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Study of the low temperature and high density states by using lattice QCD simulations

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We report our recent studies of the finite density and low temperature QCD in lattice QCD simulations with clover-improved Wilson fermions of $N_f = 2$ flavors and RG-improved gauge action. We apply the analytic continuation with imaginary chemical potentials to study the Polyakov loop at low temperatures.

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1. Introduction

It is of prime interest to understand QCD at finite temperature and density. Lattice QCD simulations with non-zero quark chemical potentials suffer from a sign problem, where the determinant of a quark matrix becomes complex and Monte Carlo simulations become unavailable. This makes it challenge to study QCD at finite densities. Several approaches have been studied to overcome or to circumvent the sign problem: multi-parameter reweighting, canonical approaches, imaginary chemical potentials, and so on.

If the chemical potential is pure imaginary, the quark determinant is real. There is no sign problem in this case, and standard Monte Carlo algorithms can be applied. QCD with imaginary chemical potential was studied in Refs. [1, 2, 3] with staggered (KS) fermions, and in [4] with standard Wilson fermions and in [16] with improved Wilson fermions.

We study QCD at low temperatures and nonzero quark chemical potentials using the imaginary chemical potential approach. We report preliminary results about analytic continuation of the Polyakov loop at $T/T_{pc} = 0.5 \sim 1$.

2. Imaginary Chemical Potential Approach

The study of the phase structure in (β, μ_I)-plane provides us with the information of the phase structure in (β, μ_R)-plane through analytic continuation. The Roberge-Weiss periodicity is important in the study of QCD with imaginary chemical potential [5]. Roberge and Weiss showed the existence of a phase transition at high temperatures, where the phase of the Polyakov loop is an order parameter, while this phase transition does not occur at low temperatures.

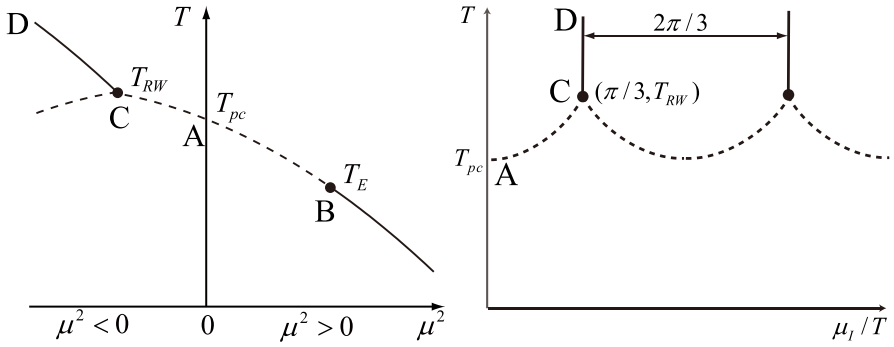

Figure 1: Schematic figures for the $N_f = 2$ QCD phase diagram in the (μ^2, T) plane (left) and $(\mu_I/T, T)$ plane (right). The figures are taken from [16].

QCD with imaginary chemical potential is free from the sign problem. Using a relation

$$(\det\Delta(\mu))^* = \det\Delta(-\mu), \quad (\mu = \mu_R + i\mu_I), \tag{2.1}$$

it is straightforward to prove that $\det\Delta(\mu)$ is real for $\mu = i\mu_I$. A partition function and its free-energy are analytic within one phase even if chemical potential is extended to complex. The analyticity holds until the occurrence of a phase transition. This validates the imaginary chemical

potential approach for the study of the QCD phase diagram. In addition, the QCD phase diagram has a unique feature in the imaginary chemical potential region, called the Roberge-Weiss periodicity [5], see Fig. 1. In the presence of the quark, the $Z(3)$ symmetry is explicitly broken. However, if chemical potential is pure imaginary, the $Z(3)$ symmetry is maintained via a translation of μ_I as

$$Z\left(T,\frac{\mu_I}{T}\right)=Z\left(T,\frac{\mu_I}{T}+\frac{2\pi}{N_c}k\right), \tag{2.2}$$

where k is an integer.

