Gluon propagators in the deep IR region and non-Abelian dual superconductivity for SU(3) Yang-Mills

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We have presented non-Abelian dual superconductivity picture in the SU(3) Yang-Mills(YM) theory, and shown evidences such as the restricted U(2)-field dominance and the non-Abelian magnetic monopole dominance in the string tension. To establish the dual superconductivity picture, the dual Meissner effect in Yang-Mills theory must be examined, and we also presented the evidence of non-Abelian dual Meissner effect by measuring chromo-electric flux tube in the last lattice conferences.

In this talk, by applying a new formulation of the YM theory on a lattice, the we further investigate the non-Abelian dual Meissner effect for SU(3) YM theory through correlation function. We examine non-abelian magnetic monopole currents as well as color flux created by the quark-antiquark source.

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1. Introduction

Quark confinement follows from the area law of the Wilson loop average. The dual super conductivity is the promising mechanism for quark confinement [1]. Based on the Abelian projection, there have been many numerical analyses such as Abelian dominance [2], magnetic monopole dominance [3][4], and center vortex dominance [5] in the string tension. However, these results are obtained only in special gauges such as the maximal Abelian (MA) gauge and the Laplacian Abelian gauge, and the Abelian projection itself breaks the gauge symmetry as well as color symmetry (global symmetry).

We have presented the lattice version of a new formulation of SU(N) Yang-Mills (YM) theory [2][11], that gives the decomposition of the gauge link variable suited for extracting the dominant mode for quark confinement in the gauge independent way. In the case of the SU(2) YM theory, the decomposition of the gauge link variable is given by a compact representation of Cho-Duan-Ge-Faddeev-Niemi (CDGFN) decomposition [6] on a lattice [8][9][10]. For the SU(N) YM theory, the new formula for the decomposition of the gauge link variable is constructed as an extension of the SU(2) case. There are several possibilities of decomposition corresponding to the stability subgroup \( \tilde{H} \) of gauge symmetry group \( G \), while there is the unique option of \( \tilde{H} = U(1) \) in the SU(2). For the case of \( G = SU(3) \), there are two possibility which we call the maximal option and the minimal option. The maximal option is obtained for the stability group \( \tilde{H} = U(1) \times U(1) \), which is the gauge invariant version of the Abelian projection in the maximal Abelian (MA) gauge [12]. The minimal one is obtained for the stability group of \( \tilde{H} = U(2) \cong SU(2) \times U(1) \), which is suitable for the Wilson loop in the fundamental representation derived from the non-Abelian Stokes’ theorem [19]. For the quark-antiquark (fundamental representation of ) static potential, we have demonstrated the gauge independent (invariant) restricted \( U(2) \)-dominance, (or conventionally called “Abelian” dominance), \( (\sigma_V/\sigma_{full}) = 93 \pm 16\% \) where the decomposed \( V \)-field (restricted \( U(2) \) field) reproduced the string tension of original YM field, and the gauge independent non-Abelian magnetic monopole dominance \( (\sigma_{mon}/\sigma_V = 94 \pm 9\%) \), where the string tension was reproduced by only the (non-Abelian) magnetic monopole part extracted from the restricted \( U(2) \) field [14][13][15][16][18].

To establish the dual superconductivity picture, we must also show the magnetic monopoles play the dominant role in quark confinement. The dual Meissner effect in Yang-Mills theory must be examined by measuring the distribution of chromo-electric field strength or color flux as well as magnetic monopole currents created by a static quark-antiquark. In \( SU(2) \) case, the extracted filed corresponding to the stability group \( \tilde{H} = U(1) \) shows the dual Meissner effect [23], which is a gauge invariant version of the Abelian projection in MA gauge. In the \( SU(3) \) case, there are many works on color flux for the Yang-Mills field by using Wilson line/loop operator, e.g., [21][22][24]. However, there is no direct measurement of the dual Meissner effect in the gauge independent (invariant) way, except for several studies based on the Abelian projection, e.g., [?] By applying our new formulation to the \( SU(3) \) YM theory, we have shown the evidence of the non-Abelian dual Meissner effect claimed by us by measuring the color flux created by a static quark-antiquark pair, and found the color flux tube.

