Lattice investigation of the tetraquark candidates $a_0(980)$ and κ

Jan Oliver Daldrop[∗], **Carsten Urbach**

Helmholtz-Institut für Strahlen und Kernphysik and Bethe Center for Theoretical Physics,
Universität Bonn, Nussallee 14-16, 53115 Bonn, Germany
E-mail: daldrop,urbach@hiskp.uni-bonn.de

Constantia Alexandrou, Mario Gravina

Department of Physics, University of Cyprus, P.O. Box 20537, 1678 Nicosia, Cyprus
Computation-based Science and Technology Research Center, Cyprus Institute,
20 Kavafi Str., Nicosia 2121, Cyprus

Mattia Dalla Brida

School of Mathematics, Trinity College Dublin, Dublin 2, Ireland

Luigi Scorzato

ECT⋆, Strada delle Tabarelle, 286, I-38123, Trento, Italy

Marc Wagner

Goethe-Universität Frankfurt am Main, Institut für Theoretische Physik,
Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany

for the European Twisted Mass Collaboration

It is a long discussed issue whether light scalar mesons have sizeable four-quark components. We present an exploratory study of this question using $N_f = 2 + 1 + 1$ twisted mass lattice QCD. A mixed action approach ignoring disconnected contributions is used to calculate correlator matrices consisting of mesonic molecule, diquark-antidiquark and two-meson interpolating operators with quantum numbers of the scalar mesons $a_0(980)$ ($1(0^{++})$) and κ ($1/2(0^+)$). The correlation matrices are analyzed by solving the generalized eigenvalue problem. The theoretically expected free two-particle scattering states are identified, while no additional low lying states are observed. We do not observe indications for bound four-quark states in the channels investigated.

[∗]Speaker.

1. Introduction

In this project, we perform first steps towards extracting the properties of the $a_0(980)$ and κ resonances from lattice QCD.

The mass and width of an infinite volume resonance can theoretically be extracted from Euclidean lattice QCD by studying the corresponding energy levels as a function of the lattice size L with an extension of Lüscher's finite volume method [1, 2, 3, 4, 5].

However, such an analysis is challenging because several energy levels have to be extracted from the lattice simulation with high accuracy. Therefore, we follow an exploratory approach: we perform a study of correlator matrices of four-quark operators in order to investigate whether bound four-quark states are observed in the $a_0(980)$ or κ channels. Recently, in such a set-up in $N_f = 2$ QCD, hints at bound tetraquark states in the σ and κ channels have been identified [6].

We compute the correlator matrices in a mixed action [7, 8] analysis of four-quark states on the lattice ignoring disconnected contributions. The goal is to answer the question whether there is a bound tetraquark or molecule state in the $a_0(980)$ or κ channel. Identifying such a state in addition to the free scattering states could be a hint on the nature of the corresponding resonance. Additionally, the overlap to different interpolating operators could be studied.

The lattice study is based on $N_f = 2 + 1 + 1$ gauge configurations generated by the European Twisted Mass (ETM) Collaboration. Details on the generation of the ensembles can be found in Ref. [9]. The analysis is performed on four ensembles with a lattice spacing of $a \approx 0.086\,\text{fm}$ and pion masses between $m_{\pi^+} \approx 280\,\text{MeV}$ and $m_{\pi^+} \approx 450\,\text{MeV}$, see Table 1.

For the computation of observables we use a twisted mass discretization for the valence s quarks, which is different from the sea s quarks to avoid the problem of mixing between s and c quarks, for details cf. Refs. [7, 8].

