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The string tension for Large N gauge theory from smeared Wilson loops

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Using smeared Creutz ratios we extract the string tension for SU(N) pure gauge theory and N=3,4,5,6,8. We employ these results to extrapolate to large N. The same methodology is applied to the single-site Twisted Eguchi Kawai model. The corresponding string tension matches perfectly within errors with the extrapolated one, providing strong evidence in favour of the twisted reduction framework. Interesting results are also obtained on the behaviour of Creutz ratios for large sizes.

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1. Introduction

Large N gauge theories are very interesting theoretical models. They have simpler properties than their finite N counterparts. At the perturbative level only planar diagrams contribute. At the non-perturbative level one has factorization, validity of the quenched approximation, stable resonances, OZI rule, etc. Furthermore, the connection with string theory is also simpler in the large N limit. On the other hand, these theories are expected to be confining and have a rich meson spectrum. Thus, it is very atractive to investigate them using lattice gauge theory methods (for a recent review see Ref. [1]). The problem is that in this context their simple character seems to be lost. Indeed, the standard pathway to obtain predictions for these theories is by extrapolating the results obtained for finite N. One possible simplification was found by Eguchi and Kawai [2]. By examining the Migdal Makeenko equations for Wilson loops, they concluded that, if the Z_N^4 symmetry of the theory remains unbroken, the large N pure gauge theory on the lattice is volume independent. Bringing this idea to the extreme they proposed a *reduced* single-site formulation called the Eguchi-Kawai model. Unfortunately, it was soon realized that in the EK model the symmetry is broken at weak coupling [3].

Since then, the question has been whether it is possible to make the reduction idea survive in the continuum limit. Very soon simple modifications were proposed [3]-[4] designed to restore the Z_N^4 symmetry of the EK model. Recently new proposals have been added to the list [5]-[6]. Alternatively, Narajanan and Neuberger [7] proposed a mixed procedure called *partial reduction*, and suggested that presumably reduction only operates beyond the deconfinement scale.

Our goal is to test whether the twisted reduction proposal, introduced by the present authors [4], is indeed capable of realizing the reduction idea in the continuum limit. This is based on the Twisted Eguchi Kawai model (TEK), a single site model whose action is given by

$$S = bN \sum_{\mu \neq \nu} \mathrm{Tr}\left(z_{\mu\nu} U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger\right) \tag{1.1}$$

where U_μ are SU(N) matrices, b is the inverse 't Hooft coupling and $z_{\mu\nu}$ are elements of the center Z_N. The choice of $z_{\mu\nu}$ is crucial for determining the behaviour of the system in the weak coupling limit (large b). Choosing $z = 1$ gives the original EK model, which has an infinite degeneracy at the classical level [8]. Although the action is invariant under Z_N^4 symmetry ($U_\mu \longrightarrow z_\mu U_\mu$), the quantum corrections favour symmetry-breaking minima.

Other choices of $z_{\mu\nu}$ lift the degeneracy of the ground state and the classical vacuum becomes invariant under a subgroup of Z_N^4. A particularly elegant choice is termed *symmetric twist* and given by ($\nu > \mu$)

$$z_{\mu\nu} = z_{\nu\mu}^* = \exp\{2\pi i \frac{k}{L}\} \tag{1.2}$$

where $N = L^2$ and k is an integer defined modulo L. With this choice, the classical vacuum is invariant under a Z_L^4 subgroup of the Z_N^4. In the large L limit (large N) this is enough to secure the reduction idea at weak coupling. Perturbative expansion around this vacuum gives rise to modified Feynman rules. The propagators become just those of a lattice field theory at finite volume L^4. This is an important information showing how the N^2 degrees of freedom of the group map fully onto effective spatial degrees of freedom. It also suggests what are the dominant finite N

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corrections to reduction. The remaining Feynman rules introduce momentum dependence at the vertices, providing a discretized version of *non-commutative field theory*.

