

# Neutron Electric Dipole Moment from Beyond the Standard Model

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We discuss the phenomenology of neutron Electric Dipole Moment from the Standard Model and beyond, and identify the matrix elements most necessary to connect the current and forthcoming experiments with phenomenology. We then describe lattice techniques for calculating these matrix elements.

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## 1. Introduction

The observed universe has  $6.1_{-0.2}^{+0.3} \times 10^{-10}$  baryons for every black body photon [1], whereas in a baryon symmetric universe, we expect no more than about  $10^{-20}$  baryons for every photon [2]. It is difficult to include such a large excess of baryons as an initial condition in an inflationary cosmological scenario [3]. The way out of the impasse lies in generating the baryon excess dynamically during the evolution of the universe. Sakharov wrote down a set of three necessary conditions for such a process to be possible: baryon number violation, CP and T violation, and out of equilibrium evolution of the universe [4]. Efforts at generating an observable baryon excess when these conditions are not satisfied have not been promising [5]. Since every Lorentz invariant field theory action needs to be symmetric under the product CPT [6], we use CP-violation and T-violation interchangeably and ignore the possibility of explicit Lorentz violations or spontaneous breaking of CPT.

CP is violated in the standard model (SM) of particle physics by a phase in the Cabibbo-Kobayashi-Maskawa quark mixing matrix [7], and possibly by a similar phase in the leptonic sector if the neutrinos are not massless [8]. The physical effects of these phases are suppressed by the smallness of the fermion masses. Baryon number is also violated in the SM by sphaleron effects in weak interactions [9], though the difference of baryon and lepton numbers is strictly conserved unless the neutrinos have a Majorana mass. At the temperatures above the electroweak transition, where the sphaleron rates are high, baryon and antibaryons, therefore, equilibrate. Since, however, the electroweak phase transition is weakly first order, the universe never goes out of equilibrium enough to generate the observed baryon density through SM processes [5].

In principle, the SM has an additional source of CP violation arising from the effect of QCD instantons. The presence of these finite action non-perturbative configurations in a non-abelian theory often leads to inequivalent quantum theories defined over various ‘ $\Theta$ ’-vacua [10]. Because of asymptotic freedom, all non-perturbative configurations including instantons are strongly suppressed at high temperatures where baryon number violating processes occur. Because of this, CP violation due to such vacuum effects do not lead to appreciable baryon number production.

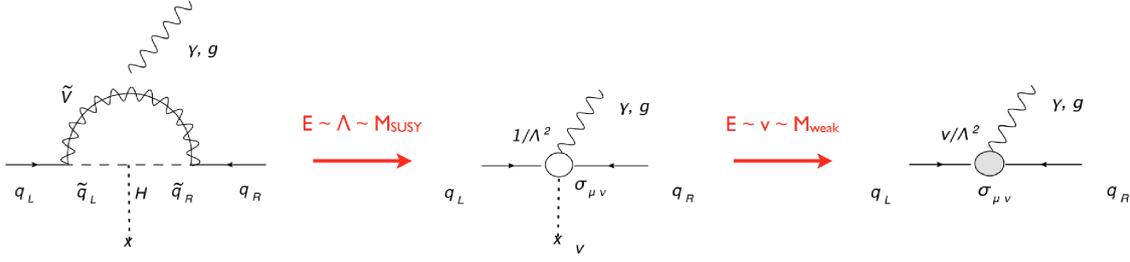
This analysis points to the need to look for CP violation from beyond the standard model (BSM). A promising experimental approach is to measure the static electric dipole moments of elementary particles, which are necessarily proportional to their spin. Since under time-reversal spin reverses sign but the electric dipole moment does not, a non-zero measurement would imply CP violation. In this work, we concentrate on the electric dipole moment of the neutron (nEDM).

## 2. Operators

In the SM, CP violation arises from (i) the CKM phase in the charged current weak interactions, and (ii) the  $\Theta$ -term multiplying the topological charge density operator in the strong interaction sector. In this section, we discuss the phenomenology of these and the leading BSM operators.

