

Analysis of the scalar mesons on the Lattice

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We study the possibility that the scalar mesons exist as four-quark states. The energy shift of two pseudoscalar mesons as a function of spatial lattice size makes a distinction between bound states and scattering states of four-quark states. We calculate the four-quark state in the quenched approximation, ignoring the two-quark annihilation diagrams and the vacuum channels. We perform a calculation of pseudoscalar meson scattering amplitudes, using $N_f = 2$ Wilson fermion and plaquette/Iwasaki gauge actions. We obtain the indication that the four-quark states in the case of the isospin zero ($I = 0$) and two ($I = 2$) channels are no bound states. And we find that the bound energy depends strongly on pion mass rather than the ratio of pion mass to rho meson mass.

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1. Introduction

Since many light scalar mesons such as $\sigma(600)$, $\kappa(800)$, $f_0(980)$ and $a_0(980)$ were found in experiments [1], a lot of theoretical studies have been devoted to investigation of their states. For the structure of light scalar mesons, in addition to the conventional two quark state from the quark model, several possibilities are proposed [2]; four-quark states, molecular states and scattering states. Because the sigma meson is considered as a chiral partner of the pion in the mechanism of hadron mass generation, it would be interesting to investigate a role of four-quark states in the mechanism. The study of four-quark states in light scalar mesons gives us insight of important QCD feature.

Since Alford and Jaffe showed the possibility that scalar mesons exist as four-quark states on the lattice [3], tetraquark search on the lattice started actively. The pioneer work of the sigma meson which is one of candidates of four-quark states was done with full QCD by SCALAR collaboration [4]. They found that disconnected diagrams which contain effectively four-quark states, glueballs and so on make the sigma meson lighter. Recently using tetraquark interpolators, not only ground states but also resonance states of scalar mesons on the lattice were reported [5].

Following the procedure which was proposed by Alford and Jaffe [3], we explore the existence of four-quark states in the scalar channel on larger lattice with a finer lattice spacing. We also investigate the dependence of bound energies for four-quark states on the ratio of pion mass and rho meson mass (m_π/m_ρ) and on the pion mass (m_π).

2. Four-quark states from Lattice QCD

We calculate four-quark correlators in two-flavor ($N_f = 2$) lattice QCD under the assumption that four-quark states of scalar mesons are consisted of bound states of the two pions. We employ the procedure which was proposed by Alford and Jaffe [3]. The four-quark correlators of isospin zero ($I = 0$) and two ($I = 2$) channels are given by,

$$J_{I=0}(t) = D(t) + \frac{1}{2}C(t) - 3A(t) + \frac{3}{2}G(t), \quad (2.1)$$

$$J_{I=2}(t) = D(t) - C(t), \quad (2.2)$$

where $D(t)$, $C(t)$, $A(t)$ and $G(t)$ are corresponding to the diagrams in Fig. 1. To evaluate the four-quark states clearly, we carry out the calculation in the quenched approximation where two-quark, multi-quark and glueball states do not mix with four-quark states as intermediate states. We drop the contribution of the diagrams for $A(t)$ and $G(t)$, ignoring the two-quark annihilation diagrams and the vacuum channels. To obtain the bound energy δE_I for the four-quark states, we construct

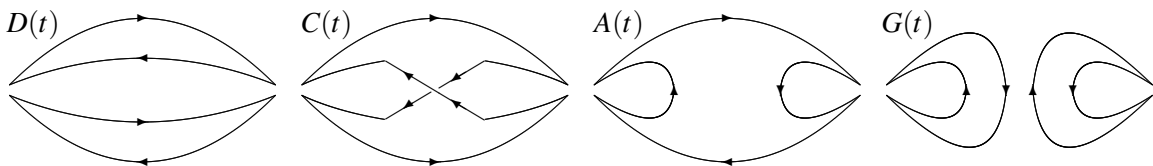


Figure 1: The diagrams for four-quark correlators.

