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Lattice study on η_{c2} and X(3872)

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Properties of 2^{-+} charmonium η_{c2} are investigated in quenched lattice QCD. The mass of η_{c2} is determined to be $3.80(3)\,\mathrm{GeV}$, which is close to the mass of D-wave charmonium $\psi(3770)$ and in agreement with quark model predictions. The transition width of $\eta_{c2} \to \gamma J/\psi$ is also obtained with a value $\Gamma = 3.8(9)\,\mathrm{keV}$. Since the possible 2^{-+} assignment to $X(3872)$ has not been ruled out by experiments, our results help to clarify the nature of $X(3872)$.

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1. Introduction

Even though the charmoium-like resonance $X(3872)$ has been established for several years [1, 2, 3, 4] with $M_X = 3871.56 \pm 0.22\,\text{MeV}$ and $\Gamma_X < 2.3\,\text{MeV}$ [5](latest Belle result gives a new bound $\Gamma_X < 1.2\,\text{MeV}$ [6]), the very nature of it has not been fully understood till now. Further analysis of its decay angular distribution also constrains its total quantum number J^{PC} to be either 1^{++} or 2^{-+}. For the decay mode $X(3872) \to \gamma J/\psi$, the BaBar and Belle collaborations reported consistent measurements [7, 8]

$$\begin{aligned} &\text{Br}(B^{\pm} \to X(3872)K^{\pm})\text{Br}(X(3872) \to J/\psi\gamma) = (2.8 \pm 0.8 \pm 0.1) \times 10^{-6} \quad \text{(BaBar)}, \\ &\text{Br}(B^{\pm} \to X(3872)K^{\pm})\text{Br}(X(3872) \to J/\psi\gamma) = (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6} \quad \text{(Belle)}. \end{aligned} \tag{1.1}$$

With the world average value $\text{Br}(B^+ \to X(3872)K^+) < 3.2 \times 10^{-4}$, one can estimate the branch ratio $\text{Br}(X(3872) \to J/\psi\gamma) > 0.9\%$ (BaBar) or 0.6% (Belle). However for the decay mode $X(3872) \to \gamma\psi'$, BaBar measured a 3.4 ± 1.4 times larger branch ratio [7], while Belle found no evidence [8]. This large discrepancy should be reconciled by further experimental measurements.

Theoretically, if we are constrained to its charmonium assignments, $X(3872)$ can be either the radial excitation of χ_{c1} (if 1^{++}), say, χ'_{c1}, or the ${}^1D_2$ charmonium η_{c2} (if 2^{-+}). The potential quark model predicts the mass of both χ'_{c1} and η_{c2} to be near X(3872). There are also many lattice studies predicting the χ'_{c1} mass ranging from 3850 to 4060 MeV [9, 10, 11, 12], and predicting η_{c2} mass in the range $3770 - 3830$ MeV, in agreement with potential quark model [13, 14, 15]. Anyway, the mass parameter should not be the unique criterion for unravelling the nature of $X(3872)$ and radiative transitions of χ'_{c1} and η_{c2} are also accessible for theoretical studies which can shed some light on the essence of $X(3872)$. In this work, we will focus on the study of the properties of η_{c2}, such as its mass and radiative transition width to J/ψ.

2. Numerical Details

In this work we aim at the lattice calculation of the radiative transition rate of η_{c2} to J/ψ. The general radiative transition width of an initial particle i to a final particle f is

$$\Gamma(i \to \gamma f) = \int d\Omega_q \frac{1}{32\pi^2} \frac{|\vec{q}|}{M_i^2} \frac{1}{2J_i + 1} \times \sum_{r_i, r_j, r_\gamma} \left| M_{r_i, r_j, r_\gamma} \right|^2 \propto \sum_k F_k^2(0), \tag{2.1}$$

where $\vec{q} = \vec{p}_i - \vec{p}_f$ is the decay momentum with the mass-on-shell value $|q| = (M_i^2 - M_f^2)/(2M_i)$; M_i and M_f being the masses of initial and final particles respectively; M_{r_i, r_f, r_γ} is the transition amplitude which could be expressed by several Lorentz invariant form factors $F_k(0)$ through the multipole decomposition.