### 2.1 CKM phase

The CKM matrix describing quark-mixing under charged current interactions is arbitrary up to quark field redefinitions that leave the rest of the Lagrangian unchanged. In the SM, this can be used to rotate away any phase in the CKM matrix unless there are at least three non-degenerate



**Figure 1:** One loop diagram involving squarks and neutralinos in a SUSY model that can give rise to a electric dipole moment or a chromo-electric dipole moment to a quark. The external wavy line denotes a generic gauge boson that can attach at various places in the diagram.  $\Lambda$  represents the heavy SUSY scale where the loop effectively becomes pointlike, whereas  $v$  is the electroweak scale where the Higgs vacuum expectation value breaks the electroweak symmetry.

generations of up and down type quarks, with non-zero values for the sines and cosines of the three mixing angles [11]. Because of this, the CKM contribution to the quark electric dipole moment (qEDM) are suppressed by the quark mass differences and mixing angles, and is only  $O(10^{-34})$  e cm because of further partial cancellations between three-loop diagrams [12]. The contribution to nEDM from weak diquark interactions within the neutron have been estimated to be a 100 times larger than this [13], but is still far below experimental sensitivity of  $O(10^{-28})$  e cm.

## 2.2 Topological Charge

Even though the QCD  $\Theta$ -term does not give rise to appreciable baryon number violation, it does contribute to the electric dipole moment of baryons. The  $U(1)_A$  axial chiral anomaly, however, allows one to simultaneously change the  $\Theta$ -term and redefine the fermion fields by a chiral phase, without changing the physics. In the SM, such a rotation can remove all physical effects of a similar term in the weak  $SU(2)$  sector that involves only the left chiral sector of the theory. For a vector theory like QCD, however, all the quarks obtain masses through the Higgs mechanism, and a rotation to remove  $\Theta$  introduces phases into this mass matrix. The fermion phases are then conventionally chosen such that the masses are positive and real. The value of  $\Theta$  under this choice of phases is called  $\bar{\Theta}$ , and is phenomenologically known to be close to zero rather than  $\pi$  [14].

For small  $\bar{\Theta}$ , its contribution to nEDM has been estimated to be  $5.2 \times 10^{-16} \bar{\Theta}$  e cm in chiral perturbation theory [14]. Since the current experimental limit is  $2.9 \times 10^{-26}$  e cm [15], this implies  $|\bar{\Theta}| \lesssim 10^{-10}$ . One can explain such an unnaturally small value by the ‘Peccei-Quinn’ (PQ) construction of elevating  $\Theta$  to a dynamical field: in the SM, the minimum of the potential is at PQ field  $\bar{\Theta} = 0$ .

## 2.3 Beyond the standard model

BSM theories can be parameterized by the effective low energy operators they introduce. Electric dipole moments of quarks are particularly interesting operators since, as discussed in Sec. 2.1, these operators arise only at three-loops in SM. In BSM, they can arise at one-loop (Fig. 1) and produce nEDMs of about  $2.9 \times 10^{-26}$  e cm [15]. Also, since they violate chirality, a phase rotation

to make the masses real typically mixes the electric and magnetic dipole operators. CP violating electric dipole moments coupling to electromagnetic, weak, and strong gauge bosons are, therefore, generic unless the dipole moment sector is precisely ‘aligned’ with the mass sector.

Though formally of mass dimension 5, such chirality violating operators can appear only due to the breaking of electroweak symmetry. One, therefore, expects them to be suppressed by the ratio of the weak scale to the scale of BSM physics. Unsuppressed operators appear at dimension 6 and typically involve four fermions or three powers of the gauge field strength tensor.