2. Interpolating operators and extraction of energy levels

To perform a four-quark analysis of $a_0(980)$, we compute correlators $C_{ij}(t) = <O_i(t)O_j^\dagger(0)>$ with interpolating operators

$$O_i = \sum_x \sum_{\mu=1,2,3} \left(\bar{d}(x)\Gamma_i^\mu s(x) \right) \left(\bar{s}(x)\Gamma_i^\mu u(x) \right) \text{ for } i = 1,2,3, \tag{2.1}$$

$$O_i = \sum_x \left[\bar{d}(x)\Gamma_i \bar{s}^T(x) \right]_a \left[s^T(x)\Gamma_i u(x) \right]_a \text{ for } i = 4,5, \tag{2.2}$$

$$O_6 = \sum_x \left(\bar{s}(x)\gamma_5 s(x) \right) \left(\bar{d}(x)\gamma_5 u(x) \right), \tag{2.3}$$

with the γ-matrices $\Gamma_1 = \gamma_5$, $\Gamma_2^\mu = \gamma^\mu$, $\Gamma_3^\mu = \gamma^\mu \gamma_5$, $\Gamma_4 = C\gamma_5$, $\Gamma_5 = C$, $C = \gamma_0 \gamma_2$. Due to the omission of disconnected diagrams the artificial pseudoscalar $\bar{s}\gamma_5 s$ state, which we call η_s in the following, will be relevant for the calculation. To ensure the correct identification of the $\eta_s \pi$ state, we explicitly included the operator O_6.

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ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	L/a	N_{conf}	smearing
A30.32	1.90	0.0030	0.150	0.190	32	672	APE
A40.20	1.90	0.0040	0.150	0.190	20	500	none
A40.24	1.90	0.0040	0.150	0.190	24	1259	APE
A80.24	1.90	0.0080	0.150	0.190	24	1225	APE

Table 1: The input parameters of the ensembles used in this project, the number of configurations and the smearing type employed.

For further clarification of the nature of the examined states, the two-meson operators

$$O_7 = \sum_x \left(\bar{d}(x)\gamma_5 s(x)\right) \sum_y \left(\bar{s}(y)\gamma_5 u(y)\right), \tag{2.4}$$

$$O_8 = \sum_x \left(\bar{s}(x)\gamma_5 s(x)\right) \sum_y \left(\bar{d}(y)\gamma_5 u(y)\right) \tag{2.5}$$

are studied as well (on a single ensemble).

In order to extract energy levels from the correlator matrices, we solve the generalized eigenvalue problem (GEVP) [3, 10, 11]. However, due to the presence of pairs of pseudoscalar meson states $|M_1>$ and $|M_2>$ coupling to our operators according to $<M_1|O_j|M_2> \neq 0$ problems arise: one of the two mesons can travel forward the other backward in time, severely complicating the analysis of our correlators. To avoid these problems, we restrict the analysis to $t \lesssim T/4$ and $T-t \lesssim T/4$, because the single-meson contributions are not relevant for sufficiently small t (or $T-t$). For a more detailed discussion we refer to an upcoming publication and to the Refs. [6, 12, 13]. Excited state contributions are taken into account by performing two-mass fits to the eigenvalues of interest.

3. Results

3.1 $a_0(980)$: four-quark and two-particle operators, a single ensemble

We start by discussing $a_0(980)$ ($I(J^{PC}) = 1(0^{++})$) results obtained using ensemble A40.20 (cf. Table 1). This ensemble with rather small spatial extent ($L \approx 1.72\,\text{fm}$) is particularly suited to distinguish two-particle states with relative momentum from states with two particles at rest and from possibly existing $a_0(980)$ four-quark states (the former have a rather large energy because one quantum of momentum is $p_{\min} = 2\pi/L \approx 720\,\text{MeV}$).

Figure 1a shows effective mass plots from a 2×2 correlation matrix with a $K\bar{K}$ molecule operator O_1 (see equation (2.1)) and a diquark-antidiquark operator O_4 (2.2). The corresponding two plateaus are around $1000\,\text{MeV}$ and, therefore, consistent both with a possibly existing $a_0(980)$ four-quark state and with two-particle $K + \bar{K}$ and $\eta_s + \pi$ states, where both particles are at rest.