We use the quenched approximation in this study. The gauge configurations are generated by the tadpole improved gauge action [16] on anisotropic lattices with relative parameters listed in Table 1.

The whole spectrum of the low-lying charmonium states we extracted in this work are illustrated in the Fig. 1 and it is seen that the effects of the finite lattice artifacts and the quenched approximation are mild.

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Table 1: Relevant input parameters and vector current renormalization factor $Z_V(a_s)$ for this work are listed. The spatial lattice spacing a_s's are determined from $r_0^{-1} = 410(20)\,\mathrm{MeV}$ by calculating the static potential.

β	ξ	a_s(fm)	La_s(fm)	$L^3 \times T$	N_{conf}	$Z_t(0,0,0)$	$Z_t(1,0,0)$	$Z_s(1,0,0)$
2.4	5	0.222	1.78	$8^3 \times 96$	1000	1.288(5)	1.299(11)	1.388(15)
2.8	5	0.138	1.66	$12^3 \times 144$	1000	1.288(5)	1.299(11)	1.388(15)

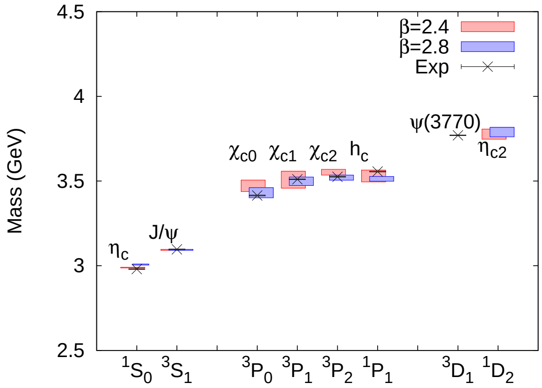

Figure 1: The $1S$, $1P$, and $1D$ charmonium spectrum. The red boxes illustrate the results for $\beta = 2.4$, and blue ones for $\beta = 2.8$. The experimental value are also plotted with points (the crosses) for comparison.

As the first step, we will emphasize on the choice of the operators for the 2^{-+} state, which is the major objective of this work. The situation for the η_{c2} is a little bit more complicated. We have tried the following types of operators:

$$|\varepsilon_{ijk}|\bar{c}(x)\Sigma_j \overleftrightarrow{D}_k c(x) \quad (D\text{−type}), \quad |\varepsilon_{ijk}|\bar{c}(x)\gamma_5 \overleftrightarrow{D}_j \overleftrightarrow{D}_k c(x) \quad (DD\text{−type}), \text{and}$$
$$|\varepsilon_{ijk}|\bar{c}(x)\gamma_j B_k c(x) \quad (F\text{−type}),$$

Since the gauge is fixed to Coulomb gauge, the gauge covariant derivative operator $\overleftrightarrow{D}$ is replaced by the ordinary difference operator $\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$ to improve the signal-to-noise in practical calculation. We finally choose the DD type operator which has the largest overlap with lower state for the related three point functions in radiative decay simulation.

For the calculation of the three point functions, in order to increase statistics, the same calculations are repeated n_t times by setting a point source on a different time slice respectively. Besides, we introduce the ratio $R^\mu(t)$ defined as,

$$R^\mu(t) = \Gamma^{(3)}(\vec{p}_f, \vec{q}, t_f, t) \times \sqrt{\frac{2E_i\Gamma_i^{(2)}(\vec{p}_i, t_f - t)}{\Gamma_i^{(2)}(\vec{p}_i, t)\Gamma_i^{(2)}(\vec{p}_i, t_f)}} \sqrt{\frac{2E_f\Gamma_f^{(2)}(\vec{p}_f, t)}{\Gamma_f^{(2)}(\vec{p}_f, t_f - t)\Gamma_f^{(2)}(\vec{p}_f, t_f)}}, \tag{2.2}$$

to reduce the contribution from excited states. From a plateau behavior of the ratio in t, the desired matrix element $\langle f(\vec{p}_f, r_f)|j^\mu(0)|i(\vec{p}_i, r_i)\rangle$ was extracted.

