Charmed baryon spectroscopy on the physical point in 2+1 flavor lattice QCD

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We investigate the charmed baryon mass spectrum using the relativistic heavy quark action on 2+1 flavor PACS-CS configurations previously generated on $32^3 \times 64$ lattice. The dynamical up-down and strange quark masses are set to the physical values by using the technique of reweighting to shift the quark hopping parameters from the values employed in the configuration generation. At the physical point, the lattice spacing equals $a^{-1} = 2.194(10)$ GeV and the spatial extent $L = 2.88(1)$ fm. Our results for the charmed baryon masses are consistent with experiments except for $\Xi_{cc}$, which has only weak experimental evidence yet. We also predict mass values for other doubly and triply charmed baryons.
1. Introduction

Recently, a lot of new experimental reports are delivered on charmed baryons [1]. BaBar and Belle give very accurate results, and precision analysis can be accomplished. In addition, new experiments, such as J-PARC, PANDA, LHCb, and Belle II are coming and expected to give further insight into charmed baryons.

Mass spectrum of the singly charmed baryons is determined in the high precision by experiments. Experimental status of the ground state is three or four-star, evaluated by the particle data group. The excited states are also investigated fairly well.

In contrast to the singly charmed baryons, experimental data for the doubly and triply charmed baryons are not sufficient. Only one candidate for the doubly charmed baryon, \( \Xi_{cc} \), has been reported by the SELEX Collaboration [2], while \( \Xi_{cc} \) is not confirmed in the other experiments such as BaBar [3] and Belle [4] groups. Experimental and theoretical cross checks are needed to establish \( \Xi_{cc} \). The other doubly charmed baryons and the triply charmed baryon have not been found by experiment, yet. Theoretical predictions for the doubly and triply charmed baryons are helpful to discover these states.

So far, almost all lattice full QCD calculations for charmed baryon spectrum have been performed with the staggered dynamical quarks [5, 6, 7, 8]. They use some technique, such as converting the staggered propagators [9] or mixed action, for the valence light quarks to deal with the tangled flavor structure of the staggered quarks. Although these approaches have a correct continuum limit, it breaks the unitarity and complicates the continuum extrapolation. It is preferred to use the other type of dynamical quarks which is simple in flavor. Another point is that their chiral extrapolations suffer from large higher order corrections. The pion masses are limited to 220 – 290 MeV. NLO SU(2) heavy baryon chiral perturbation theory is employed to extrapolate their data to the physical point, but it shows a bad convergence even with \( m_\pi = 220 \) MeV. It is desirable to perform a simulation directly on the physical point.

The ETMC group studied the charmed baryons with \( N_f = 2 \) dynamical twisted mass quarks and Osterwalder-Seiler strange and charm valence quarks [10]. They found \( \Xi_{cc} = 3.513(23)(14) \) GeV, which agrees with the SELEX experimental value \( \Xi_{cc}^{SELEX} = 3.519(1) \) GeV. This is the only result that is consistent with the SELEX experiment. The other lattice QCD simulations show deviations from it. This disagreement in lattice QCD must be resolved. A subtle issue in the ETMC calculation is that the heavy quark mass correction may not be under control at their lattice spacings, \( a = 0.09 – 0.06 \) fm. Their results for charmed baryons, especially for the triply charmed baryon \( \Omega_{ccc} \), do not show a clear scaling behavior. It is better to employ a heavy quark action that handles mass dependent lattice artifacts in the formulation. Chiral extrapolation of ETMC data from \( m_\pi = 260 \) MeV using the NLO heavy baryon chiral perturbation theory is also problematic.

In Ref. [11], we have shown that the charm quark mass corrections are under control at \( a^{-1} = 2.194(10) \) GeV by adopting the relativistic heavy quark action of Ref. [12]. It removes the leading cutoff errors of \( O((m_Q a)^n) \) and the next to leading effects of \( O((m_Q a)^n(a A_{QCD})) \) for arbitrary order \( n \). We calculated the spectra of mesons involving charm quarks using the relativistic heavy quark action on the 2+1 dynamical flavor PACS-CS configurations of \( 32^3 \times 64 \) lattice reweighted to the physical point for up, down and strange quark masses. We found our results are consistent with experiment at a percent level, and so are those for the decay constants with a few percent accuracy.
The aggregate of 2000 MD time units were generated at the hopping parameter given by (2.1) and the Iwasaki gauge action at $\beta = 1.90$. The aggregate of 2000 MD time units were generated at the hopping parameter given by $(\kappa_{ud}, \kappa_s) = (0.13778500, 0.13660000)$, and 80 configurations at every 25 MD time units were used for measurements. We then reweight those configurations to the physical point given by $(\kappa_{ud}, \kappa_s) = (0.13779625, 0.13663375)$. The reweighting shifts the masses of $\pi$ and $K$ mesons from $m_\pi = 152(6)$ MeV and $m_K = 509(2)$ MeV to $m_\pi = 135(6)$ MeV and $m_K = 498(2)$ MeV, with the cutoff at the physical point estimated to be $a^{-1} = 2.194(10)$ GeV. Our parameters and statistics at the physical point are collected in Table 1.

