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Hyperon-Nucleon Interactions and the Composition of Dense Nuclear Matter

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Low-energy $n\Sigma^-$ interactions determine, in part, the role of the strange quark in dense matter. Our Lattice QCD calculations, performed at a pion mass of $m_\pi \sim 389$ MeV in two large lattice volumes, and at one lattice spacing, are extrapolated to the physical pion mass by diagonalizing the leading order finite-volume Hamiltonian in NN effective field theory. The interactions determined from QCD are consistent with those extracted from hyperon-nucleon experimental data within uncertainties.

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The interactions between hyperons and nucleons are important for understanding the composition of dense matter. In high-density baryonic systems, the large values of the Fermi energies may make it energetically advantageous for some of the nucleons to transform into hyperons via the weak interactions, with the increase in rest mass being compensated for by the decrease in combined Fermi energy of the system. This is speculated to occur in the interior of neutron stars, but a quantitative understanding of this phenomenon depends on knowledge of the hyperon-nucleon (YN) interactions in the medium. In recent work [1], we used $n\Sigma^-$ scattering phase shifts in the ${}^1S_0$ and ${}^3S_1$ spin-channels calculated with Lattice QCD (LQCD) to quantify the energy shift of the Σ^- hyperon in dense neutron matter, as might occur in the interior of a neutron star.

Existing experimental information about the YN interaction comes from the study of hypernuclei [2, 3], the analysis of associated Λ-kaon and Σ-kaon production in NN collisions near threshold [4, 7, 5, 6, 8, 9], hadronic atoms [10], and from charge-exchange production of hyperons in emulsions and pixelated scintillation devices [11]. There are only a small set of cross-section measurements of the YN processes and, as a result, the extracted scattering parameters are not accurately known. The potentials developed by the Nijmegen [12, 13] and Jülich [14, 15, 16] groups are two examples of phenomenological models based on meson exchange, with couplings fit to the available YN data. In Refs. [12, 13], for example, six different models are constructed, each describing the available YN cross-section data equally well, but predicting different values for the phase shifts. Effective field theory (EFT) descriptions have also been developed [17, 18, 19, 20, 21] and have the advantage of being model independent.

Several years ago, the NPLQCD Collaboration performed the first $n_f = 2+1$ LQCD calculations of YN interactions [22] (and NN interactions [23]) at unphysical pion masses. Quenched and dynamical calculations were subsequently performed by the HALQCD Collaboration [24] and by NPLQCD [25]. Recent work by NPLQCD [26, 27, 28] and HALQCD [29, 30] has shown that the $S = -2$ H-dibaryon is bound for pion masses larger than those of nature, and NPLQCD [28] has shown that the same is true for the $\Xi^-\Xi^-$ with $S = -4$. We have used the results of LQCD calculations to determine leading-order (LO) couplings of the YN EFT with Weinberg power counting [21] which led to a determination of YN interactions at the physical pion mass.

Lüscher's method [31, 32, 33, 34] can be employed to extract two-particle scattering amplitudes below inelastic thresholds. For a single scattering channel, the deviation of the energy eigenvalues of the two-hadron system in the lattice volume from the sum of the single-hadron masses is related to the scattering phase shift $\delta(q)$. The form of the interpolating operators and the methodology used for extracting the energy shifts are discussed in detail in Ref. [35]. By computing the masses of the particles and the ground-state energy of the two-particle system, one obtains the squared momentum q^2, which can be either positive or negative. For s-wave scattering below inelastic thresholds, q^2 is related to the real part of the inverse scattering amplitude through Lüscher's eigenvalue equation [32], enabling a LQCD determination of the value of the phase shift at the momentum $\sqrt{q^2}$.

Determining the ground-state energy of a system in multiple lattice volumes allows for bound states to be distinguished from scattering states. A bound state corresponds to a pole in the S matrix, and in the case of a single scattering channel, is signaled by $\cot\delta(q) \to +i$ in the large-volume limit. With calculations in two or more lattice volumes that both have $q^2 < 0$ and $q\cot\delta(q) < 0$ it is possible to perform an extrapolation to infinite volume to determine the binding energy of the

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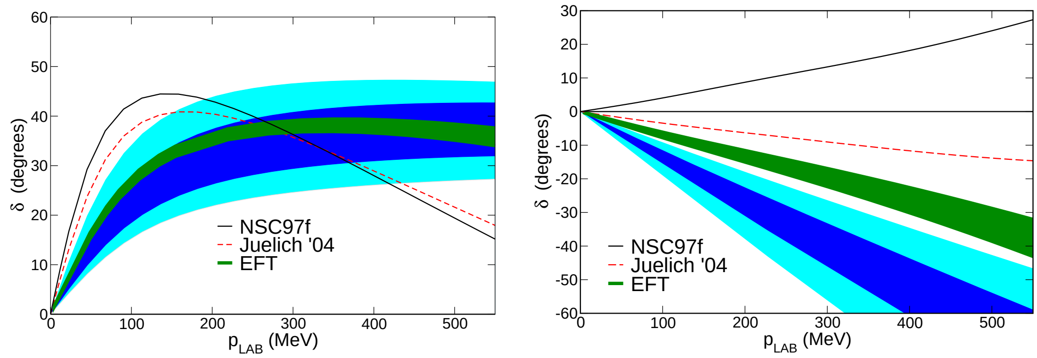