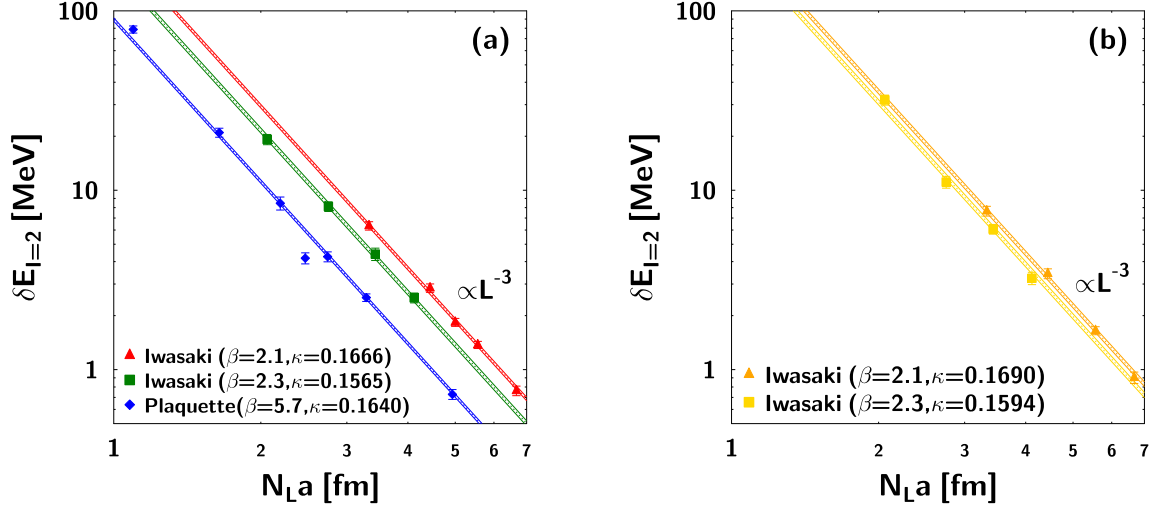


Figure 2: The physical lattice size $L(=N_L a)$ dependence of the energy shifts $\delta E_{I=2}$. In left figure (a) (right figure (b)), the values of m_π/m_ρ (m_π) are fixed. In both figures the lines are proportional to L^{-3} . In left figure (a), the three symbols are on the different L^{-3} lines, though they have the same values of m_π/m_ρ . In right figure (b), the two symbols which have the same values of m_π are almost on the same L^{-3} line, though they have the different values of m_π/m_ρ .

the ratio of the four-quark correlator $J_I(t)$ and the pion correlator $P(t)$ and fit it to an exponential at large t ,

$$R_I(t) = \frac{J_I(t)}{(P(t))^2} \xrightarrow{t \rightarrow \infty} \frac{Z_I}{Z_\pi^2} \exp(-\delta E_I t) + \dots \quad (2.3)$$

We discuss the possibility that the four-quark states exist as bound states from the N_L dependence of the bound energies.

If a four-quark state is a bound state, δE_I is negative and would approach to a negative constant in the large N_L region. On the other hand if a four-quark state is a scattering state, it is expected that δE_I obeys the scattering formula of a two-particle state in a cube box of size L with periodic boundary condition in Ref. [6],

$$\delta E_I = E_I - 2m_\pi = \frac{T_I}{L^3} \left[1 + 2.8373 \left(\frac{m_\pi T_I}{4\pi L} \right) + 6.3752 \left(\frac{m_\pi T_I}{4\pi L} \right)^2 \right] + \mathcal{O}(L^{-6}), \quad (2.4)$$

where E_I is the total energy, and T_I is the scattering amplitude which can be written by the scattering length a_I as

$$T_I = -\frac{4\pi a_I}{m_\pi}. \quad (2.5)$$

From Eq.(2.4), we find that δE_I is proportional to L^{-3} in the region where the physical spatial lattice size $L = N_L a$ (a is the lattice spacing) is enough large.