In this talk, we further study the non-Abelian dual Meissner effect and the correlation functions (propagator) for the original YM field and the decomposed variables.

2. Method

We introduce a new formulation of the lattice YM theory of the minimal option, which extracts the dominant mode of the quark confinement for \( SU(3) \) YM theory [18][16], since we consider the quark confinement in the fundamental representation. Let \( U_{x,\mu} = X_{x,\mu}V_{x,\mu} \) be the decomposition of YM link variable,
where $V_{x,\mu}$ could be the dominant mode for quark confinement, and $X_{x,\mu}$ the remainder part. The YM field and the decomposed new-variables are transformed by full SU(3) gauge transformation $\Omega_x$ such that $V_{x,\mu}$ is transformed as the gauge link variable and $X_{x,\mu}$ as the site available:

$$U_{x,\mu} \rightarrow U'_{x,\mu} = \Omega_x U_{x,\mu} \Omega_{x+\mu}^\dagger, \quad (2.1a)$$
$$V_{x,\mu} \rightarrow V'_{x,\mu} = \Omega_x V_{x,\mu} \Omega_{x+\mu}^\dagger, \quad (2.1b)$$

The decomposition is given by solving the defining equation:

$$D_\mu^g[V]h_x := \frac{1}{g} \left[ V_{x,\mu} h_{x+\mu} - h_x V_{x,\mu} \right] = 0, \quad (2.2a)$$
$$g_x := e^{2\pi q_x/N} \exp(-ia_x h_x - i\sum_{j=1}^N a_x^{(j)} u_x^{(j)}) = 1, \quad (2.2b)$$

where $h_x$ is an introduced color field $h_x = \xi (\lambda^8 / 2) \xi^\dagger \in [SU(3)/U(2)]$ with $\lambda^8$ being the Gell-Mann matrix and $\xi$ the SU(3) gauge element. The variable $g_x$ is undetermined parameter from Eq.(2.2a), $u_x^{(j)}$’s are $su(2)$-Lie algebra values, and $q_x$ an integer value $0, 1, \cdots, N - 1$. These defining equations can be solved exactly [17], and the solution is given by

$$X_{x,\mu} = \tilde{L}_{x,\mu} \det(\tilde{L}_{x,\mu})^{1/N} g_x^{-1}, \quad V_{x,\mu} = X_{x,\mu}^\dagger U_{x,\mu} = g_x \tilde{L}_{x,\mu} U_{x,\mu}, \quad (2.3a)$$
$$\tilde{L}_{x,\mu} = \left( L_{x,\mu} L_{x,\mu}^\dagger \right)^{-1/2} L_{x,\mu}, \quad (2.3b)$$
$$L_{x,\mu} = \frac{N^2 - 2N + 2}{N} \mathbf{1} + (N - 2) \sqrt{\frac{2(N-1)}{N}} (h_x + U_{x,\mu} h_{x+\mu} U_{x,\mu}^\dagger) + 4(N-1) h_x U_{x,\mu} h_{x+\mu} U_{x,\mu}^\dagger. \quad (2.3c)$$

Note that the above defining equations correspond to the continuum version: $D_\mu[Y]h(x) = 0$ and $\text{tr}(h(x) \mathcal{D}_\mu(x)) = 0$, respectively. In the naive continuum limit, we have the corresponding decomposition $A_\mu(x) = V_\mu(x) + X_\mu(x)$ in the continuum theory[11] as

$$V_\mu(x) = A_\mu(x) - \frac{2(N-1)}{N} \left[ h(x), [h(x), A_\mu(x)] \right] - \frac{2(N-1)}{N} \left[ \partial_\mu h(x), h(x) \right], \quad (2.4a)$$
$$X_\mu(x) = \frac{2(N-1)}{N} \left[ h(x), [h(x), A_\mu(x)] \right] + i\frac{1}{2} \frac{2(N-1)}{N} \left[ \partial_\mu h(x), h(x) \right]. \quad (2.4b)$$