3. Framework

3.1 Formulation and parameter set up

We employ the renormalization-group improved gauge action [9] given by

$$S_g=\frac{\beta}{6}\left[c_0\sum(1\times 1\text{ loop})+c_1\sum(1\times 2\text{ loop})\right], \tag{3.1}$$

with $c_1=-0.331$ and $c_0=1-8c_1$ and the clover-improved Wilson fermion given by

$$\begin{aligned}\Delta(x,y)=\delta_{x,x'} &- \kappa\sum_{i=1}^{3}\left[(1-\gamma_i)U_i(x)\delta_{x',x+\hat{i}}+(1+\gamma_i)U_i^{\dagger}(x')\delta_{x',x-\hat{i}}\right]\\ &- \kappa\left[e^{+\mu}(1-\gamma_4)U_4(x)\delta_{x',x+\hat{4}}+e^{-\mu}(1+\gamma_4)U_4^{\dagger}(x')\delta_{x',x-\hat{4}}\right]\\ &- \kappa C_{SW}\delta_{x,x'}\sum_{\mu\le\nu}\sigma_{\mu\nu}F_{\mu\nu}.\end{aligned} \tag{3.2}$$

We determine C_{SW} using a result obtained from an one-loop perturbation theory [10]: $C_{SW}=(1-0.8412\beta^{-1})^{-3/4}$. Here μ is the quark chemical potential in lattice unit.

We calculate the Polyakov loop $\langle L\rangle=|L|\exp(i\phi)$ for various chemical potentials, and determine the chemical potential dependence of $|L|$. We will explain this point in the next subsection.

In order to study the low temperature region, we carried out lattice simulations for three set of lattice sizes: $N_s^3\times N_t=(8^3\times 4)$, $(8^3\times 6)$, and $(8^3\times 8)$. We used a fixed value of $\beta=1.86$, and three cases correspond to $T/T_c=1,2/3$ and $1/2$ for $N_t=4,6$, and 8, respectively. The value of the hopping parameter was fixed with $\kappa=0.139838$, which was determined by the lines of constant physics with $m_{PS}/m_V=0.8$ at $\mu=0$ [11, 12].

We used a step size of the molecular dynamics of $\delta\tau=0.02$, the number of the molecular dynamics of 50, which gives the length of one trajectory of one. As we will discuss below, the Polyakov loop is quite insensitive to quark chemical potential at low temperetures. Hence, high statistics simulations are required to study the low temperature region. We generated 95,000, 100,000 and 70,000 trajectories for $8^3\times 4$, $8^3\times 6$ and $8^3\times 8$, respectively. For all the ensemble, the first 20,000 trajectories were removed as thermalization. The Polyakov loop was measured for each trajectory.

3.2 Fitting Function

Now we consider a function describing the chemical potential dependence of the Polyakov loop. First, we consider a canonical formalism, where the partition function $Z(\mu)$ is a function of $\xi=$

$\exp(-\mu/T)$. With $Z(\mu) = Z(-\mu)$ implying a symmetry under $\xi \leftrightarrow 1/\xi$, $Z(\xi)$ is described by

$$Z(\xi) = \sum_m C(3m) e^{-3m\alpha} (\xi^{3m} + \xi^{-3m}). \tag{3.3}$$

Then, an observable O is given by

$$\langle \mathcal{O} \rangle (\xi) = \frac{\sum_m C'(3m) e^{-3m\alpha} (\xi^{3m} + \xi^{-3m})}{\sum_m C(3m) e^{-3m\alpha} (\xi^{3m} + \xi^{-3m})}, \tag{3.4}$$

α, C and C' are parameters. We consider the sum up to $m = 3$.