Figure 1: LQCD-predicted $^1S_0$ $n\Sigma^-$ (left panel) and $^3S_1$ $n\Sigma^-$ (right panel) phase shift versus laboratory momentum at the physical pion mass (blue bands), compared with other determinations, as discussed in the text.

bound state $B_\infty = \gamma^2/m$, where γ is the binding momentum [32, 33, 34]. The range of nuclear interactions is determined by the pion mass, and therefore the use of Lüscher's method requires that $m_\pi L \gg 1$ to strongly suppress the contributions that depend exponentially upon the volume, $e^{-m_\pi L}$ [36]. However, corrections of the form $e^{-\gamma L}$, where γ^{-1} is approximately the size of the bound state, must also be small for the infinite volume extrapolation to rapidly converge.

Our results are from calculations on two ensembles of $n_f = 2+1$ anisotropic clover gauge-field configurations [37, 38] at a pion mass of $m_\pi \sim 389$ MeV, a spatial lattice spacing of $b_s \sim 0.123(1)$ fm, an anisotropy of $\xi \sim 3.5$, and with spatial extents of 24 and 32 lattice sites, corresponding to spatial dimensions of $L \sim 3.0$ and 3.9 fm, respectively, and temporal extents of 128, and 256 lattice sites, respectively. A detailed analysis demonstrates that the single-baryon masses in these lattice ensembles are effectively in the infinite-volume limit [39], and that exponential volume corrections can be neglected in this work. Lüscher's method assumes that the continuum single-hadron energy-momentum relation is satisfied over the range of energies used in the eigenvalue equation. As discussed in Refs. [26, 28], the uncertainties in the energy-momentum relation translate to a 2% uncertainty in the determination of q^2.

We focus on $^1S_0$ and $^3S_1$ $n\Sigma^-$ interactions, $N\Sigma$ in the $I = 3/2$ channel, and do not consider the $I = 1/2$ $N\Sigma$-$N\Lambda$ coupled channels. Calculations in the $I = 1/2$ channel are complicated by the proximity in energy of the ground and first-excited levels in the finite volume. In the limit of SU(3) flavor symmetry, the $^1S_0$-channels are in symmetric irreducible representations of $\mathbf{8}\otimes\mathbf{8}$, and $n\Sigma^-$ transforms as a **27**. YN and NN scattering data along with the leading SU(3) breaking effects, arising from the light-meson and baryon masses, suggest that these channels are attractive at the physical pion mass, and that $\Xi^-\Xi^-$ and $\Sigma^-\Sigma^-$ are bound [40, 41, 42]. In contrast, the $^3S_1$-channel of $n\Sigma^-$ scattering transforms in the **10** of SU(3), and is therefore unrelated to NN interactions. Hence, this channel is quite uncertain, with disagreements among hadronic models as to whether the interaction is attractive or repulsive.

The low-energy $n\Sigma^-$ interactions can be described by an EFT of nucleons, hyperons and pseudoscalar mesons (π, K and η), constrained by chiral symmetry [17, 20, 21]. At leading order (LO) in the expansion, the $n\Sigma^-$ interaction is given by one-meson exchange together with a contact operator describing the low-energy effect of short-distance interactions. As these contact operators are independent of the light-quark masses, at LO the quark-mass dependence of the $n\Sigma^-$ interactions is dictated by the meson masses.

At $m_\pi \sim 389$ MeV, using a volume extrapolation as discussed above, we find that the ${}^1S_0$ $n\Sigma^-$ channel has a bound state, with binding energy $B = 25 \pm 9.3 \pm 11$ MeV. The quality of the LQCD data in this channel is comparable to that of its **27**-plet partner $\Xi^-\Xi^-$, analyzed in detail in Ref. [28] (see also [43]). In the EFT, the coefficient of the LO contact operator in this channel is determined by tuning it to reproduce the LQCD-determined binding energy. We find that this channel becomes unbound at $m_\pi \lesssim 300$ MeV, in agreement with Ref. [44], which constrained the LO contact operator using experimental data. In Fig. 1, we show the predicted ${}^1S_0$ $n\Sigma^-$ phase shift at the physical pion mass — (dark, light) blue bands correspond to (statistical, systematic) uncertainties — and compare with the EFT constrained by experimental data [21], the Nijmegen NSC97f model [12], and the Jülich '04 model [16]. The systematic uncertainties on our predictions include those arising from the LQCD calculation (see [43]) as well as estimates of omitted higher-order effects in the EFT.

