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Form factors for semi-leptonic B decays

Ran Zhou, Steven Gottlieb*
Department of Physics, Indiana University, Bloomington, IN 47405, USA
E-mail: sg@indiana.edu

Jon A. Bailey
Department of Physics and Astronomy, Seoul National University, Seoul 151-747, South Korea

Daping Du, Aida X. El-Khadra, R.D. Jain
Physics Department, University of Illinois, Urbana, Illinois 61801, USA

Andreas S. Kronfeld, Ruth S. Van de Water
Fermi National Accelerator Laboratory, Batavia IL 60510, USA

Yuzhi Liu, Yannick Meurice
Department of Physics and Astronomy, University of Iowa, Iowa City, IA 52240, USA

(Fermilab Lattice and MILC Collaborations)

We report on form factors for the $B \to K l^+ l^-$ semi-leptonic decay process. We use several lattice spacings from $a = 0.12$ fm down to 0.06 fm and a variety of dynamical quark masses with 2+1 flavors of asqtad quarks provided by the MILC Collaboration. These ensembles allow good control of the chiral and continuum extrapolations. The b-quark is treated as a clover quark with the Fermilab interpretation. We update our results for $f_{\|}$ and $f_{\perp}$, or, equivalently, f_+ and f_0. In addition, we present new results for the tensor form factor f_T. Model independent results are obtained based upon the z-expansion.

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1. Motivation and theoretical background

The rare B meson decay $B \to Kll$ is mediated by a flavor changing neutral current (FCNC). The Standard Model (SM) contribution occurs through a penguin diagram for the $b \to sll$ process. Since FCNCs are small in the SM, this decay presents an opportunity to detect beyond-the-Standard-Model (BSM) physics. This decay has been studied by several experiments including BaBar [1], Belle [2], CDF [3], and LHCb [4]. We expect that LHCb will continue to improve its precision, and new B factories such as SuperB and SuperKEKB will greatly add to our understanding of this decay.

Recently, the CDF Collaboration published results for the $B^+ \to K^+\mu^+\mu^-$ differential branching ratio [3]. They compared their measurement with a decade-old prediction of the decay form factors based on light cone sum rule (LCSR) calculations [4]. There is considerable uncertainty in the LCSR form factors at small momentum transfer q^2. Unless the uncertainty in the form factors is reduced, our ability to search for BSM physics will be severely limited by lack of knowledge of the SM decay rate.

Lattice QCD enables first-principles calculations of the form factors with controlled errors that are systematically improvable. In fact, the first study of this decay, using the quenched approximation, was done over 15 years ago [5]. Since then, there have been three additional studies using the quenched approximation [6, 7, 8]. Recently, there have been two groups studying this decay using $2+1$ flavor improved staggered quark configurations from the MILC collaboration [9, 10].

2. Ensembles

Table 1 shows the ensembles that were used in the current calculation. The MILC ensembles [11, 12, 13] were generated using three flavors of dynamical quarks. The quarks are asqtad-improved staggered quarks and a Symanzik improved gauge action is used. The common mass (in lattice units) of the two light sea quarks is denoted am_l and that of the strange sea quark is am_s. We use four coarse ($a \approx 0.12$ fm), five fine (($a \approx 0.09$ fm) and two super-fine ($a \approx 0.06$ fm) ensembles, in order to control the chiral and continuum extrapolations. The ratio of light-to-strange sea-quark masses varies from 0.4 to 0.05. On each configuration, we use four source times to increase statistics.