The parameter α can be absorbed into C and C'. However, in the case, the determination of C and C' may suffer from numerical errrors, because they rapidly decrease with m. The parameter α is introduced to keep C and C' measurable. To estimate α, we consider a canonical approach,

$$Z(T,\mu) = \sum_{n=-N}^{N} Z_n(T) \xi^n, \tag{3.5}$$

where $Z_n(T)$ is a canonical partition function with a fixed quark number n. N is the maximum number of the quark. $Z(T,\mu)$ is the grand canonical partition function given by

$$Z(T,\mu) = \int \mathcal{D}U [\det\Delta(\mu)]^{N_f} e^{-S_G}. \tag{3.6}$$

Using a reduction formula [14], $\det\Delta(\mu)$ is expanded in terms of fugacity

$$\det\Delta(\mu) = C_0 \sum_{n=-N_{red}/2}^{N_{red}/2} c_n \xi^n, \tag{3.7}$$

where $N_{\rm red} = 4N_c N_s^3$. Using the Glasgow method in μ, the canonical partition function is given by

$$Z_n \equiv Z_C(n) = \left\langle \frac{C_0^2 d_n}{(\det\Delta(0))^2} \right\rangle_0 \tag{3.8}$$

Here d_n are the fugacity coefficients of the two-flavor determinant. $\langle . \rangle_0$ denotes an ensemble average for gauge configurations generated at $\mu = 0$.

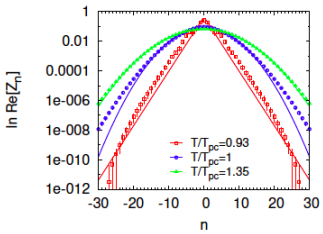

Figure 2: The canonical partition functions Z_n for three temperatures. Data were obtained from lattice simulations on $8^3 \times 4$ lattice. [15]

We have studied Z_n in Ref. [15], which is shown in Fig. 1. We determine the parameter α at $N_t = 4$ using the result for $T = 0.93 T_{pc}$. We employ $Z_C(n) = A\exp(-a|n|)$, then we obtain

$A = 0.438633 \pm 0.0382$, $a = 0.845268 \pm 0.0145$. Here n is the quark number. Physically relevant terms are $n \in \{n|\mod(n,3) = 0\}$. For instance, if there are three quarks, then there is one baryon: $Z_C(n=3) = Z_C(n_B = 1) = A\exp(-3a)$. We estimate values of α at $N_t = 6$, and 8, following a relation $Z_C(n) \sim \exp(-F_n/T)$ with the free energy F_n.

4. Results

Scatter plots of the Polyakov loop in the complex plane are shown Fig. 3. The magnitude of the Polyakov loop decreases with temperature. The Polyakov loop depends on μ_I at $N_t = 4$, and it is almost insensitive to μ_I at $N_t = 6, 8$.