### 3. Model Estimates

As explained in Sec. 2.3, after choosing the field basis to make the mass terms real and positive, the major contributions to the nEDM come from the quark electric and chromoelectric dipole moment operators in addition to topological charge effects induced by  $\Theta$ :

$$\begin{aligned} \mathcal{LCP} = & -i\bar{\Theta} \frac{g^2}{16\pi^2} G^{\mu\nu a} \tilde{G}_{\mu\nu a} \\ & + i e d_u^\gamma \bar{L} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} \frac{\tilde{H}}{\tilde{v}} U + i e d_d^\gamma \bar{L} \sigma_{\mu\nu} \gamma_5 F^{\mu\nu} \frac{H}{v} D \\ & + i g d_u^G \bar{L} \sigma_{\mu\nu} \gamma_5 \lambda^a G^{\mu\nu a} \frac{\tilde{H}}{\tilde{v}} U + i g d_d^G \bar{L} \sigma_{\mu\nu} \gamma_5 \lambda^a G^{\mu\nu a} \frac{H}{v} D + \dots, \end{aligned} \quad (3.1)$$

where  $e$  and  $g$  are the electromagnetic and strong couplings,  $F$  and  $G$  represent the electromagnetic and gluonic field strengths,  $L$  represents the left handed quark fields,  $U$  and  $D$  represent the right handed up-type and down-type quarks,  $H$  and  $\tilde{H}$  represent possibly distinct Higgs doublets whose vacuum expectation values,  $v$  and  $\tilde{v}$  (assumed real), break the electroweak symmetry, and  $d_{u,d}^{\gamma,G}$  represent the electric and chromoelectric dipole moments of the up and down quarks.

Model estimates of the effect of these terms show

$$d_n \approx \frac{8\pi^2}{M_n^3} \left[ -\frac{2m_*}{3} \frac{\partial \langle \bar{q}\sigma q \rangle_F}{\partial F} \left( \bar{\Theta} + g \frac{\langle \bar{q}G\sigma q \rangle}{2\langle \bar{q}q \rangle} \sum \frac{d_q^G}{m_q} \right) \right] \quad (3.2a)$$

$$+ \frac{\langle \bar{q}q \rangle}{3} (4d_d^\gamma - d_u^\gamma) + g \frac{\langle \bar{q}G\sigma q \rangle}{6\langle \bar{q}q \rangle} \left( 4d_d^G \frac{\partial \langle \bar{d}\sigma d \rangle_F}{\partial F} - d_u^G \frac{\partial \langle \bar{u}\sigma u \rangle_F}{\partial F} \right) \quad (3.2b)$$

$$\approx \left( \frac{4}{3} d_d^\gamma - \frac{1}{3} d_u^\gamma \right) - \frac{2e\langle \bar{q}q \rangle}{M_n f_\pi^2} \left( \frac{2}{3} d_d^G + \frac{1}{3} d_u^G \right), \quad (3.2c)$$

where  $M_n$  is the nucleon mass,  $m_*$  is the reduced light quark mass,  $f_\pi$  is the pion decay constant,  $\langle \bar{q}q \rangle$  and  $\langle \bar{q}G\sigma q \rangle$  represent the quark condensate and the mixed quark-gluon condensate, respectively, and  $\langle \bar{d}\sigma d \rangle_F$ ,  $\langle \bar{u}\sigma u \rangle_F$  and  $\langle \bar{q}\sigma q \rangle_F$  represent the up, down and average tensor light quark condensates induced by an external uniform electromagnetic field  $F$  [16]. In the simplified expression one assumes that the term in Eq. (3.2a) vanishes by the PQ mechanism. Numerically, one finds:

$$d_n(\bar{\Theta}) \approx (1 \pm 0.5) \frac{|\langle \bar{q}q \rangle|}{(225\text{MeV})^3} \bar{\Theta} (2.5 \times 10^{-16} \text{ e cm}) \quad (3.3a)$$

$$d_n(d_q^{\gamma,G}) \approx -d_n \left( \bar{\Theta} \approx \sum \frac{d_q^G/m_q(\text{MeV})}{(3.1 \times 10^{-17} \text{ e cm})} \frac{|m_0^2|}{(0.8\text{GeV})^2} \right) + \quad (3.3b)$$

$$(1 \pm 0.5) \frac{|\langle \bar{q}q \rangle|}{(225 \text{MeV})^3} [1.1 (d_d^G + 0.5 d_u^G) e + 1.4 (d_d^Y - 0.25 d_u^Y)], \quad (3.3c)$$

where  $m_0^2 \equiv \langle \bar{q}G\sigma q \rangle / \langle \bar{q}q \rangle$  and Eq. (3.3b) is a contribution that is cancelled in PQ theory because of the non-zero  $\Theta$  at the minimum of the BSM potential [16]. The quark dipole moments are often of the order of

$$\kappa_q \equiv \frac{m_q}{16\pi^2 M_\Lambda^2} = 1.3 \times 10^{-25} \text{e cm} \frac{m_q}{1 \text{MeV}} \left( \frac{1 \text{TeV}}{M_\Lambda} \right)^2, \quad (3.3d)$$

where  $M_\Lambda$  is the scale of new physics.