3. Form factors

Semileptonic decays of a heavy-light pseudoscalar meson to a pseudoscalar or vector meson are characterized by form factors that describe matrix elements of a hadronic current between initial and final states. We concentrate here on the decay $B \to Kll$ for which we need two matrix elements $\langle K(p_K)|i\bar{s}\gamma^\mu b|B(p_B)\rangle$ and $\langle K(p_K)|i\bar{s}\sigma^{\mu\nu} b|B(p_B)\rangle$, where the B meson momentum is p_B and that of the kaon is p_K. For the vector current matrix element, we define two form factors f_+ and f_0, while for the tensor operator, we only need a single form factor f_T:

$$\langle K|i\bar{s}\gamma^\mu b|B\rangle = f_+(q^2)\left(p_B^\mu + p_K^\mu - \frac{M_B^2 - M_K^2}{q^2} q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_K^2}{q^2} q^\mu, \tag{3.1}$$

$\approx a$(fm)	size	am_l/am_s	N_{meas}
0.12	$20^3 \times 64$	0.02/0.05	2052
0.12	$20^3 \times 64$	0.01/0.05	2259
0.12	$20^3 \times 64$	0.007/0.05	2110
0.12	$20^3 \times 64$	0.005/0.05	2099
0.09	$28^3 \times 96$	0.0124/0.031	1996
0.09	$28^3 \times 96$	0.0062/0.031	1931
0.09	$32^3 \times 96$	0.00465/0.031	984
0.09	$40^3 \times 96$	0.0031/0.031	1015
0.09	$64^3 \times 96$	0.00155/0.031	791
0.06	$48^3 \times 144$	0.0036/0.018	673
0.06	$64^3 \times 144$	0.0018/0.018	827

Table 1: Ensembles of asqtad $N_f = 2+1$ configurations analyzed.

$$\langle K|i\bar{s}\sigma^{\mu\nu}b|B\rangle = \frac{2f_T(q^2)}{M_B+M_K}(p_B^\mu p_K^\nu - p_B^\nu p_K^\mu), \tag{3.2}$$

In these equations, $q^\mu = p_B^\mu - p_K^\mu$ is the momentum transfer, and q^2 is the outgoing dilepton invariant mass squared. We can alternatively describe the vector current form factors by $f_\parallel$ and $f_\perp$. We define

$$\langle K|i\bar{s}\gamma^\mu b|B\rangle = \sqrt{2M_B}\left[v^\mu f_\parallel(E_K) + p_\perp^\mu f_\perp(E_K)\right], \tag{3.3}$$

where $v^\mu = p_B^\mu/M_B$ is the four-velocity of the B meson and $p_\perp^\mu = p_K^\mu - (p_K \cdot v)v^\mu$.

In lattice QCD, it is convenient to work in the rest frame of the B meson. The new form factors are considered to be functions of the kaon energy E_K:

$$f_\parallel(E_K) = \frac{\langle K|i\bar{s}\gamma^0 b|B\rangle}{\sqrt{2M_B}}, \tag{3.4}$$

$$f_\perp(E_K) = \frac{\langle K|i\bar{s}\gamma^i b|B\rangle}{\sqrt{2M_B}p_K^i}, \tag{3.5}$$

$$f_T(E_K) = \frac{M_B+M_K}{\sqrt{2M_B}}\frac{\langle K(k)|ib\sigma^{0i}s|B(p)\rangle}{\sqrt{2M_B}p_K^i}, \tag{3.6}$$

where there is no sum on the index i in the equations for $f_\perp$ and f_T. In the B-meson rest frame, $q^2 = (p_B - p_K)^2 = M_B^2 + M_K^2 - 2M_BE_K$, so small E_K corresponds to large q^2.

4. Chiral-continuum extrapolation

Because we work with light quark masses larger than in Nature and at non-zero lattice spacing, we must take the chiral and continuum limits of our results. We use heavy-light meson staggered chiral perturbation theory (HMSχPT) [14] to perform extrapolations. We work to zeroth order in $1/m_b$ and next-to-leading order in the light-quark masses, kaon recoil energy, and lattice spacing.