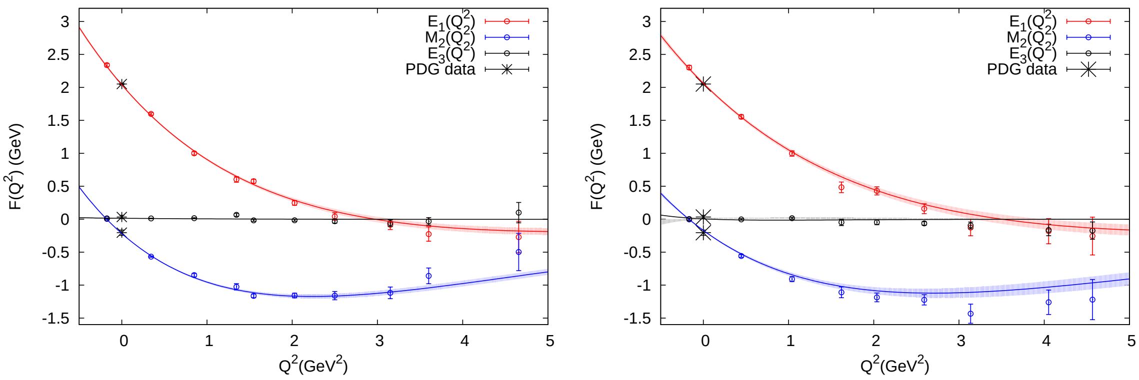

Figure 2: Plotted are the extracted form factors $E_1(Q^2)$, $M_2(Q^2)$, and $E_3(Q^2)$ versus Q^2 for the two lattices of $\beta = 2.4$ (the upper panel) and $\beta = 2.8$ (the lower panel), respectively, where the points are the simulation data, the line the fit function, and the error bands the jackknife ones. The PDG values of $E_1(0)$ and $M_2(0)$ are also plotted for comparison.

2.1 $\chi_{c2} \to \gamma J/\psi$ transition

In the practical study, we set the tensor charmonium (T) to be at rest and let the vector (V) moving with different spatial momenta $\vec{p} = 2\pi\vec{n}/L$. 27 momentum modes of $\vec{n} = (n_1, n_2, n_3)$ ranging from $(0,0,0)$ to $(2,2,2)$ are calculated for V.

The transition width of $\chi_{c2} \to \gamma J/\psi$ for an on-shell photon ($Q^2 = 0$) involves only the form factors $E_1(0)$, $M_2(0)$ and $E_3(0)$,

$$\Gamma(\chi_{c2} \to \gamma J/\psi) = \frac{16\alpha|\vec{k}|}{45M_{\chi_{c2}}^2}(|E_1(0)|^2 + |M_2(0)|^2 + |E_3(0)|^2) \tag{2.3}$$

where $|\vec{q}| = (M_{\chi_{c2}}^2 - M_{J/\psi}^2)/2M_{\chi_{c2}}$ is the decaying energy of the photon, $\alpha = e^2/(4\pi)$ the fine structure constant. We adopt the fitting functional form inspired by non-relativistic quark model to interpolate the simulation data to $Q^2 = 0$ [17],

$$F_k(Q^2) = F_k(0)(1 + \lambda_k Q^2)e^{-\frac{Q^2}{16\beta_k^2}} \tag{2.4}$$

which has been applied successfully in previous works. The curves plotted in Fig. 2 with jackknife error bands are the extracted form factors $E_1(Q^2)$, $M_2(Q^2)$, and $E_3(Q^2)$ versus Q^2 for the two lattices of $\beta = 2.4$ (the left panel) and $\beta = 2.8$ (the right panel) respectively, and the data points are the simulated results with jackknife errors.

Table 2 lists the results of the interpolation, where the continuum limit extrapolation is also given. It is seen that the electric dipole (E_1) contribution is dominant in the transition $\chi_{c2} \to \gamma J/\psi$, and the contribution of other form factors are drastically suppressed, as depicted by the ratio

$$a_{2,3} = \frac{M_2(0)(\text{or } E_3(0))}{\sqrt{E_1(0)^2 + M_2(0)^2 + E_3(0)^2}}, \tag{2.5}$$

for which $a_2 = -0.067(7)$ and $a_3 = -0.003(6)$, which is consistent with the PDG data $a_2 = -0.100 \pm 0.015$ and $a_3 = 0.016 \pm 0.013$ [5].