Most of the early studies of the TEK model were done for $k = 1$ and showed that the Z_L^4 symmetry remained unbroken at intermediate couplings. A few years ago it was found [9]-[10]-[11] that at intermediate couplings the symmetry breaks down for $N>N_c$=100. The critical value of N depends on k roughly as $N_c(k) \sim 90k^2$. Thus, in Ref. [12] we proposed that the correct large N limit has to be taken keeping k/L approximately fixed. Our goal is to test this idea and use the reduced model to obtain physical information from the large N continuum gauge theory at infinite volume. We are currently involved in this task and the present talk reports part of these results.

It must be said that we have been able to simulate the model up to N=1369 without finding any indication of symmetry breaking. Notice that this amounts to effective lattice sizes of 37^4 which are normally considered large enough in ordinary lattice gauge theory simulations. To try to understand the interplay between finite N and finite volume in a more detailed form we have analysed in certain detail the 3 dimensional version of the theory. Some results from this work, done in collaboration with Margarita García Pérez, has been reported by her at this conference [13]. The results support our claims that, if the flux k is chosen judiciously, the physical results essentially depend on Nl, the product of torus linear size l and group rank. This is indeed consistent with the volume independence achieved at infinite N.

The present work reports another form of evidence in favour of the twisted reduction idea. We set ourselves the goal of computing the continuum string tension at large N using the reduced model. To make the comparison free of systematics associated with the methodology, we decided to use exactly the same procedure to extract the string tension for ordinary SU(N) lattice gauge theory, and then extrapolate these results to infinite N. The main results have appeared [14] recently. Here we will show our updated results which include those for SU(4).

2. Numerical results on the string tension

The basic data is made up several independent lattice gauge theory simulations for different values of N and lattice gauge coupling b and a spatial volume of 32^4. Each simulation contains 260 configurations separated by 100 sweeps formed by 1 heat-bath and 5 overrelaxation updates, after discarding 4000 sweeps to reach thermalization. We simulated the SU(N) gauge groups for N=3,4, 5, 6 and 8. For each group we made independent simulations at different values of b (7 for N=3 and 4, and 5 for N=5,6,8). This huge information can be used to determine the string tension and to extrapolate to the continuum limit.

In addition, we made simulations of the TEK model for $N = 29^2 = 841$, $k = 9$ and 7 values of b. The number of configurations in that case was around 6000 with similar updating procedure and separation of sweeps.

In all cases we computed $T \times R$ rectangular Wilson loops $W(T,R)$ computed from smeared links, and from them we extracted the Creutz ratios defined as

$$\chi(T,R) = -\log \frac{W(T+0.5,R+0.5)W(T-0.5,R-0.5)}{W(T+0.5,R-0.5)W(T-0.5,R+0.5)} \tag{2.1}$$

Errors were computed by jack-knife. Details of the procedure will be given elsewhere [14]-[17]. Here we will focus upon the results.

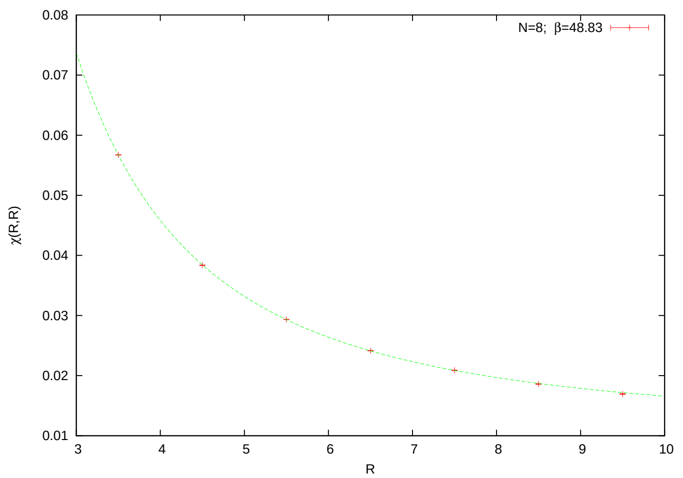

Figure 1: $\chi(R,R)$ compared to the best fit Eq. (2.2) for N=8 and b=0.3815

Our goal is to obtain the continuum string tension for all gauge groups. This is a two step procedure in which one first obtains the lattice string tension as $\kappa = \lim_{R,T\longrightarrow\infty} \chi(T,R)$, and then one makes a scaling analysis of the results to take the continuum limit. The first part can be done by fitting the square Creutz ratio data (R=T) to the formula