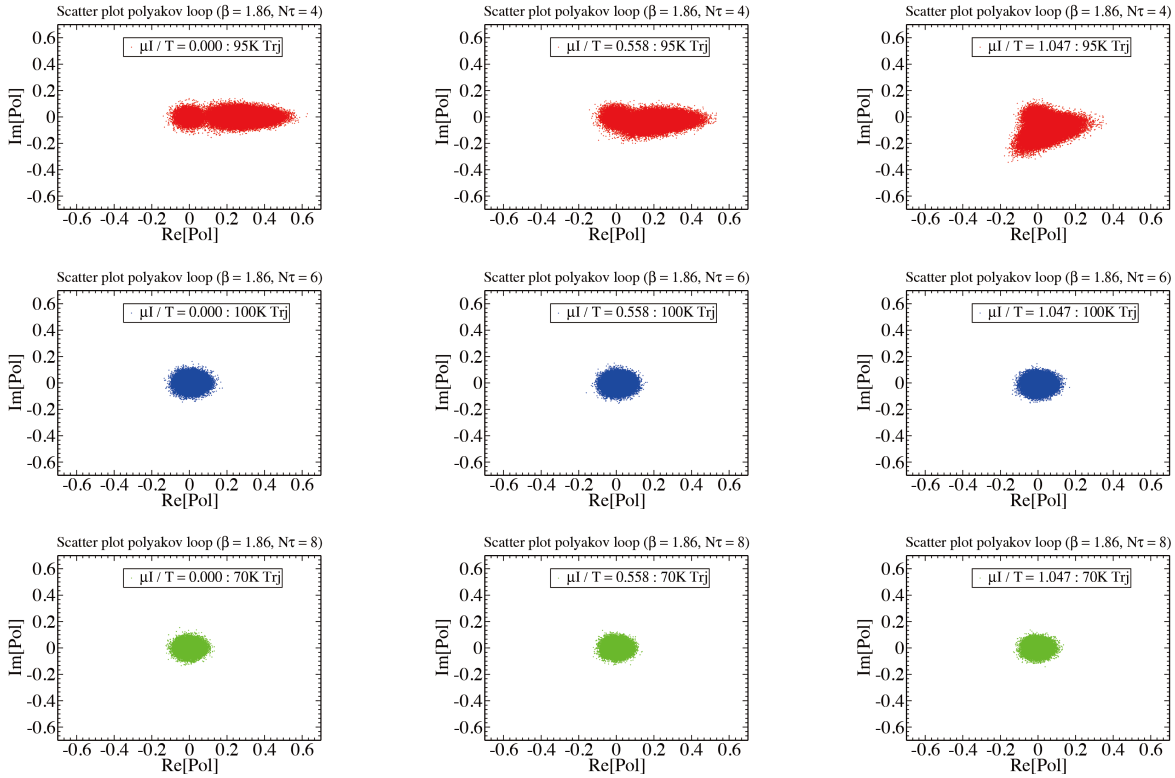

Figure 3: Scatter plots of the Polyakov loop. $T/T_{pc} = 1.0$, 0.75, and 0.5 from top to bottom, and $\mu_I/T = 0.0$, 0.558 and 1.047 from left to right.

Figure 4 shows preliminary results of the chemical potential dependence of the Polyakov loop. The left panels show the Polyakov loop as a function of μ_I/T; here each panel containing the data and fit function. The right panels show the Polyakov loop as a function of μ/T obtained from analytic continuation of the fit function in the left panels. The signal-to-noise ratio was quite small at low temperatures. Although we have generated more than 50 000 configurations, the determination of the μ dependence of the Polyakov loop suffers from this problem. For instance, the fit functions are away from some data points at the left panels, and a value of μ_R/T at which the Polyakov loop is saturated decreases with increasing N_t. This is inconsistent with a naive expectation of the μ dependence of the confinement/deconfinement phase boundary.