Increasing this correlation matrix to 4×4 by adding two-particle $K + \bar{K}$ and $\eta_s + \pi$ operators (equations (2.4) and (2.5)) yields the effective mass plot shown in Figure 1b. The same two low-lying states are resolved, however, with significantly better quality. Two additional states are observed, whose plateaus are around $1500\,\text{MeV} - 2000\,\text{MeV}$. From this 4×4 analysis we conclude the following:
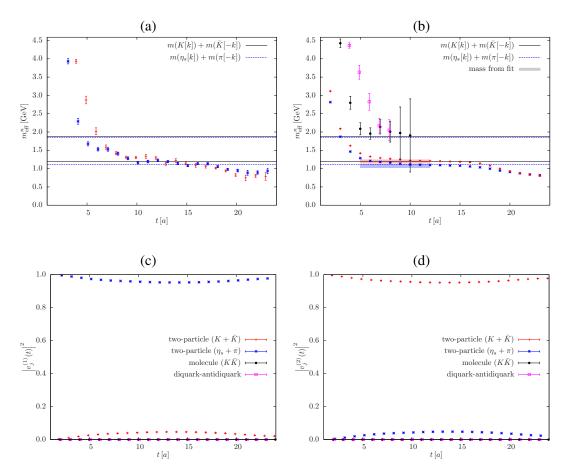
Figure 1: $a_0(980)$ sector, A40.20 ensemble. **(a)** Effective masses as functions of the temporal separation, 2×2 correlation matrix (local operators: $K\bar{K}$ molecule, diquark-antidiquark). Horizontal lines indicate the expected two-particle $K + \bar{K}$ and $\eta_s + \pi$ energy levels. **(b)** 4×4 correlation matrix (local operators: $K\bar{K}$ molecule, diquark-antidiquark, two-particle $K + \bar{K}$, two-particle $\eta + \pi$). **(c)**, **(d)** Squared eigenvector components of the two low-lying states from **(b)** as functions of the temporal separation.

1. We do not observe a third low-lying state around $1000\,\mathrm{MeV}$, even though we provide operators, which are of four-quark type as well as of two-particle type. This suggests that the two low-lying states are the expected two-particle $K + \bar{K}$ and $\eta_s + \pi$ states, while no additional stable $a_0(980)$ four-quark state does exist in the A40.20 ensemble.

2. The effective masses of the two low-lying states are of much better quality in Figure 1b than in Figure 1a. We attribute this to the two-particle $K + \bar{K}$ and $\eta_s + \pi$ operators, which presumably create larger overlap to those states than the four-quark operators. This in turn confirms the interpretation of the two low-lying states as two-particle states.

3. To investigate the overlap in a more quantitative way, we show the squared eigenvector components of the two low-lying states in Figure 1c and Figure 1d (cf. Ref. [8] for a more detailed discussion of such eigenvector components). Clearly, the lowest state is of $\eta_s + \pi$ type, whereas the second lowest state is of $K + \bar{K}$ type. On the other hand, the two four-quark operators are essentially irrelevant for resolving those states. These eigenvector plots give additional strong support of the

above interpretation of the two low lying states as two-particle states.

4. The estimated energy levels of two-particle excitations with one relative quantum of momentum are consistent with the effective mass plateaus of the second and third excitation in Figure 1b.

Figure 1a and Figure 1b also demonstrate that two-particle states can be resolved by four-quark operators, i.e. two-particle operators are not necessarily needed, to extract the full spectrum. Since we are mainly interested in possibly existing states with a strong four-quark component, we restrict the correlation matrices computed for other ensembles to four-quark operators.

3.2 $a_0(980)$: four-quark operators, many ensembles

We have analyzed the three additional ensembles listed in Table 1 with respect to $a_0(980)$ in a similar way as explained in the previous subsection. The main difference is that this time we exclusively use four-quark operators, but no two-particle operators. To be able to resolve more than two low-lying states, we use the operators O_1 to O_6.

