles, like analyticity, unitarity, and asymptotic freedom. In this fit $|V_{ub}|$ appears as a free parameter and can be determined with a precision of around 15%.

With future more precise experimental data and including the $O(1/m_b)$ terms in our lattice analysis, we hope to reduce the uncertainty on $|V_{ub}|$ to the $5-10\%$ level. Of course, eventually we will need to consider $N_f > 2$, but we would first like to verify with $N_f = 2$ simulations that the extrapolation to the continuum limit can be carried out within our framework of non-perturbatively renormalized HQET.

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