$$\chi(R,R) = \kappa + \frac{2\gamma}{R^2} + \frac{\eta}{R^4} \tag{2.2}$$

Different ranges have been examined, and order 1 chi squares per degree of freedom are achieved using the data $R = T \in [3.5, 8.5]$. As an example, we show in Fig. 1 one such fit.

Once the different values of $\kappa(N,b)$ are extracted from the fits, one must perform a scaling analysis. This is done by taking the limit

$$\sigma(N) = \lim_{a(N,b)\longrightarrow 0} \frac{\kappa(N,b)}{a^2(N,b)} \tag{2.3}$$

where $a(N,b)$ gives the lattice spacing (in certain units) as a function of the coupling b. Different choices of $a(N,b)$ correspond to different renormalization schemes. A good choice makes the approach to the $a = 0$ limit smoother. We have tried several expressions found in the literature for $a(N,b)$. In all cases, the approach seems to be linear in a^2 with slopes depending on the scheme and on the rank of the group N. A class of schemes uses perturbation theory and has the advantage of expressing the lattice spacing in units of $\Lambda_{\overline{\mathrm{MS}}}$. We choose here the choice taken in Ref. [15] which has been tested and used for similar purposes as ours.

Finally, the resulting string tensions $\sigma(N)$ are studied as a function of N and extrapolated to large N. In Fig. 2 we display the ratio $\Lambda_{\overline{\mathrm{MS}}}/\sqrt{\sigma(N)}$ as a function of $1/N^2$. The data shows a nice linear pattern that allows an extrapolation to infinite N. The best fit value gives $\Lambda_{\overline{\mathrm{MS}}}/\sqrt{\sigma(\infty)} = 0.525(2)$. The result obtained from the TEK model is $\Lambda_{\overline{\mathrm{MS}}}/\sqrt{\sigma(\infty)} = 0.523(5)$. The agreement is very good and provides a nice test that reduction is operative in the continuum within the confinement regime of the theory. The $1/N^2$ dependence displayed by the data matches perfectly with the result obtained in Ref. [15]. Our estimate of $\sigma(\infty)$, however, disagrees with the one given in