The decomposition is uniquely obtained as the solution of Eqs.(2.3), if color fields $\{h_x\}$ are obtained. To determine the configuration of color fields, we use the reduction condition which makes the theory written by new variables $(X_{x,\mu}, V_{x,\mu})$ equipollent to the original YM theory. Here, we use the reduction function

$$F_{\text{red}}[h_x] = \sum_{x,\mu} \text{tr} \left\{ (D_\mu^g[U_{x,\mu}] h_x) \cdot (D_\mu^g[U_{x,\mu}] h_x) \right\}, \quad (2.5)$$

and color fields $\{h_x\}$ are obtained by minimizing the functional. It should be noticed that the gauge invariant magnetic monopole $k_\mu$ is defined by using $V$-field:

$$\Theta_{\mu \nu}^g := - \text{arg} \left[ \left( \frac{1}{3} \mathbf{1} - \frac{2}{\sqrt{3}} h_x \right) V_{x,\mu} V_{x+\nu,\mu} V_{x+\nu,\nu}^\dagger \right], \quad (2.6a)$$
$$k_\mu = 2\pi n_\mu := \frac{1}{2} \epsilon_{\mu \nu \lambda} \partial_\nu \Theta_{\lambda \beta}^g, \quad (2.6b)$$

which are derived from the non-Abelian Stokes’ theorem [19,20] and the Hodge decomposition of the field strength $\mathcal{F}_\mu[V]$. It should be also noticed that, this magnetic current (monopole) is non-Abelian magnetic monopole, which is defined by $V$ field corresponding to the stability group $\tilde{H} = U(2)$. 

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Figure 1: (Left) The connected correlator \((U_pLWL)^\dagger\) between a plaquette and the Wilson loop. (Right) Measurement of \(E_z\) component of the color flux in the y-z plane for the restricted \(U(2)\) field.

Then, to investigate the color flux, we use the gauge invariant correlation function. The color flux created by a quark-antiquark pair is measured by using gauge invariant connected correlator of the Wilson loop \(25\):

\[
\rho_W := \frac{\langle \text{tr}(U_pLWL) \rangle}{\langle \text{tr}(W) \rangle} - \frac{1}{N} \frac{\langle \text{tr}(U_p) \text{tr}(W) \rangle}{\langle \text{tr}(W) \rangle},
\]

where \(W\) represents a pair of quark and antiquark settled by Wilson loop in Z-T plain, \(U_p\) a plaquette variable as the probe operator to measure field strength, and \(L\) the Wilson line connecting the source \(W\) and probe \(U_p\), and \(N\) the number of color \((N = 3)\) (see the right panel of Fig. 1). The symbol \(\langle \mathcal{O} \rangle\) denotes the average of the operator \(\mathcal{O}\) in the space and the ensemble of the configurations. Note that this is sensitive to the field strength rather than the disconnected one. Indeed, in the naive continuum limit, the connected correlator \(\rho_W\) is given by \(\rho_W \approx g^2 \langle F_{\mu\nu} \rangle q\bar{q} = \frac{\langle \text{tr}(gL^2F_{\mu\nu}LWL) \rangle}{\langle \text{tr}(W) \rangle} + O(e^4)\). Thus, the color filed strength is given by \(F_{\mu\nu} = \sqrt{\frac{\beta}{2N}} \rho_W\).

3. Result

We generate YM gauge configurations using the standard Wilson action on a \(24^4\) lattice with \(\beta = 6.2\). The gauge link decomposition is obtained by the formula given in the previous section, i.e., the color field configuration is obtained by solving the reduction condition of minimizing the functional eq(2.5) for each gauge configuration, and then the decomposition is obtained by using the formula eq(2.3). In the measurement of the Wilson loop, we apply the APE smearing technique to reduce noises. To investigate the non-Abelian dual Meissner effect as the mechanism of quark confinement, we measure correlators of the restricted \(U(2)\) field, \(V_{x,\mu}\), as well as the original YM field. Note again that this restricted \(U(2)\)-field and the non-Abelian magnetic monopole extracted from it reproduce the string tension in the quark–antiquark potential \([16][18]\).