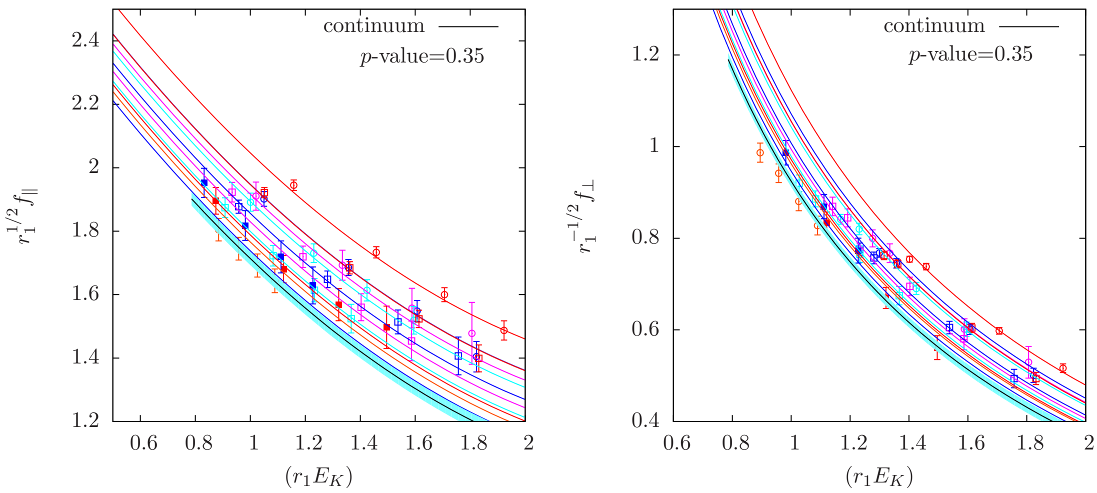

Figure 1: The form factors $f_\parallel$ (left) and $f_\perp$ (right) as a function of E_K in lattice units. The legend in Fig. 2 explains the meaning of all the symbols.

The form factors $f_\parallel$ and $f_\perp$ are fit to the following forms:

$$f_\parallel = \frac{C_\parallel^{(0)}}{f_\pi}\left[1+\text{logs}+C_\parallel^{(1)}m_l+C_\parallel^{(2)}(2m_l+m_s)+C_\parallel^{(3)}E_K+C_\parallel^{(4)}E_K^2+C_\parallel^{(5)}a^2\right], \tag{4.1}$$

$$\begin{aligned} f_\perp &= \frac{C_\perp^{(0)}}{f_\pi}\left[\frac{1}{E_K+\Delta^*_{B_s}+\text{logs}}+\frac{\text{logs}}{E_K+\Delta^*_{B_s}}\right] \\ &\quad+\frac{C_\perp^{(0)}}{f_\pi(E_K+\Delta^*_{B_s})}\left[C_\perp^{(1)}m_l+C_\perp^{(2)}(2m_l+m_s)+C_\perp^{(3)}E_K+C_\perp^{(4)}E_K^2+C_\perp^{(5)}a^2\right], \end{aligned} \tag{4.2}$$

where the notation in Ref. [15] is used, and $\Delta^*_{B_s} = m_{B^*_s} - M_B$. The 1-loop $SU(2)$ chiral logarithms (denoted as "logs" above) are non-analytic functions of the pseudoscalar meson masses. We use the same form for f_T as for $f_\perp$, because the Isgur-Wise relation [16] requires that $f_T = f_+$ in the large M_B limit, and the shape of f_+ is dominated by $f_\perp$ at low recoil. Figures 1 and 2 show our results for $f_\parallel$, $f_\perp$ and f_T including the continuum, physical quark-mass curve plotted as a black line with a cyan error band.