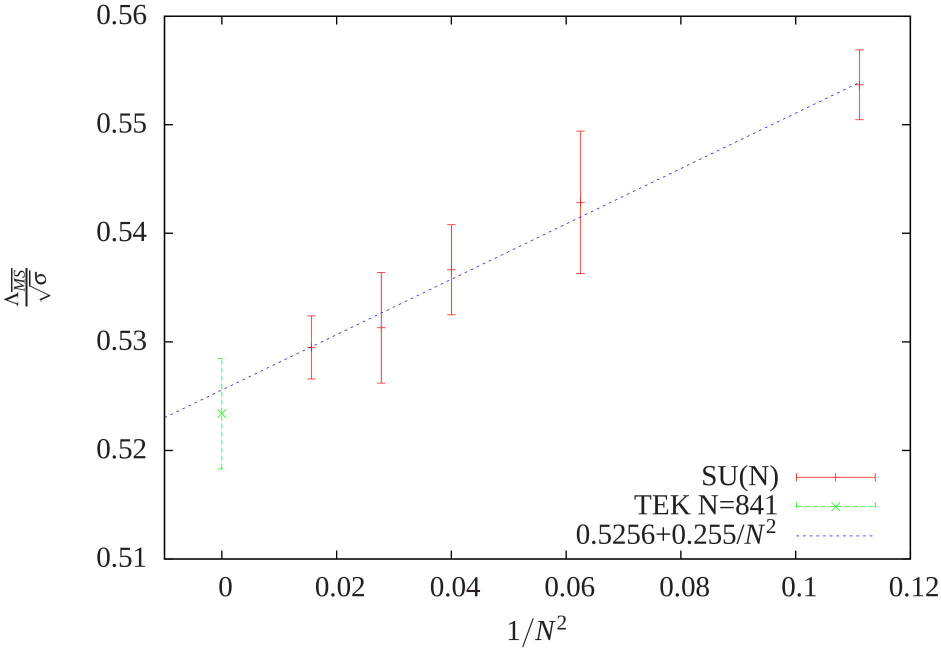

Figure 2: $\Lambda_{\overline{\rm MS}}/\sqrt{\sigma}$ as a funtion of $1/N^2$

that reference within statistical errors, although not if the systematic errors given in Ref. [15] are taken into account. A new estimate of the large N string tension has appeared recently [16]. The methodology is also based upon smeared Wilson loops but the volumes and values of N used sit in between our conventional and fully reduced model. It is interesting to point out that their result is in agreement with ours and differs from the best estimate of Ref. [15].

We believe that the main source of systematic error in our estimate of $\Lambda_{\overline{\rm MS}}/\sqrt{\sigma(\infty)}$ is indeed sitting in the scale itself. For example, using other perturbative schemes one gets differences of the order of two percent. For that purpose it is better to use a non-perturbative renormalization scheme based on the data itself. This can be achieved by inverting the order of the limits $R \longrightarrow \infty$ and $a \longrightarrow 0$. That means that we will take the continuum limit of the Creutz ratios. This is possible since Creutz ratios are free from corner and perimeter divergences. Notice, however, that Creutz ratios have intrinsic scaling violations due to its mere definition. Thus, in the continuum limit one should have

$$\chi(T,R) = a^2(b)\tilde{F}(t,r) + a^4(b)\tilde{H}(t,r) + \dots \tag{2.4}$$

where $t = Ta(b)$, $r = Ra(b)$ and $\tilde{F}(t,r)$ and $\tilde{H}(t,r)$ are well-defined continuum functions. Indeed, $\tilde{F}(t,r)$ is given by

$$\tilde{F}(t,r) = -\frac{\partial^2 \log(\mathscr{W}(t,r))}{\partial r \partial t} = \sigma + \gamma(r/t)\left(\frac{1}{r^2} + \frac{1}{t^2}\right) + \dots \tag{2.5}$$

where $\mathscr{W}(t,r)$ is the continuum Wilson loop (divergences drop from the formula). The expression on the right-hand side is the asymptotic prediction from an effective string theory description of the

Wilson loop expectation value. Indeed, the Nambu-Goto theory predicts

$$\gamma_{NG}(z) = -\frac{1}{1+1/z^2}\frac{\partial}{\partial z}\left(z\frac{\partial \log(\eta(iz))}{\partial z}\right) \tag{2.6}$$

where $\eta(iz)$ is the Dedeking eta function. Indeed, perturbation theory is also compatible with the form given in Eq. 2.5, with $\sigma = 0$ and

$$\gamma_{PT}(z) = \frac{(N^2-1)g^2}{4\pi^2 N}\frac{1+z\mathrm{atan}(z)+\mathrm{atan}(1/z)/z}{z+1/z} \tag{2.7}$$

The continuum function $\tilde{F}(t,r)$ can be used to fix a physical scale $\bar{r}$ as follows:

$$\bar{r}^2\tilde{F}(\bar{r},\bar{r}) = 1.65 \tag{2.8}$$

The choice of 1.65 is conventional and suggested by the analogy between our scheme and the so-called Sommer scale.