An effective mass plot for the A30.32 ensemble (cf. Table 1) is shown in Figure 2 together with the expected energy levels of the relevant two-particle states and the masses extracted by fits. The effective mass plots for the other ensembles will be shown in an upcoming publication.

On a qualitative level our findings agree for all ensembles, i.e. are as reported in the previous subsection: there are always two low-lying states, whose masses are consistent with the expected masses of the two-particle $K + \bar{K}$ and $\eta_s + \pi$ states; higher excitations (the third, forth, etc. extracted state) are in all cases significantly heavier and consistent with two-particle excitations with one relative quantum of momentum.
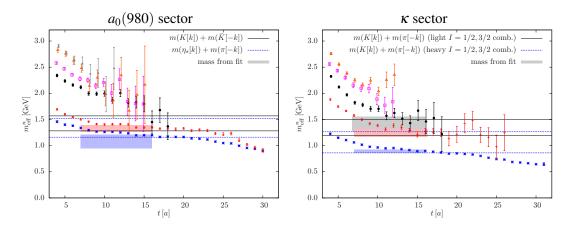

Figure 2: Effective masses as functions of the temporal separation for the $a_0(980)$ and κ sector for the A30.32 ensemble. Horizontal lines indicate the expected two-particle $K + \bar{K}$ and $\eta_s + \pi$ energy levels. Additionally, the masses extracted from fits are included.

3.3 κ: four-quark operators, many ensembles

The analysis for the κ sector ($I(J^P) = 1/2(0^+)$) closely parallels the analysis of the $a_0(980)$ sector presented above. We consider 5×5 correlation matrices containing a $K\pi$ molecule operator

analogue to O_1 and further operators corresponding to O_2 to O_5 with the appropriate valence quark content, so that our operators are essentially identical to those considered in Ref. [6].

In twisted mass lattice QCD the isospin I is not a quantum number. Therefore, it is not sufficient to only resolve $I = 1/2$ two-particle $K + \pi$ states. One has to take into account also mixing with $I = 3/2$ two-particle $K + \pi$ states, i.e. it is necessary to resolve these low-lying two-particle states at the same time (details will be discussed in an upcoming publication).

An effective mass plot for the A30.32 ensemble is shown in Figure 2 together with the expected energy levels of two-particle $K + \pi$ states and the masses extracted by fits. While effective mass plateaus are consistent with these expected two-particle energy levels, there is no indication of any additional low lying state, i.e. of a possibly existing bound four-quark κ state. While this is suggested by experimental data, it contradicts the findings of the similar recent lattice study of κ [6].

4. Summary and Outlook

We computed the low-lying spectrum in the $a_0(980)$ and κ sectors by employing trial states designed to have a substantial overlap with both two-particle and possibly existing tetraquark states. With our ensembles, we did not see additional states beside those that can be identified with the expected two-particle spectrum. The next states appear roughly consistent with excitations of the first quantum of momentum ($2\pi/L$) on top of those thresholds. This is somewhat difficult to reconcile with the additional state found in Ref. [6] in the κ channel, despite the rather similar lattice setups.

We find that the low lying spectrum has essentially exclusively overlap to two-particle trial states. This suggests that the states that we see are, indeed, the expected two-particles states at the threshold and not tightly bound states either of molecular type or diquark-antidiquark type.

These conclusions can be strengthened by studying more volumes, by introducing twisted boundary conditions [14] and by studying further trial states of different type. As for the latter, it will be crucial to combine four quarks with traditional quark-antiquark operators including disconnected diagrams. As for the volume dependence, we plan to use the finite volume formulae of Lüscher [1, 2, 3, 4, 5] and their extensions to multiple channels developed in Refs. [15, 16, 17, 18]. At present, our limited number of volumes is insufficient for such an analysis. Corresponding computations are in progress.

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