5. *z*-expansion

Because only relatively small kaon momenta are accessible in the lattice calculation, the range of q^2 directly calculable is close to the maximum value. However, the experimental results are best at low q^2. We use the z-expansion to extend the kinematic range [17]. This expansion is based on the field theoretic principles analyticity and crossing symmetry. It is systematically improvable by adding more orders to the expansion. The q^2 region is mapped to z by

$$z(q^2,t_0) = \frac{\sqrt{t_+-q^2}-\sqrt{t_+-t_0}}{\sqrt{t_+-q^2}+\sqrt{t_+-t_0}}, \tag{5.1}$$

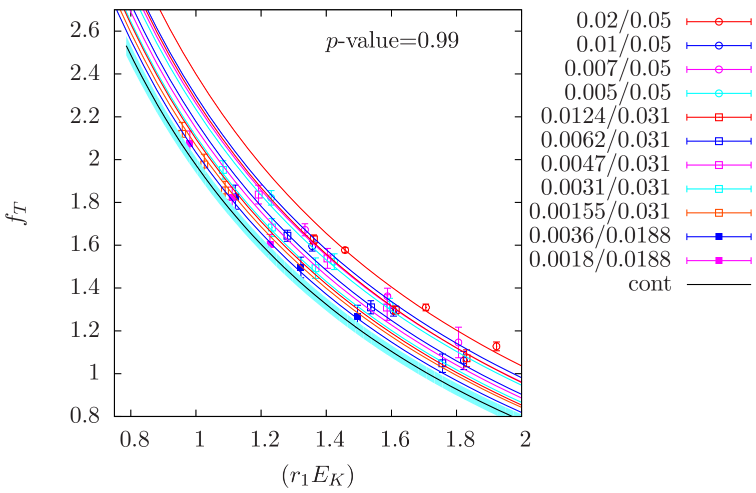

Figure 2: The form factor f_T as a function of E_K in lattice units.

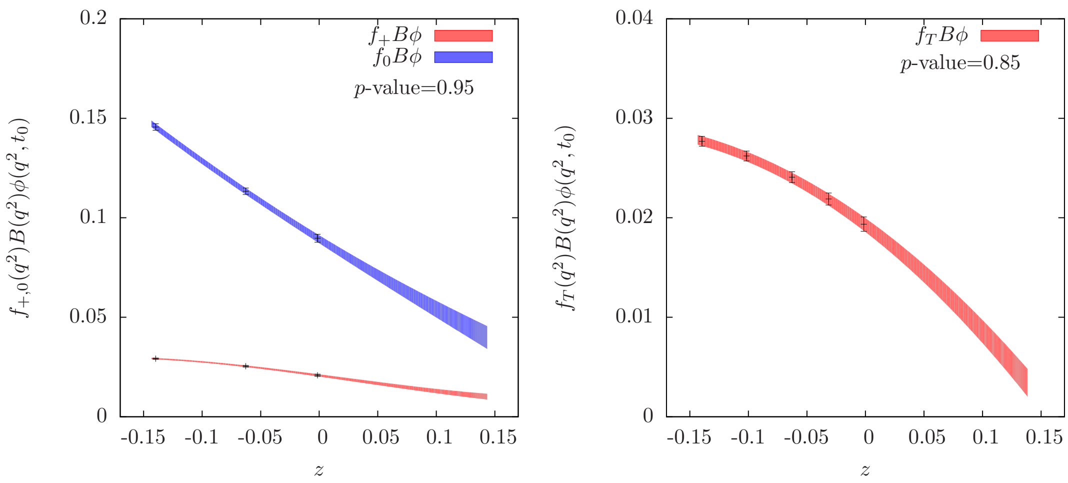

Figure 3: Fit of form factor multiplied by Blaschke factor and outer function to a polynomial expansion in z. Only statistical errors are shown.

where $t_\pm = (M_B \pm M_K)^2$. We are allowed to choose t_0 as we wish to minimize the range of $|z|$. It is convenient to choose $t_0 = t_+\left(1-\sqrt{1-\frac{t_-}{t_+}}\right)$. This results in the full range of q^2 for this decay being mapped to $|z| < 0.16$. The form factors are then expressed as a function of z:

$$f(q^2) = \frac{1}{B(z)\phi(z)} \sum_{k=0}^{\infty} a_k z^k, \tag{5.2}$$

where the Blaschke factor $B(z) = z(q^2, m_R^2)$ is used to account for the pole structure of the form factor, and the outer function $\phi(z)$ is selected such that $\sum_{k=0}^{\infty} a_k^2 \leq 1$ [18].