Rough estimates of the other dimension 6 operators, the Weinberg operator  $(w/3)f_{abc}G^{\mu\nu,a}G_\mu^{\rho,b}\tilde{G}_{\rho\nu}^c$  and the four-quark operators  $iC_{ij}(\bar{q}_i^a q_i^a)(\bar{q}_j^b \gamma_5 q_j^b)$ , are also similar [16]:

$$|d_n(w)| \approx (4.4 \times 10^{-22} \text{e cm}) \frac{w(\mu)}{(1 \text{TeV})^{-2}} \Big|_{\mu=1 \text{GeV}} \quad (3.3e)$$

$$|d_n(C)| \approx (1.2 \times 10^{-24} \text{e cm}) \frac{C_{bd}(\mu) + C_{db}(\mu)}{(1 \text{TeV})^{-2}} \Big|_{\mu=m_b}. \quad (3.3f)$$

#### 4. Lattice Methods

In this section we discuss the calculation of the contribution of three CP violating operators—the topological charge, the electric dipole moment of the quark, and the chromo-electric dipole moment of the quark—to nEDM. Since nEDM changes the energy of the neutron in an external electric field, there are two general ways of calculating it—as the energy difference between two spin states of a neutron in an external electric field, or by relating it to the CP-violating form factor  $F_3$ :

$$\langle n | J_\mu^{\text{EM}} | n \rangle \Big|_{\mathcal{CP}} = \frac{F_3(q^2)}{2M_n} \bar{n} q_\nu \sigma^{\mu\nu} \gamma_5 n \quad d_n = \lim_{q^2 \rightarrow 0} \frac{F_3(q^2)}{2M_n}, \quad (4.1)$$

where  $J_\mu^{\text{EM}}$  is the electromagnetic current,  $\bar{n}$  and  $n$  are appropriate spinors for the neutron, and  $q$  is the momentum transfer in the process. We will only consider the second method here.

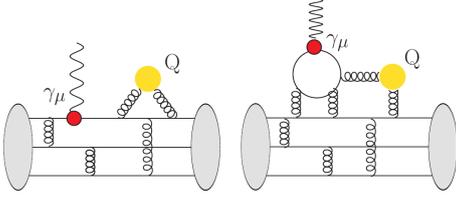
In either of these cases, we need to calculate lattice correlation functions in the presence of a CP violating operator. This is technically difficult because the CP violating operator is complex, so, in practice one needs to expand the action for small values of the CP-violation parameter.

##### 4.1 Topological Charge

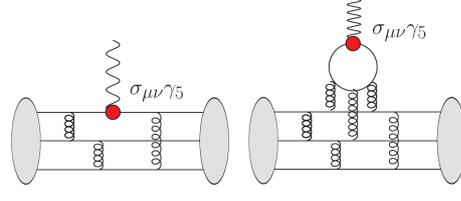
For the  $\Theta$ -term, the CP violating part of the action is  $\Theta \int d^4x G_{\mu\nu} \tilde{G}^{\mu\nu} = \Theta Q$ , where  $Q$  is the topological charge. Hence, we need to evaluate

$$\begin{aligned} \langle n | J_\mu^{\text{EM}} | n \rangle \Big|_{\mathcal{CP}} &= \Theta \left\langle n \left| \left( \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d \right) Q \right| n \right\rangle \\ &= \frac{\Theta}{2} \langle n | \bar{q} \gamma_\mu q Q | n \rangle + \frac{\Theta}{6} \langle n | \bar{q} \gamma_\mu \tau_3 q Q | n \rangle, \end{aligned} \quad (4.2)$$

where the formulae are written for a mass-degenerate two-flavour theory with a doublet ( $q$ ) consisting of up ( $u$ ) and down ( $d$ ) quark fields and all matrix elements on the rhs are calculated in a CP conserving background lattices generated with  $\Theta = 0$ . Since the topological charge is independent



**Figure 2:** The lattice matrix elements that are needed to calculate the contribution of the  $\Theta$ -term to nEDM. The circle labeled ‘Q’ represents that each diagram is to be weighted by the topological charge of the configuration.