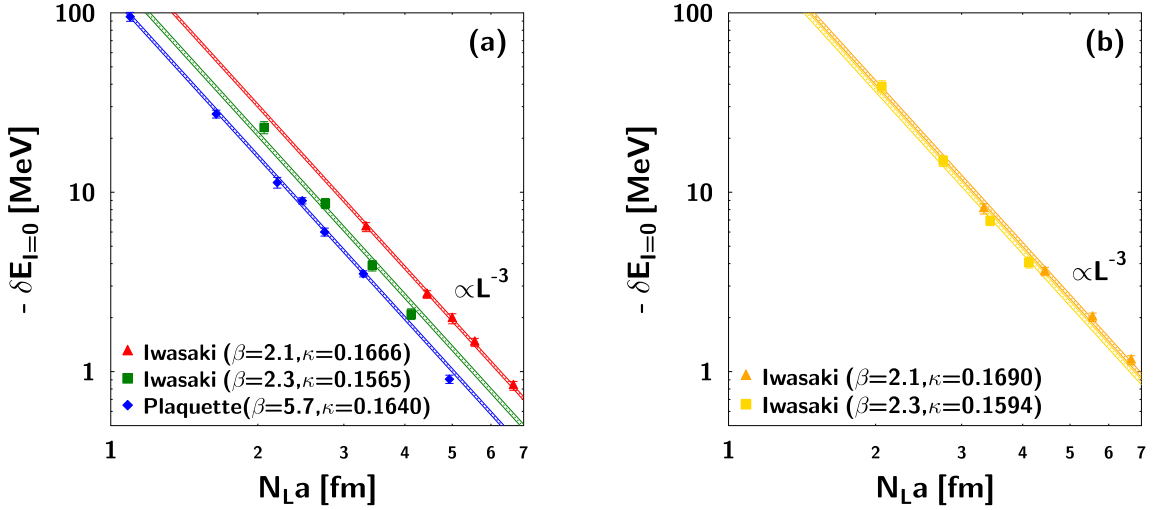


Figure 3: The physical lattice size $L(=N_L a)$ dependence of the energy shifts $\delta E_{I=0}$. In left figure (a) (right figure (b)), the values of m_π/m_ρ (m_π) are fixed. In both figures the lines are proportional to L^{-3} . In left figure (a), the three symbols are on the different L^{-3} lines, though they have the same values of m_π/m_ρ . In right figure (b), the two symbols which have the same values of m_π are almost on the same L^{-3} line, though they have the different values of m_π/m_ρ .

3. Results

In the work by Alford et al. [3], they observed that the $I = 2$ channel is clearly the scattering state but found the possibility that the $I = 0$ channel is the four-quark bound state. However to get the conclusive result for the four-quark bound state in the $I = 0$ channel, further calculations on a larger lattice with finer lattice spacing are required.

We calculate the four-quark correlators on a larger lattice with $N_f = 2$ Wilson fermion using configurations which are produced with both plaquette and Iwasaki gauge actions. The lattice parameters are shown in Tables 1 and 2. We impose the Dirichlet boundary condition in the temporal direction on the quark fields.

The results for $I = 2$ channel are shown in Fig. 2. In Fig. 2 (a), the quark masses are set as the values of the ratio of pion mass to rho meson mass, $m_\pi/m_\rho \sim 0.74$ become close to those in Ref. [3] (See Table 4). In spite of the same values of m_π/m_ρ , the values of m_π are different among them, which comes from the lattice artifact due to the large lattice spacings. In Fig. 2 (b), the bound energies are obtained under the same values of $m_\pi \sim 370$ MeV. We can see that in all cases of Fig.2 (a) and (b) the symbols are on the L^{-3} lines, which is the same as that in Ref. [3]. On the other hand the results for $I = 0$ channel are shown in Fig. 3. In Fig. 3 (a), the quark masses are set as the values of the ratio of m_π/m_ρ become close to those in Ref. [3] (See Table 4). In Fig. 3 (b), the bound energies are obtained under the same values of $m_\pi \sim 370$ MeV. Again we find that in all cases of Figs.3 (a) and (b) the symbols are on the L^{-3} lines, which is contradicted with the result of Ref. [3]. We can not observe the suggestion of the existence of the bound state in the $I = 0$ channel.