Table 2: Here lists the results of the interpolation and the corresponding widths, where the continuum limit extrapolation is also given. All the results are in the physical units. The widths can be compared with the PDG data $\Gamma = 384(38)\,\mathrm{keV}$

β	E_1(GeV)	M_2(GeV)	E_3(GeV)	Γ(keV)
2.4	2.04(2)	−0.218(4)	0.014(3)	347 ± 20
2.8	2.08(2)	−0.171(10)	0.005(8)	352 ± 11
Cont.	2.11(2)	−0.141(15)	−0.007(12)	361 ± 9

As shown in Table 2, we obtain the following results for the partial decay width: $\Gamma(\chi_{c2} \to J/\psi\ \gamma)$=$347\pm20$ keV or 352 ± 11 keV for the two lattices respectively. The continuum extrapolation gives $\Gamma = 361\pm9\,\mathrm{keV}$. All these results can be compared with the PDG average of 384(38) keV.

2.2 $\eta_{c2} \to J/\psi\gamma$ transition

With real photons in the transition $\eta_{c2} \to \gamma J/\psi$, only three multipoles are contributing: the magnetic dipole (M_1), the electric quadrupole (E_2), and M_3. The transition width is given by,

$$\Gamma(\eta_{c2} \to \gamma J/\psi) = \frac{16\alpha|\vec{q}|}{45M^2_{\eta_{c2}}}(|M_1(0)|^2 + |E_2(0)|^2 + |M_3(0)|^2). \tag{2.6}$$

Unlike the χ_{c2} case, each multipole form factor here could be expressed as a series of $\Omega/(m_V^2 m_T^2)$ with a pre-factor $\sqrt{\Omega}/m_T$ which is absent in the E_1 dominated decay channels. In the rest frame of T (as is the case in our calculation), the expression of Ω is simplified to $\Omega = (m_T|\vec{p}_V|)^2$, such that $\Omega/(m_V^2 m_T^2) = v^2$ with $v = |\vec{p}_V|/m_V$ being the spatial velocity of V. Since v is small enough, we have used the following simplified expression of the form factors M_1, E_2, and M_3,

$$\begin{aligned} F_i(v) &= Av + Bv^3 + Cv^5 + O(v^6)(F_i \to M_1,\ E_2) \\ F_i(v) &= Bv^3 + Cv^5 + Dv^7 + O(v^9)(F_i \to M_3), \end{aligned} \tag{2.7}$$

The extracted form-factors $M_1(Q^2)$, $E_2(Q^2)$, and $M_3(Q^2)$ and the interpolations are shown in Fig 3. One can see that at $v = 0$ (corresponding to $Q^2 = -(m_T - m_V)^2 \sim 0.5\,\mathrm{GeV}^2$) the form factors M_1, E_2, and M_3 are surely consistent with zero. The fits using Eq. 2.7 are also shown as curves with jackknife error bands. The interpolated values of these form factors at $Q^2 = 0$ for both $\beta = 2.4$ and $\beta = 2.8$ are listed in Table 3, where the resultant transition widths and the corresponding continuum limits are also given. After a naive continuum extrapolation using the data from the two lattices in this work, we get the continuum results of the form factors as follows,

$$M_1 = 0.104(10)\mathrm{GeV},\ E_2 = -0.071(20)\mathrm{GeV},\ M_3 = -0.132(10)\mathrm{GeV}. \tag{2.8}$$

Applying these results to Eq. 2.6, the transition width of $\eta_{c2} \to \gamma J/\psi$ is predicted to be

$$\Gamma(\eta_{c2} \to \gamma J/\psi) = 3.8 \pm 0.9\,\mathrm{keV}. \tag{2.9}$$

There have also been several phenomenological studies on this transition, one of which is in the

