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Current Status of Indirect CP Violation in Neutral Kaon System

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In the standard model (SM), the CP violation is introduced through a single phase in the CKM matrix. The neutral kaon system is one of the most precise channels to test how the SM theory describes the experiment data such as ε_K accurately. The indirect CP violation is parametrized into ε_K, which can be calculated directly using lattice QCD. In this calculation, the largest uncertainty comes from two sources: one is $\hat{B}_K$ and the other is V_{cb}. We use the lattice results of $\hat{B}_K$ and exclusive V_{cb} to calculate the theoretical estimate of ε_K, which turns out to be 3.1σ away from its experimental value. Here, the error is evaluated using the standard error propagation method.

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1. Introduction

The neutral kaon system has two kinds of CP violation: indirect and direct CP violation. Indirect CP violation is parametrized into ε_K. The experimental value of ε_K is very well known [1]:

$$\varepsilon_K = (2.228 \pm 0.011) \times 10^{-3} \times e^{i\phi_\varepsilon}, \qquad \phi_\varepsilon = 43.51 \pm 0.05. \tag{1.1}$$

We can also calculate ε_K directly from the SM using B_K, V_{cb}, and other input parameters, which are determined from other experiments and the SM theory. By comparing these two values, we can test a fundamental ansatz of the SM, the unitarity of the CKM matrix.

Here, we calculate ε_K directly from the SM using the known parameters with their errors in control. Two of the most important input parameters are $\hat{B}_K$ and V_{cb}, which dominate the statistical and systematic uncertainty in ε_K. During a past decade, lattice QCD has reduced the B_K error dramatically down to less than 5% level [2–4] as well as V_{cb} [5].

There are two independent methods to determine V_{cb}: inclusive channels and exclusive channel. There exists about 2 3σ difference in V_{cb} between inclusive and exclusive methods. We address this issue on how this have an effect on ε_K.

Here, we use the Wolfenstein parameters for the CKM matrix to calculate ε_K mainly because they are convenient.

Let us define a parameter Δ which test the unitarity ansatz directly as

$$\Delta \equiv V \cdot V^\dagger - I.$$

Here, we calculate ε_K and Δ to test the SM. The CKM matrix elements are accurate up to $\mathcal{O}(\lambda^5) \approx 3 \times 10^{-4}$. We use the standard error propagation method to estimate the errors.

2. Review of the Neutral Kaon Mixing: ε_K

The neutral kaon system forms a two dimensional Hilbert space. In this subspace, the time evolution of the neutral kaon state vectors can be described by the effective Hamiltonian $\hat{H}_{\text{eff}}$

$$i\frac{d}{dt}|K(t)\rangle = \hat{H}_{\text{eff}}|K(t)\rangle, \quad \hat{H}_{\text{eff}} = \hat{M} - i\frac{\hat{\Gamma}}{2}. \tag{2.1}$$

$\hat{M}$ ($\hat{\Gamma}$) is the dispersive (absorptive) part. Here, the dispersive part gives the mass eigenvalues of K_S and K_L, and the absorptive part represents the decay rates of K_S and K_L. Let us take the basis with the CP even $|K_1\rangle$ and odd $|K_2\rangle$ states,

$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right), \quad |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right). \tag{2.2}$$

In this basis, the matrix elements can be written as the followings:

$$M = \begin{pmatrix} M_1 & +im' \\ -im' & M_2 \end{pmatrix}, \Gamma = \begin{pmatrix} \Gamma_1 & +i\gamma' \\ -i\gamma' & \Gamma_2 \end{pmatrix}. \tag{2.3}$$

By construction of the formal perturbation theory in quantum field theory known as the Wigner-Weisskopf theory [6], M and Γ are hermitian matrices. In addition, if we assume that the CPT invariance is exactly respected, then m' and γ' must be real.

Solving the eigenvalue problem with this matrix representation for $H_{\rm eff}$, the eigenstates

$$|K_S\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}(|K_1\rangle + \tilde{\varepsilon}|K_2\rangle), \quad |K_L\rangle = \frac{1}{\sqrt{1+|\tilde{\varepsilon}|^2}}(|K_2\rangle + \tilde{\varepsilon}|K_1\rangle) \tag{2.4}$$

have the mass $M_{S,L}$ and the decay rate $\Gamma_{S,L}$, respectively. The small CP impurity $\tilde{\varepsilon}$ satisfies the following equation

$$\tilde{\varepsilon} = \tilde{\varepsilon}_{(0)}(1+\tilde{\varepsilon}^2). \tag{2.5}$$