The relativistic heavy quark formalism [12] is designed to reduce cutoff errors of $O((m_Qa)^n)$ with arbitrary order $n$ to $O(f(m_Qa)(a\Lambda_{QCD})^2)$, once all of the parameters in the relativistic heavy quark action are determined nonperturbatively, where $f(m_Qa)$ is an analytic function around the massless point $m_Qa = 0$. The action is given by

$$S_Q = \sum_{x,y} \overline{Q}_x D_{x,y} Q_y,$$

$$D_{x,y} = \delta_{xy} - \kappa_Q \sum_i [(r_s - \nu \gamma_i)U_{x,i} \delta_{x+1,y} + (r_s + \nu \gamma_i)U_{x,i}^\dagger \delta_{x,y+1}]$$

$$- \kappa_Q \left[ (1 - \gamma_4)U_{x,4} \delta_{x+4,y} + (1 + \gamma_4)U_{x,4}^\dagger \delta_{x,y+4} \right]$$

$$- \kappa_Q \left[ c_B \sum_{i,j} F_{ij}(x) \sigma_{ij} + c_E \sum_i F_{id}(x) \sigma_{id} \right] \delta_{xy}. \tag{2.2}$$

The parameters $r_s, c_B, c_E$ and $\nu$ have been adjusted in Ref. [11]. It should be noticed that the parameter $\nu$ is determined non-perturbatively to reproduce the relativistic dispersion relation for the spin-averaged $1S$ state of the charmonium. The heavy quark hopping parameter $\kappa_Q$ is set to reproduce the experimental value of the mass for the spin-averaged $1S$ state. Our parameters for the relativistic heavy quark action are summarized in Table 2.

We use the relativistic operators to obtain charmed baryon spectrum, because the relativistic heavy quark action is employed in our calculation. Charmed baryons can be classified under $4 \times 4 \times 3$.

Based on this result, we extend our calculations to the charmed baryon sector. The notable point is that our measurements are performed at the physical point. We are free from the convergence problem of the heavy baryon chiral perturbation theory. We check if our masses of singly charmed baryons reproduce the experimental values to confirm validity of our calculation. We also evaluate the doubly and triply charmed baryon spectrum as our predictions.

### Table 1: Simulation parameters. MD time is the number of trajectories multiplied by the trajectory length.

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$\kappa_{ud}$</th>
<th>$\kappa_s$</th>
<th># conf</th>
<th>MD time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.90</td>
<td>0.13779625</td>
<td>0.13663375</td>
<td>80</td>
<td>2000</td>
</tr>
</tbody>
</table>

2. Set up

Our calculation is based on a set of $2 + 1$ flavor dynamical lattice QCD configurations generated by the PACS-CS Collaboration [13] on a $32^3 \times 64$ lattice using the nonperturbatively $O(a)$-improved Wilson quark action with $\gamma_0^{SW} = 1.715$ [14] and the Iwasaki gauge action at $\beta = 1.90$. The parameters

$$m_D = a \kappa_D,$$

$$m_S = a \Lambda_{QCD} \kappa_S,$$


The parameters $r_s, c_B, c_E$ and $\nu$ have been adjusted in Ref. [11]. It should be noticed that the parameter $\nu$ is determined non-perturbatively to reproduce the relativistic dispersion relation for the spin-averaged $1S$ state of the charmonium. The heavy quark hopping parameter $\kappa_Q$ is set to reproduce the experimental value of the mass for the spin-averaged $1S$ state. Our parameters for the relativistic heavy quark action are summarized in Table 2.

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4 = 20 + 20_1 + 20_2 + \bar{4}. In addition to \( J = 3/2 \) decuplet-type 20-plet, there are \( J = 1/2 \) octet-type 20-plet and \( \bar{4} \)-plet.

\( J = 1/2 \) octet-type baryon operators are given by

\[
O^{fgh}_a(x) = \epsilon^{abc}((q^f_a(x))^T \Gamma q^b(x))q^c_{h1}(x),
\]

\( C = \gamma_5 \gamma_2 \),

where \( f, g, h \) are quark flavors and \( a, b, c \) are quark colors. \( \alpha = 1, 2 \) labels the \( z \)-component of the spin. The \( \Sigma \)-type and \( \Lambda \)-type are distinguished as

\[
\Sigma\text{-type} : - \frac{O^{fghg} + O^{[gh]f}}{\sqrt{2}},
\]

\[
\Lambda\text{-type} : \frac{O^{fghg} - O^{[gh]f} - 2O^{[fgh]}\gamma_5}{\sqrt{6}},
\]

where \( O^{fgh} = O^{fgh} - O^{[fgh]} \).