The ${}^3S_1$-${}^3D_1$ $n\Sigma^-$ coupled channel is found to be highly repulsive in the s-wave at $m_\pi \sim 389$ MeV, requiring interactions with a hard repulsive core of extended size. Such a core, if large enough, would violate a condition required to use Lüscher's relation, namely $R \ll L/2$ where R is the range of the interaction. We have determined the EFT interactions directly by solving the 3-dimensional Schrödinger equation in finite volume to reproduce the energy levels obtained in the LQCD calculations. This is accomplished by evaluating the interaction in the momentum space defined by the plane waves in the lattice volume, and then numerically diagonalizing the Hamiltonian matrix in this basis. The repulsive core is found to be large, and formally precludes the use of Lüscher's relation, but both methods lead to phase shifts that agree within uncertainties, indicating that the exponential corrections to Lüscher's relation are small. In Fig. 1, we show the predicted ${}^3S_1$ $n\Sigma^-$ phase shift at the physical pion mass.

The $n\Sigma^-$ interactions are an important ingredient in calculations that address whether Σ^-'s appear in dense neutron matter. A result due to Fumi for the energy shift due to a static impurity in a non-interacting Fermi system [45]:

$$\Delta E \ = \ -\frac{1}{\pi\mu}\int_0^{k_f} dk\, k \left[\frac{3}{2}\delta_{{}^3S_1}(k) \ + \ \frac{1}{2}\delta_{{}^1S_0}(k) \right] , \tag{1}$$

where μ is the reduced mass in the $n\Sigma^-$ system, is used to provide an estimate of the impact of the calculated YN interactions in dense neutron matter. Using our LQCD determinations of the phase shifts, and allowing for a 30% theoretical uncertainty, the resulting energy shift and uncertainty band is shown in Fig. 2. At neutron number density $\rho_n \sim 0.4$ fm^{-3}, which may be found in the interior of neutron stars, the neutron chemical potential is $\mu_n \sim M_N + 150$ MeV due to neutron-neutron interactions, and the electron chemical potential, $\mu_{e^-} \sim 200$ MeV [46]. Therefore $\mu_n + \mu_{e^-} \sim 1290$ MeV, and consequently, if $\mu_{\Sigma^-} = M_\Sigma + \Delta E \lesssim 1290$ MeV, that is, $\Delta E \lesssim 100$ MeV, then the Σ^-, and hence the strange quark, will play a role in the dense medium. We find using Fumi's theorem that $\Delta E = 46 \pm 13 \pm 24$ MeV at $\rho_n = 0.4$ fm^{-3}. Corrections due to correlations among neutrons are difficult to estimate and will require future many-body calculations. Despite this caveat, the results shown in Fig. 2 indicate that the repulsion in the $n\Sigma^-$ system is inadequate to exclude the presence of Σ^-'s in neutron star matter, a conclusion that is consistent with previous phenomenological modeling (for a review, see Ref. [47]).

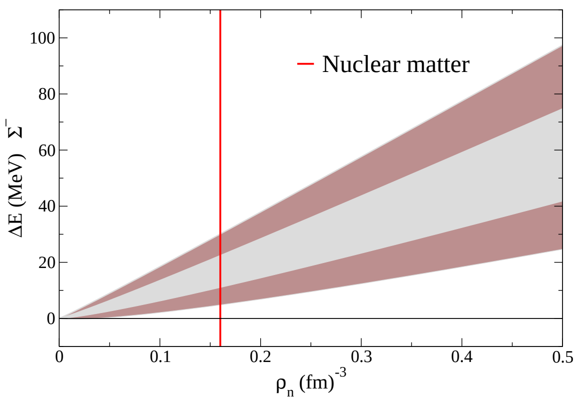

Figure 2: The energy shift versus neutron density of a single Σ^- in a non-interacting Fermi gas of neutrons as determined from Fumi's theorem in Eq. (1). The inner (outer) band encompasses statistical (systematic) uncertainties.

To conclude, we have performed the first LQCD predictions for hypernuclear physics, the ${}^1S_0$ and ${}^3S_1$ $n\Sigma^-$ scattering phase shifts shown in Fig. 1. While the calculations have been performed at a single lattice spacing, lattice-spacing artifacts are expected to be smaller than the other systematic uncertainties. We anticipate systematically refining the analysis as greater computing resources become available. The $n\Sigma^-$ interaction is critical in determining the relevance of hyperons in dense neutron matter, and we have used the LQCD predictions of the phase shifts to estimate the Σ^- energy shift in the medium. Our calculation suggests that hyperons are important degrees of freedom in dense matter, consistent with expectations based upon the available experimental data and hadronic modeling. It is important that more sophisticated many-body techniques be combined with the interactions determined in this work to obtain a more precise determination of the energy shift of the Σ^- in medium.

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