Here, the $\tilde{\varepsilon}_{(0)}$ parameter is defined as

$$\tilde{\varepsilon}_{(0)} \equiv \frac{-i\left(m' - \frac{i}{2}\gamma'\right)}{\left(M_1 - M_2\right) - \frac{i}{2}\left(\Gamma_1 - \Gamma_2\right)} = e^{i\theta}\sin\theta\left(\frac{m'}{\Delta M_K} - i\cot\theta\frac{\gamma'}{\Delta\Gamma_K}\right) + \mathcal{O}\left(\tilde{\varepsilon}_{(0)}^3\right), \tag{2.6}$$

where

$$\Delta M_K = M_L - M_S,\ \Delta\Gamma_K = \Gamma_S - \Gamma_L,\ \tan\theta = \frac{2\Delta M_K}{\Delta\Gamma_K}. \tag{2.7}$$

The solution of $\tilde{\varepsilon}$ can be obtained by iteration. Since $\tilde{\varepsilon}_{(0)}$ is of the order of 10^{-3}, we may expand $\tilde{\varepsilon}$ perturbatively as follows,

$$\tilde{\varepsilon} = \tilde{\varepsilon}_{(0)} + \tilde{\varepsilon}_{(0)}^3 + 2\tilde{\varepsilon}_{(0)}^5 + 5\tilde{\varepsilon}_{(0)}^7 + \cdots. \tag{2.8}$$

In Eq. (2.6), $M_{1,2}$ and $\Gamma_{1,2}$ can be safely replaced by the eigenvalues $M_{S,L}$ and $\Gamma_{S,L}$ which are experimental observables. Note that this approximation makes an error of the size $\mathcal{O}(\tilde{\varepsilon}_{(0)}^3) \approx 10^{-9}$ which is of no interest to us. In the case of $\gamma'/\Delta\Gamma_K$, we presume the following assumptions:

- First, we make the approximation $\Delta\Gamma_K \cong \Gamma_1$ which is good up to the precision of 10^{-3}.
- Second, we assume that the contribution from the two pion state is dominant in γ', Γ_1 which is good in the precision level of 10^{-3}.
- Third, we assume that the contribution from the $I = 0$ two pion state is dominant in γ', Γ_1 compared with that from the $I = 2$ state. This approximation is good up to the precision of 10^{-7}.

Using these assumptions, we can approximate the $\gamma'/\Delta\Gamma_K$ as follows,

$$\frac{\gamma'}{\Delta\Gamma_K} = \xi_0 + \mathcal{O}\left(10^{-7}\right), \quad \xi_0 \equiv \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}. \tag{2.9}$$

Then, we can express ε_K approximately as follows,

$$\varepsilon_K \equiv \frac{\langle\pi\pi(I=0)|H_W|K_L\rangle}{\langle\pi\pi(I=0)|H_W|K_S\rangle} = \tilde{\varepsilon}_{(0)} + i\xi_0 + \mathcal{O}\left(\tilde{\varepsilon}_{(0)}^3\right) = e^{i\theta}\sin\theta\left(\frac{m'_{(6)}}{\Delta M_K} + \xi_0\right) + \Delta\varepsilon_K\,. \tag{2.10}$$

The correction of $\mathcal{O}(\tilde{\varepsilon}_{(0)}^3)$ in Eq. (2.10) is smaller than both the current experiment precision and the size of the long distance contributions of the m' [7]. The last expression in Eq. (2.10) is obtained by substituting $\tilde{\varepsilon}_{(0)}$ with Eq. (2.6) and Eq. (2.9). The correction of $\Delta\varepsilon_K$ contains both short-distance (SD) contribution and long-distance (LD) contribution, which are expected to be about $\approx 5\%$[7]. Here, we also neglect this contribution from $\Delta\varepsilon_K$, mainly because it is not known to a sufficient precision theoretically.

In this analysis, we take into account only the short-distance contribution from the dimension 6 operators $m'_{(6)}$. In the SM, this part can be calculated from the box diagram [8]. Here, we follow the notations in [8]. Then, we can obtain the following master formula which will be used in this analysis:

$$|\varepsilon_K^{\rm SM}| = \sqrt{2}\sin\theta\left(C_\varepsilon \hat{B}_K X + \xi_0\right), \tag{2.11}$$

where

$$X = \bar{\eta}\lambda^2|V_{cb}|^2 \times \left[|V_{cb}|^2(1-\bar{\rho})\eta_2 S_0(x_t) + \eta_3 S_0(x_c,x_t) - \eta_1 S_0(x_c)\right] \tag{2.12}$$

$$C_\varepsilon = \frac{G_F^2 F_K^2 m_{K^0} M_W^2}{6\sqrt{2}\pi^2 \Delta M_K}. \tag{2.13}$$

where we use the experimental value for ΔM_K. The input ξ_0 has been taken from the lattice calculation which accounts the long distance contribution.