**Figure 3:** The lattice matrix elements that are needed to calculate the contribution of the quark electric dipole moment to nEDM.

of the quark propagators, this reduces to weighted sums of three-point functions in each topological sector.

To isolate the CP-violating form factor, these matrix elements have to be calculated at non-zero momentum, and then the limit of zero momentum taken. The two lattice diagrams contributing to this process are shown in Figure 2.

#### 4.2 Quark Electric Dipole Moment

When the up and down quarks have non-zero electric dipole moments they directly give extra CP violating contributions to the electromagnetic current. As a result, the nEDM is given by

$$\begin{aligned} \langle n | J_\mu^{\text{EM}} | n \rangle_{\mathcal{CP}} &= \langle n | (d_u^\gamma \bar{u} \sigma_{\mu\nu} u + d_d^\gamma \bar{d} \sigma_{\mu\nu} d) q^\nu | n \rangle \\ &= q^\nu \frac{d_u^\gamma + d_d^\gamma}{2} \langle n | \bar{q} \sigma_{\mu\nu} q | n \rangle + q^\nu \frac{d_u^\gamma - d_d^\gamma}{2} \langle n | \bar{q} \sigma_{\mu\nu} \tau_3 q | n \rangle. \end{aligned} \quad (4.3)$$

Even though Eq. (4.3) suggests that the calculation needs injection of non-zero momentum ( $q^\nu$ ) at the operator, note that the form factor is also multiplied by the same factor in Eq. (4.1). As a result, this calculation can be performed directly at zero momentum.

The effect of the quark electric dipole moments, therefore, turn out to be related to the isoscalar and iso-vector tensor charges of the nucleon that have been studied in other contexts in the past [17]. The two lattice diagrams are shown in Figure 3.

#### 4.3 Quark Chromoelectric Dipole Moment

The contribution of the Chromoelectric dipole moments are more difficult to evaluate, since they naïvely need us to evaluate a four point function, the technology for which has not yet been tested on the lattice. We can, however, evaluate it using the Feynman-Hellmann theorem:

$$\begin{aligned} \langle n | J_\mu^{\text{EM}} | n \rangle_{\mathcal{CP}} &= \langle n | J_\mu^{\text{EM}} (d_u^G \bar{u} \sigma_{\nu\kappa} u + d_d^G \bar{d} \sigma_{\nu\kappa} d) \tilde{G}^{\nu\kappa} | n \rangle \\ &= \frac{\partial}{\partial A_\mu(E)} \Big|_{E=0} \langle n | (d_u^G \bar{u} \sigma_{\nu\kappa} u + d_d^G \bar{d} \sigma_{\nu\kappa} d) \tilde{G}^{\nu\kappa} | n \rangle_E, \end{aligned} \quad (4.4)$$

where the subscript  $E$  refers to matrix elements calculated in the presence of an external electric field  $E$ , and  $A_\mu(E)$  refers to the corresponding vector potential. Since the background electric field breaks translational invariance, no momentum needs to be explicitly introduced at the operator.

## 5. Conclusions

In this article, we have studied the calculation of the nEDM due to the leading operator contributing to CP-violation and arising from beyond the standard model physics. Prior work [18] had concentrated on the effects and renormalization of the so-called  $\Theta$ -term. Here, we show that the effect of the quark electric dipole moment is related to the tensor charge of the neutron, which has also been previously studied [17]. We also describe the technique for calculating the effect of the quark chromo-electric moment, and postpone the discussion of the renormalization of this term. Some of the four-fermion operators recently discussed in the literature [19] are also calculable with similar effort, but we do not discuss them here.

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