Once the scale is fixed, one can use the data to obtain a determination of $\tilde{F}(r,t)$ for each N using all the Creutz ratios for all values of R, T and b. In particular, our result for $\bar{r}^2\tilde{F}(r,r)$ and N=8 is given in Fig. 3. The data are very well fitted to by a second degree polynomial in $\frac{\bar{r}^2}{r^2}$ with a fairly small quadratic term. Similar behaviour applies for other values of N and for the TEK model. From the data we estimate $\sigma(\infty)\bar{r}^2 = 1.105(10)$ and $\gamma(1) = 0.272(5)$. The latter value differs from the prediction of Nambu-Goto theory $\gamma_{NG}(1) = 0.16\ldots$. Similarly, we analysed the behaviour of $\gamma(z)$ for z close to one. The data is well described by a parametrization $\gamma(z) = \gamma(1)(1+\tau\frac{(z-1)^2}{2z})$. with $\tau = 0.31(6)$. This value is fairly close to perturbative result 0.39 obtained from $\gamma_{PT}(z)/\gamma_{PT}(1)$, and differs significantly from the value ~ 2 obtained from γ_{NG}. The result might indicate that one has to supplement the string contribution with one coming from one gluon exchange.

3. Conclusions

The main conclusions of our work are:

- We have presented a new method to determine the string tension from smeared Creutz ratios.
- The results scale smoothly to the continuum limit.
- The continuum string tension extrapolates linearly in $1/N^2$ towards the large N limit. The result matches with that obtained from the TEK reduced model. This gives a strong support for the validity of continuum reduction.
- The data satisfy nice scaling properties, that allow the reconstruction of a finite continuum function of Wilson loops, called $\tilde{F}(r,t)$.
- The function gives rise to the definition of a new non-perturbative renormalization scheme, and accompanying scale $\bar{r}$.
- The subleading behaviour of $\tilde{F}(r,t)$ differs from the prediction of Nambu-Goto theory.

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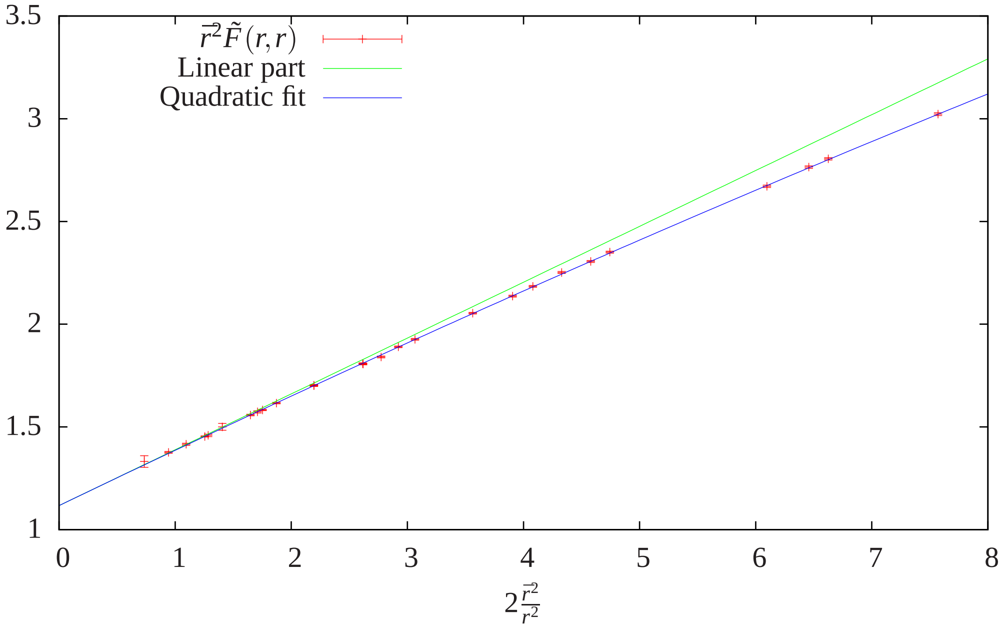

Figure 3: The continuum function $\vec{r}^2\tilde{F}(r,r)$ for N=8

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