G_F	$1.16637(1)\times 10^{-5}$ GeV^{-2}	[1]
M_W	80.399(23) GeV	[1]
$m_c(m_c)$	1.25(9) GeV	[10]
$m_t(m_t)$	162.7(1.3) GeV	[10]
η_1	1.43(23)	[10]
η_2	0.5765(65)	[10]
η_3	0.47(4)	[10]
θ	$43.51(5)^\circ$	[1]
m_{K^0}	497.614(24) MeV	[1]
ΔM_K	$3.483(6)\times 10^{-12}$ MeV	[1]

(a)

A	0.808(22)	[1] CKMfitter
	0.832(17)	[1] UTfit
λ	0.2253(7)	[1] CKMfitter
	0.2246(11)	[1] UTfit
$\bar{\rho}$	$0.132^{+0.022}_{-0.014}$	[1] CKMfitter
	0.130(18)	[1] UTfit
$\bar{\eta}$	0.341(13)	[1] CKMfitter
	0.350(13)	[1] UTfit

(b) Wolfenstein Parameters

F_K	156.1(0.2)(0.8)(0.2) MeV	[1]
	156.1(1.1) MeV	[11] LAT.AVG.
$\hat{B}_K$	0.7674(99)	[11] LAT.AVG.
	0.727(4)(38)	[2] SWME
ξ_0	$-1.63(19)(20)\times 10^{-4}$	[13]

(c) Lattice Calculation

$\|V_{cb}\|$	$41.85(42)(9)(59)\times 10^{-3}$	[12] $(X_c l\nu + X_s\gamma)_{\rm Kin}$
	$41.68(44)(9)(58)\times 10^{-3}$	[12] $(X_c l\nu)_{\rm Kin}$
	$41.87(25)\times 10^{-3}$	[12] $(X_c l\nu + X_s\gamma)_{\rm 1S}$
	$42.31(36)\times 10^{-3}$	[12] $(X_c l\nu)_{\rm 1S}$
	$41.5(7)\times 10^{-3}$	[1] Incl.PDG.AVG.
	$39.5(1.0)\times 10^{-3}$	[11] Excl.

(d) V_{cb}

Table 1: Input Parameters

3. Input Parameters

The parameters, $m_c, m_t, \eta_1, \eta_2, \eta_3$ depend on the renormalization scheme, and so are taken from the single reference for consistency (Table 1a[1]). The CKMfitter and UTfit results in Table 1b are obtained by their own global fit method using the same PDG inputs.

In $\hat{B}_K$ calculation in Table 1c, BMW quotes the smallest systematic error. It dominates the smallness of the lattice average error. RBC-UKQCD collaboration calculates ImA_2 on the lattice.

[1] In Ref. [9], they reported results of η_1 up to NNLO but end up with a noticeably larger error bar. Hence, we decide to use the NLO value.

Using this value, they determine ξ_0 through the relation

$$\mathrm{Re}\Big(\frac{\varepsilon'_K}{\varepsilon_K}\Big) = \frac{1}{\sqrt{2}|\varepsilon_K|}\frac{\mathrm{Re}A_2}{\mathrm{Re}A_0}\Big(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \xi_0\Big). \tag{3.1}$$

Other inputs such as $\mathrm{Re}A_0, \mathrm{Re}A_2, \varepsilon_K$, and $\varepsilon'_K/\varepsilon_K$ are taken from experiments.

Inclusive V_{cb} can be extracted from global fit of measured moments (lepton energy, hadronic mass, and photon energy) of the decay channels:

$$B \to X_c l \nu, \qquad B \to X_s \gamma.$$

We use the PDG average value as the representative of the inclusive V_{cb}. The quoted exclusive V_{cb} is the average of two semi-leptonic decay channels:

$$B \to D^* \ell \nu\,, \qquad B \to D \ell \nu\,.$$

For each scalar and vector channel, HFAG result is combined with FNAL/MILC lattice QCD calculation of the zero recoil form factor.