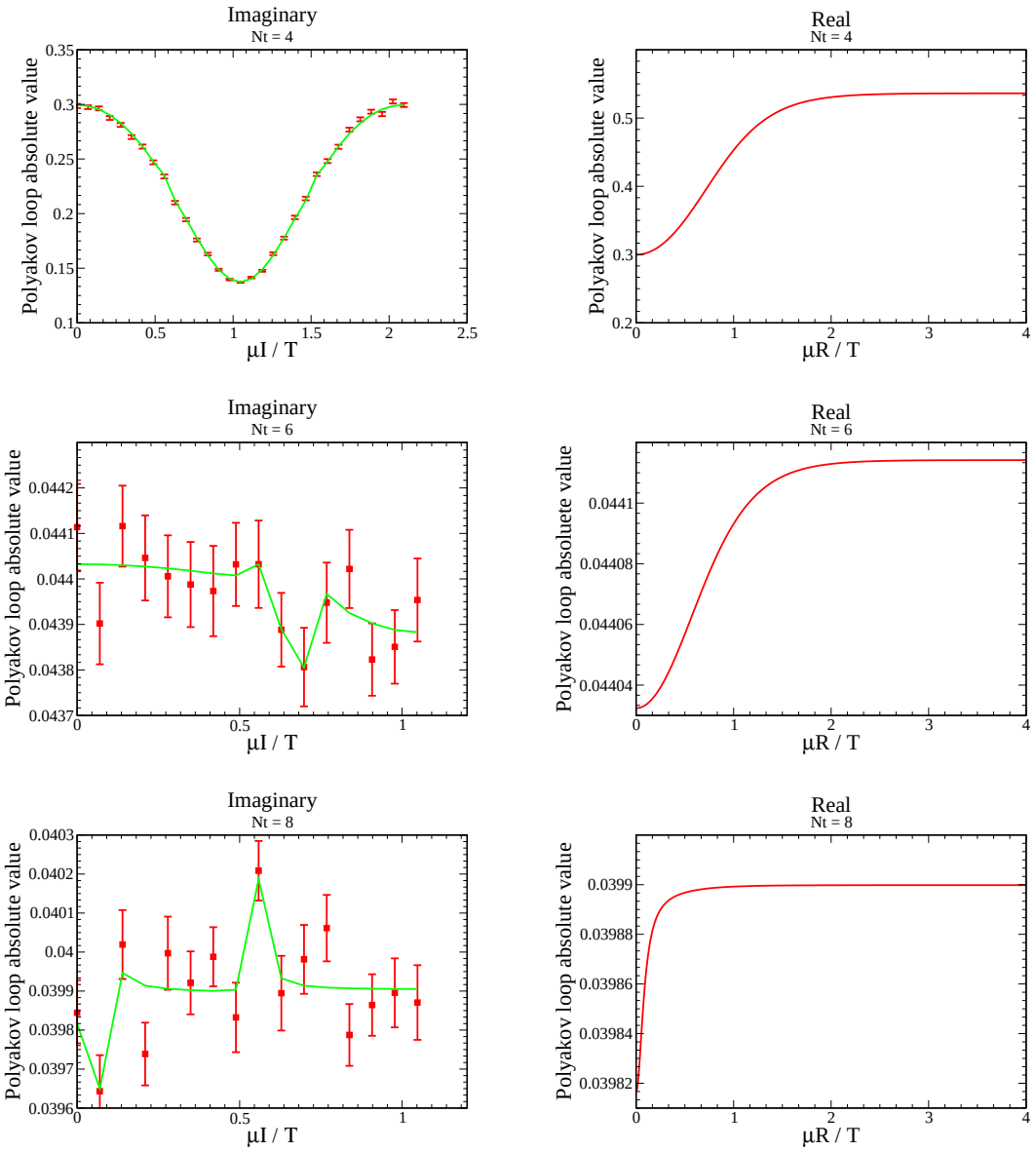

Figure 4: Chemical potential dependence of the magnitude of the Polyakov loop. Left : imaginary chemical potential, right : real chemical potential. $T/T_{pc} = 1.0$, 0.75, and 0.5 from top to bottom.

5. Summary

We have considered QCD with imaginary chemical potentials to obtain the chemical potential dependence of the Polyakov loop at low temperatures. We employed $N_f = 2$ flavors QCD with the renormalization-group improved gauge action and the clover-improved Wilson quark action. We performed simulations on $8^3 \times 4$, $8^3 \times 6$ and $8^3 \times 8$ lattice and calculated the Polyakov loop at low temperatures. We considered a function of fugacity to fit the chemical potential dependence of the Polyakov loop, and determined the parameters with the simulation data. The results were extended

to real chemical potential through analytic continuation.

It turns out that the Polyakov loop is quite insensitive to chemical potentials at low temperatures although some structures are seen. Although our simulations were done with high statistics of more than 50 000 configurations, the determination of the chemical potential dependence of the Polyakov loop suffers from significantly small signal-to-noise ratio.

In order to increase the precision of the results, we plan to consider several future works : using other observables such as quark number density, including smearing of the Polyakov loop, increasing lattice volume etc.

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