In Fig. 3, we show our polynomial fits to the form factors multiplied by the Blaschke factor and outer function, *i.e.*, $f(q^2)B(z)\phi(z)$. To generate these plots, we calculate synthetic data at selected values of z in the range accessible to our simulations based on the chiral-continuum extrapolation

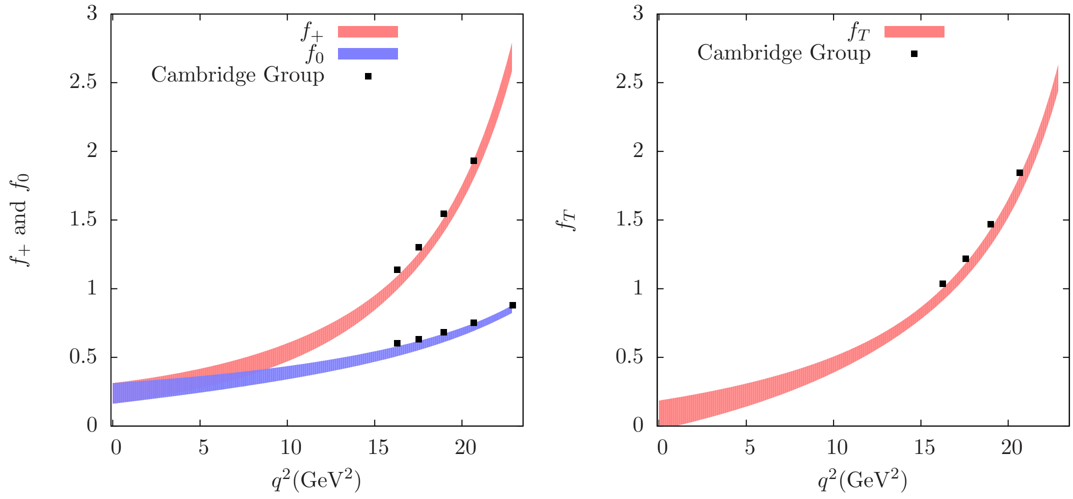

Figure 4: Preliminary results for form factors as a function of q^2, including comparison with Ref. [9].

of the lattice data in Figs. 1–2. We see that the negative z region corresponds to maximum q^2, and that is the region in which the lattice calculation is done. We impose the kinematic constraint $f_+(q^2=0) = f_0(q^2=0)$ in the z-fit. We use terms up to z^3 for f_+ and f_0, and z^2 for f_T. The form factors as a function of q^2 are shown in Fig. 4. Both statistical and systematic errors are shown, but these results are preliminary and do not include the final renormalization factors or all the ensembles that we will analyze.

At large q^2, where we can directly compute the form factors, the error is about 5%. As q^2 decreases, the errors from statistics, chiral fit and z-expansion grow to more than 30% for f_+ and f_0 and to about 100% for f_T. One reason that the relative errors grow at small q^2 is that the form factors themselves are getting much smaller.

6. Conclusions

We have presented some preliminary results for the rare decay $B \to Kll$ form factors. There are additional ensembles not yet included in the analysis. We still need to include the perturbative renormalization factors, which are expected to be close to unity and will not change our results significantly. Our results for $f_{\parallel}$, $f_{\perp}$ and f_T can be used to calculate the SM $B \to Kll$ branching fraction or to make predictions for this process in BSM theories. Other semileptonic decays currently being analyzed include $B \to \pi l \nu$ and $B_s \to K l \nu$.

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