4. Error Estimate

For the function with N arguments, the error propagation formula gives the combined error σ_f in terms of the errors of each arguments:

$$\sigma_f^2 = \sum_{j,k=1}^{N} C_{jk} \frac{\partial f(\mathbf{x})}{\partial x_j}\bigg|_{\langle \mathbf{x}\rangle} \frac{\partial f(\mathbf{x})}{\partial x_k}\bigg|_{\langle \mathbf{x}\rangle} \sigma_{x_j}\sigma_{x_k}, \tag{4.1}$$

where C_{jk} denotes the normalized correlation between the parameters x_j and x_k, and $|C_{ij}| \le 1$. Especially the diagonal components $C_{ii} = 1$. We turn off the correlation and so $C_{ij} = \delta_{ij}$. In the case of asymmetric error, $\bar{\rho}$ given by CKMfitter, we take a larger error and treat it as a symmetric error.

For $\varepsilon_K^{\mathrm{SM}}$,

$$f(\mathbf{x}) = |\varepsilon_K(\mathbf{x})| \tag{4.2}$$

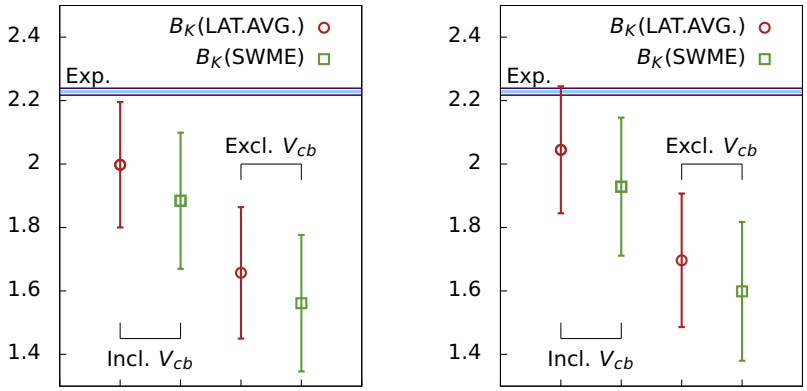
$$\mathbf{x} = (\theta, G_F, F_K, m_{K^0}, M_W, \Delta M_K, \xi_0, \hat{B}_K, \lambda, \bar{\rho}, \bar{\eta}, |V_{cb}|, \eta_1, \eta_2, \eta_3, x_c, x_t). \tag{4.3}$$

To check the unitarity of the CKM matrix,

$$f_{ij}(\mathbf{x}) = [V(\mathbf{x})V^\dagger(\mathbf{x}) - I]_{ij} \equiv \Delta_{ij} \tag{4.4}$$

$$\mathbf{x} = (A, \lambda, \rho, \eta; |V_{cb}|). \tag{4.5}$$

We use the Wolfenstein parametrization to evaluate each elements of the CKM matrix V_{ij}, except for V_{cb} itself. Real and imaginary part of Δ_{ij} are separately treated.



(a) CKMfitter (b) UTfit

Figure 1: $\varepsilon_K(\times 10^3)$

5. Results

The Wolfenstein parameter set, $\hat{B}_K$, and V_{cb} has a multiple choice. It forms 8 input parameter sets. We calculate $\varepsilon_K^{\rm SM}$ for these sets as shown in Fig. 1. We find out that $|\varepsilon_K|$ shows 3.1σ tension between $|\varepsilon_K^{\rm Exp}|$ and $|\varepsilon_K^{\rm SM}|$, using exclusive V_{cb} and SWME calculation of B_K as shown in Fig. 1a. With the UTfit Wolfenstein parameters, the tension is slightly reduced to 2.9σ [Fig. 1b].

The deviation matrix Δ provides a test of the Wolfenstein parameters, outputs of the global fit. These are inputs for the $\varepsilon_K^{\rm SM}$, as well. So testing the compatibility between the global fit results and the V_{cb} is needed to interpret the difference between $\varepsilon_K^{\rm SM}$ and $\varepsilon_K^{\rm SM}$. We find that the numerical size of Δ_{ij} has the following hierarchy

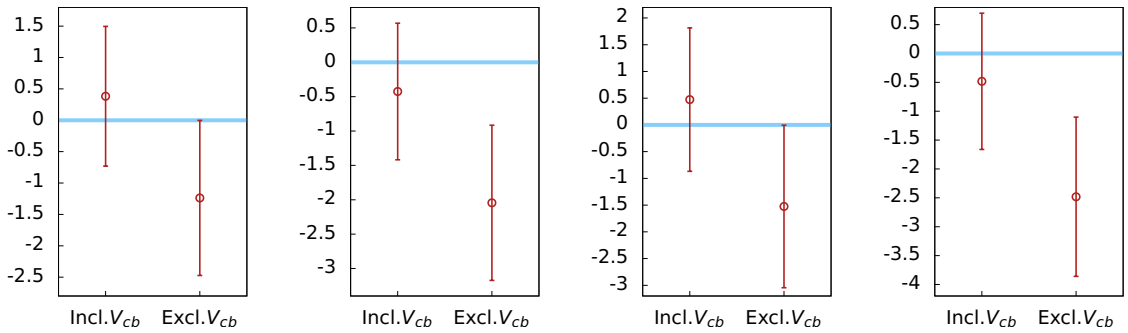
$$10^2\mathscr{O}\left(|{\rm Re}\Delta_{12}|\right) = 10\mathscr{O}\left(|{\rm Re}\Delta_{22}|\right) = \mathscr{O}\left(|{\rm Re}\Delta_{23}|\right). \tag{5.1}$$