The right panel of Fig. 1 shows the result of measurements of the color flux obtained from the restricted \(U(2)\)-field, where the gauge link variable \(V_{x,\mu}\) is used in eq(2.7) instead of \(U_{x,\mu}\). Here the quark and antiquark source is introduce as \(8 \times 8\) Wilson loop \((W)\) in the Z-T plane, and the probe \((U_p)\) is set at the center of Wilson loop and moved along the Y-direction. The \(E_z\) component only has non-zero value as well as the original YM field, and it decreases quickly as away from the Wilson loop. To know the shape of the color flux in detail, we explore the distribution of color flux in the 2-dimensional plane. The right panel of Fig. 2 shows the measurement of \(E_z\) component of the color flux, where the quark-antiquark source as \(9 \times 11\) Wilson loop
where $F$ denotes the field strength 2-form of the Yang-Mills field, $d$ the exterior derivative and $^*$ denotes the Hodge dual. Note that non-zero magnetic current is a signal of the monopole condensation because of violation of the Bianchi identity. (Since the field strength is given by the exterior derivative of the Yang-Mills field, the magnetic monopole (current) can be calculated as

$$k = ^* dF[V],$$

(3.1)

where $F[V]$ is the field strength 2-form of the $V_\mu$ field, $d$ the exterior derivative and $^*$ denotes the Hodge dual.

Therefore, we observed the monopole condensation.

Finally, we devote to study the 2-point correlation functions (propagators) of the new variables and the original Yang-Mills field which are defined by

$$D_{0\alpha}(x-y) := \langle O^\alpha_\mu(x) O^\alpha_\mu(y) \rangle \text{ for } O^\alpha_\mu(y) \in \{V_{\nu,\mu}, X_{\nu,\mu}, A_{\nu,\mu}\},$$

(3.2)

where an operator $O^\alpha_\mu(x)$ is defined by linear type, e.g., $A_{\nu,\mu} := (U_{\nu,\mu} - U^\dagger_{\nu,\mu})/2g\varepsilon$. Fig[4] shows the logarithmic plot of the scaled propagators $r^{-3/2}D_{0\alpha}(r)$ with $r = |x-y|$ in lattice Landau gauge (LLG), where the correlaton function is in the unit if string tension $\sigma_{\text{phys}}$, and data of lattice spacing is from the TABLE I in Ref.[27]. The correlation function $D_{VV}$ damps slowly and has almost the same damping as $D_{AA}$, while the $D_{XX}$ damps quickly. Thus, from the viewpoint of the propagator, the $V$-field plays the dominant role in the deep IR region.
Here, we estimate the mass of the propagator $D_{OO}(r)$ by using the massive propagator in the Euclidian space, i.e., the Fourier transformation of behaves for large $M_X$

$$D_{OO}(r) = \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{3}{k^2 + M_O^2} \simeq \frac{3\sqrt{M_O}}{(2\pi)^{3/2}} e^{-M_Or} r^{-3/2} \text{ (M_Or} \gg 1), \quad (3.3)$$

and hence the scaled propagator $r^{3/2}D_{OO}(r)$ should be proportional to $\exp(-M_Or)$. For parameter fitting of $M_O$ for $O = \{V_{\nu,\mu}, A_{\nu,\mu}\}$, we use data in the region $[2,4,5]$ and eliminated near the midpoint of the lattice to eliminate the finite volume effect, and for $O = X_{\nu,\mu}$ we use the region $[1,4]$. When we use $\sigma_{\text{phys}} = 440MeV$, the preliminary result shows

$$M_A \approx 0.76GeV, \quad M_V \simeq 0.73GeV, \quad M_X \simeq 1.15GeV, \quad (3.4)$$

which should be compared with result of the maximal case$[28]$ in LLG, and also result of the Abelian projection in the maximal Abelian gauge$[29]$.  

4. Summary and outlook

We have studied the dual superconductivity for $SU(3)$ YM theory by using our new formulation of YM theory on a lattice. We have extracted the restricted $U(2)$ field from the YM field which plays a dominant role in confinement of quark (fermion in the fundamental representation), i.e., the restricted $U(2)$ dominance and the non-Abelian magnetic monopole dominance in the string tension, and measured the color flux for both the original YM field and the restricted $U(2)$ field to confirm the non-Abelian dual superconductivity picture. We have observed the dual Messier effect of $SU(3)$ YM theory, i.e., the flux tube, magnetic monopoles created by quark and antiquark pair. It is important to determine the dual superconductivity, i.e., type I or type II, which should be compared with border of type I and II to the SU(2) YM theory.

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References