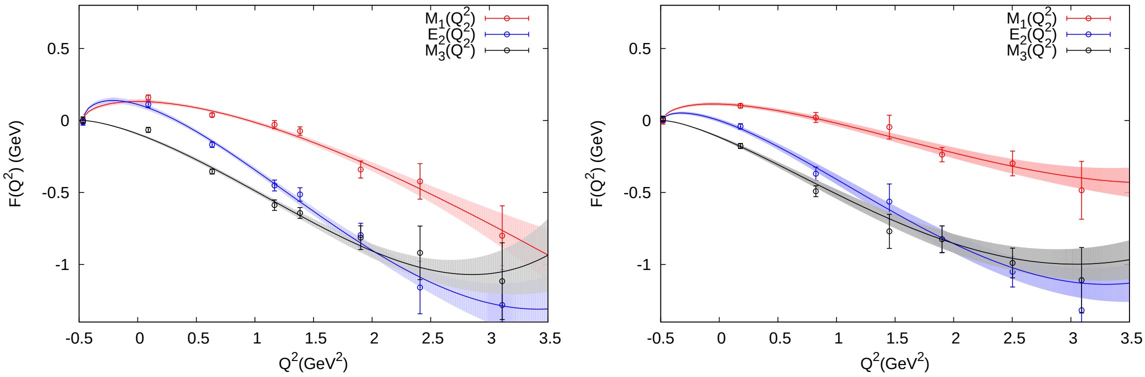

Figure 3: Plotted are the $\eta_{c2} - J/\psi$ transition form factors $M_1(Q^2)$, $E_2(Q^2)$, and $M_3(Q^2)$ versus Q^2 for the two lattices of $\beta = 2.4$ (the upper panel) and $\beta = 2.8$ (the lower panel), respectively. The points are the simulation data, and the lines illustrate the fit function with jackknife error bands.

Table 3: Listed are the interpolated values of these form factors at $Q^2 = 0$ for both $\beta = 2.4$ and $\beta = 2.8$. The resultant transition widths and the corresponding continuum limits are also given.

β	M_1(GeV)	E_2(GeV)	M_3(GeV)	Γ(keV)
2.4	0.133(13)	0.111(17)	$-0.093(9)$	4.4 ± 0.9
2.8	0.115(11)	$-0.0007(14)$	$-0.117(9)$	3.1 ± 0.6
Cont.	0.104(10)	$-0.071(20)$	$-0.132(10)$	3.8 ± 0.9

framework of light-front quark model(LFQM) [18], where the corresponding form factors as,

$$M_1 = 0.079(2)\text{GeV}, E_2 = -0.086(2)\text{GeV}, M_3 = -0.125(3)\text{GeV}, \tag{2.10}$$

which gives a width Γ=3.54(12) keV. It is seen that our lattice results and the LFQM results are in excellent agreement.

The other phenomenological study [19] applying the non-relativistic QCD (NRQCD) gives the transition width as about 4 keV, and

$$M_1 \sim 0.026 - 0.045\,\text{GeV}, E_2 \simeq M_3 \simeq -0.13\,\text{GeV}. \tag{2.11}$$

which are also in reasonable agreement with our results.

3. Conclusion

Within the quenched approximation, we study the properties of $J^{PC} = 2^{-+}$ charmonium η_{c2}. We find that its radiative transition width to J/ψ to be 3.8(9) keV, which is in agreement with the phenomenological studies. In addition, we calculate the transition width of $\chi_{c2} \to \gamma J/\psi$, and get the result 361 ± 9 keV, which is in good agreement with the experimental value of 384 ± 38 keV. Both of these facts manifest the small systematic uncertainties due to the quenched approximation and the finite lattice spacings.

Taking the branch ratio $\mathrm{Br}(X(3872) \to J/\psi\gamma) > 0.9\%$ (BaBar) or 0.6% (Belle), the full width of $X(3872)$ is estimated to be $< 440-700$ keV, which is much smaller than, but not contradicts with the experimental upper limit $\Gamma_X < 2.3\,\mathrm{MeV}$. Obviously, a reliable calculation of the partial width $\eta_{c2} \to \psi'\gamma$ is also crucial for the $^1D_2$ charmonium assignment of $X(3872)$, which of course is more challenging on the lattice.

However, we can still infer some useful information from the calculation of $\eta_{c2} \to J/\psi\ \gamma$. For the decay channel $\eta_{c2} \to \psi'\ \gamma$, the relative form factors are suppressed by several orders of kinematic factor from smaller $v = \sqrt{\Omega}/(m_V m_T)$. With this fact in mind, the η_{c2} assignment of $X(3872)$ can be ruled out if the BaBar's observation of $\mathrm{Br}(X(3872) \to \gamma\psi')/\mathrm{Br}(X(3872) \to \gamma J/\psi) = 3.4 \pm (1.4)$ is confirmed.

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