${\rm Re}\Delta_{22}$ shows that the difference between exclusive V_{cb} and Wolfenstein parameters from CKMfitter (UTfit) are about 1.0σ (1.8σ) as shown in Fig. 2a and 2b. ${\rm Re}\Delta_{23}$ shows the difference about 1.0σ (1.8σ) as shown in Fig. 2c and 2d. Other components of Δ, which does not depend on the choice of inclusive or exclusive V_{cb}, are so small as to be consistent with the unitary ansatz.

In Table 2, V_{cb} dominates the error of $\varepsilon_K^{\rm SM}$ regardless of the inclusive or exclusive determination. In case of the SWME calculation of B_K and inclusive V_{cb}, both contribute to the total error in comparable size. In the case of the lattice average of B_K, V_{cb} becomes an extremely dominant error and the subdominant error comes from $\bar{\eta}$.

W.P.	V_{cb}	B_K	m_c	η_1	η_3	F_K	B_K	ξ_0	$\bar{\rho}$	$\bar{\eta}$	V_{cb}
CKMfitter	Incl.	LAT.AVG.	10.62	5.24	12.50	2.36	1.98	1.85	4.29	17.29	41.43
		SWME	8.10	4.00	9.53	1.80	25.06	1.57	3.28	13.19	31.61
	Excl.	LAT.AVG.	7.94	3.92	9.34	1.53	1.28	1.68	2.63	11.17	58.99
		SWME	6.61	3.26	7.78	1.27	17.67	1.56	2.19	9.29	49.10

Table 2: Error Fractions $\sigma_i^2/\sum\sigma_j^2$. The UTfit Wolfenstein parameters(W.P) show the same tendency.



(a) ReΔ_{22}(CKMfitter) (b) ReΔ_{22}(UTfit) (c) ReΔ_{23}(CKMfitter) (d) ReΔ_{23}(UTfit)

Figure 2: Re$\Delta_{22}(\times 10^4)$ and Re$\Delta_{23}(\times 10^3)$ to test the CKM unitarity.

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References

[1] K. Nakamura *et. al.* (Particle Data Group), *J.Phys.G* **G37** (2010) 075021.

[2] T. Bae *et. al.*, *Phys.Rev.Lett.* **109** (2012) 041601 [hep-lat/1111.5698].

[3] Y. Aoki *et. al.*, *Phys. Rev.* **D84** (2011) 014503 [hep-lat/1012.4178].

[4] C. Aubin *et. al.*, *Phys. Rev.* **D81** (2010) 014507 [hep-lat/0905.3947].

[5] J. Bailey *et. al.*, PoS(LATTICE2010)311 [hep-lat/1011.2166].

[6] N. Christ, in *Proceedings of Lattice2011* PoS(LATTICE2011)277 [hep-lat/1201.2065].

[7] A. J. Buras, D. Guadagnoli and G. Isidori, *Phys. Lett.* **B688** (2010) 309 [hep-ph/1002.3612].

[8] A. J. Buras, *Weak Hamiltonian, CP violation and rare decays* in *Probing the Standard Model of Particle Interactions*, F. David and R. Gupta, eds., Elsevier Science B.V. (1998) [hep-ph/9806471].

[9] J. Brod and M. Gorbahn, *Phys.Rev.Lett* **108** (2012) 121801 [hep-ph/1108.2036].

[10] A. J. Buras, and D. Guadagnoli, *Phys.Rev.* **D78** (2008) 033005 [hep-ph/0805.3887].

[11] J. Laiho, and E. Lunghi, and R. S. Van de Water, *Phys.Rev.* **D81** (2010) 034503 [hep-ph/0910.2928] http://latticeaverages.org/.

[12] J. Laiho, and B. D. Pecjak, and C. Schwanda, in *Proceedings of CKM2010* (2011) [hep-ex/1107.3934].

[13] T. Blum *et. al.*, *Phys.Rev.Lett.* **108** (2012) 141601 [hep-lat/1111.1699].