The decuplet-type \( J = 3/2 \) baryon operators are expressed as,

\[
D^{fgh}_{3/2}(x) = \epsilon^{abc}((q^f_a(x))^T \Gamma q^b(x))q^c_{h1}(x),
\]

\[
D^{fgh}_{1/2}(x) = \epsilon^{abc}(((q^f_a(x))^T \Gamma q^b(x))q^c_{h1}(x) - ((q^f_a(x))^T \Gamma q^b(x))q^c_{h2}(x))/3,
\]

\[
D^{fgh}_{-1/2}(x) = \epsilon^{abc}(((q^f_a(x))^T \Gamma q^b(x))q^c_{h1}(x) - ((q^f_a(x))^T \Gamma q^b(x))q^c_{h2}(x))/3,
\]

\[
D^{fgh}_{-3/2}(x) = \epsilon^{abc}((q^f_a(x))^T \Gamma q^b(x))q^c_{h2}(x),
\]

\[
\Gamma_\pm = (\gamma_1 \mp i\gamma_2)/2, \Gamma_0 = \gamma_3.
\]

The baryon correlators are calculated with exponentially smeared sources and a local sink. The smearing function is given by \( \Psi(r) = A \exp(-Br) \) at \( r \neq 0 \) and \( \Psi(0) = 1 \). We set \( A = 1.2, B = 0.07 \) for the \( ud \) quark, \( A = 1.2, B = 0.18 \) for the strange quark, and \( A = 1.2, B = 0.55 \) for the charm quark. The number of source points is octupled and polarization states are averaged to reduce statistical fluctuations. Statistical errors are analyzed by the jackknife method with a bin size of 100 MD time units (4 configurations), as in the light quark sector \([\text{I3}]\). We extract charmed baryon masses by fitting correlators with exponential functions.

### 3. Singly charmed baryon spectrum

Our results for the singly charmed baryon spectrum at the physical point are summarized in Fig. \( \text{[I]} \). All our values for the charmed baryon masses are predictions, because the physical
charm quark mass has already been fixed with the charmonium spectrum. We found the predicted spectrum is in reasonable agreement with experiment. We also compare our value for $\Lambda_c$ with other results by recent lattice QCD simulations using the dynamical staggered quarks [5, 6, 7], and the twisted mass quarks [10]. All results are consistent with each other.

Fig. 1 displays several mass differences. We have consistent results with experiments in 2 $\sigma$ accuracy. The decomposition of $J = 1/2$ $\Sigma$-type and $\Lambda$-type baryons, as well as that of $J = 3/2$ and $J = 1/2$ charmed baryons, are successful.

It is noted that several systematic errors have not been evaluated, yet. One is finite size effects. Though NLO heavy baryon chiral perturbation theory predicts that finite size effects for charmed baryons are less than 1%, higher order terms can give significant contributions. A direct lattice QCD check by comparing spectrum on different lattice volumes is desirable. Another aspect is that strong decays such as $S_c \rightarrow L_c p$ are kinematically prohibited on our lattice. Our estimates should be considered as the lower mass limits for unstable baryons. Moreover, we have not performed the continuum extrapolation. A naive order counting implies that the cutoff effects of $O(\alpha_s^2 f(m_Q a)(a\Lambda_{QCD})) f(m_Q a)(a\Lambda_{QCD})$ from the relativistic heavy quark action are at a percent level. Additional calculations are needed to remove these systematic errors.

4. Doubly and triply charmed baryon spectrum

For doubly and triply charmed baryons, an experimental value has been reported only for $X_{cc}$, although the experimental status is controversial. In the other channels, lattice QCD gives predictions to experiments.

Fig. 3 shows our results for the doubly charmed baryons. Our estimate for $m_{X_{cc}}$ clearly deviates from the experimental value by SELEX Collaboration [2]. The difference is 4$\sigma$. We compare our result for $m_{X_{cc}}$ with those by other lattice QCD calculations. We have a consistent value with other lattice QCD calculations, except for that by ETMC.

Similarly, Fig. 4 displays lattice QCD results for the triply charmed baryon from several groups. Our prediction agrees with those by others, except for ETMC value. We also plot $m_{\Omega_{ccc}}$.
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Figure 2: Comparison of mass differences of $\Sigma_c - \Lambda_c$ types (upper left panel), $\Sigma_c^* - \Sigma_c$ types (upper right panel), $\Sigma_c^* - \Lambda_c$ types (lower panel).

$\frac{1}{3} m_{J/\Psi}$ mass difference. A slight discrepancy is observed. For a more definite comparison, evaluation of systematic errors is necessary. We need to take the continuum extrapolation.

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References


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Figure 3: Our results for the doubly charmed baryon spectrum (left panel), and comparison of $\Xi_{cc}$ with other lattice QCD results (right panel).

Figure 4: Comparison of $\Omega_{ccc}$ (left panel), and $\Omega_{ccc} - 3/2 J/\psi$ (right panel).