In Fig. 3 (a) the three symbols which have the same values of m_π/m_ρ are on the different L^{-3} lines, but in Fig. 3 (b) the two symbols which have the same values of m_π are almost on the same

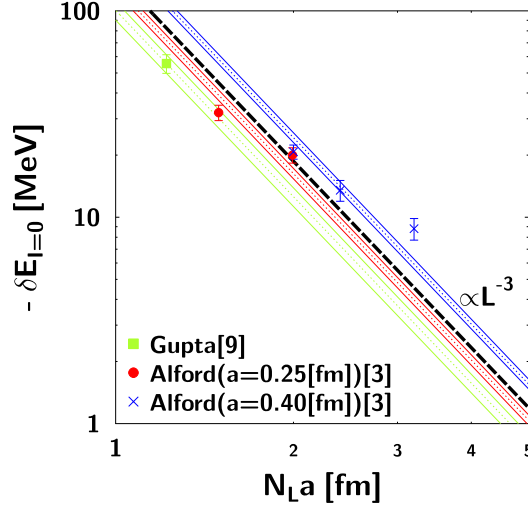


Figure 4: The physical lattice size $L(=N_L a)$ dependence of the energy shifts $\delta E_{I=0}$. The lines are proportional to L^{-3} . The color difference corresponds to m_π difference. The green, red and blue solid lines stand for $m_\pi = 940, 840$ and 790 MeV, respectively. The black dashed line was drawn in Fig.5 of Ref.[3].

L^{-3} line. It implies that δE_I depends strongly on m_π rather than m_π/m_ρ , which is deduced from Eq.(2.5). Since the scattering amplitude is proportional to the inversion of m_π , the absolute value of δE_I decreases with the increasing amount of m_π .

In Ref. [3] they insisted that they found the deviation from the L^{-3} line (the black dashed line in Fig. 4) in the physical lattice size dependence of energy shifts in the $I = 0$ channel. However we can give another explanation to it. Because of the m_π differences among data plots, we can draw the three different L^{-3} lines (the green, red and blue solid lines) instead of one L^{-3} line (the black dashed line) in Fig. 4. It does not suggest that the symbols deviate from the L^{-3} line (the black dashed line), but suggests that the symbols are on the three different L^{-3} lines, respectively. It means that there are no bound states in the $I = 0$ channel.

4. Summary

We have investigated a four-quark state as a bound state from the spatial lattice size dependence of the bound energies for a four-quark state. We found that the four-quark states in the cases of both $I = 0$ and $I = 2$ channels exist as no bound states, i.e., all bound energies for the four-quark states as a function of spatial lattice size L are on the L^{-3} lines, which is contradicted with the previous work [3]. The symbols which have the same values of m_π/m_ρ are on the different L^{-3} lines, on the other hand the symbols which have the same values of m_π are almost on the same L^{-3} line. This results suggest that the absolute value of bound energy depends directly on m_π rather than m_π/m_ρ .

This work is a starting point of study of four-quark states in the light scalar mesons. To reach the conclusive results for the light scalar meson states, further investigation is needed; full QCD calculation with light quark masses on larger lattice size, application of four-quark interpolators, state-of-the-art action and so on.

Acknowledgments

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Table 1: Lattice parameters. κ is the hopping parameter. We adjust parameters, β and κ to fix the values of $m_\pi/m_\rho \sim 0.74$. See Table 4.

Gauge action	β	a [fm]	κ	$N_L^3 \times N_T$	# Conf.
Iwasaki	2.100	0.278(3) [7]	0.1666	$12^3 \times 20$	150
				$16^3 \times 20$	120
				$18^3 \times 20$	78
				$20^3 \times 20$	120
				$24^3 \times 20$	78
Iwasaki	2.300	0.172(4) [7]	0.1565	$12^3 \times 20$	120
				$16^3 \times 20$	60
				$20^3 \times 20$	60
				$24^3 \times 20$	54
Plaquette	5.700	0.137(2) [8]	0.1640	$8^3 \times 20$	1008
				$12^3 \times 20$	240
				$16^3 \times 20$	102
				$18^3 \times 20$	210
				$20^3 \times 20$	192
				$24^3 \times 20$	300
				$36^3 \times 20$	66

Table 2: Lattice parameters. We adjust parameters, β and κ to fix the values of $m_\pi \sim 370$ MeV. See Table 4.

Gauge action	β	a [fm]	κ	$N_L^3 \times N_T$	# Conf.
Iwasaki	2.100	0.278(3) [7]	0.1690	$12^3 \times 20$	150
				$16^3 \times 20$	120
				$20^3 \times 20$	120
				$24^3 \times 20$	60
Iwasaki	2.300	0.172(4) [7]	0.1594	$12^3 \times 20$	300
				$16^3 \times 20$	180
				$20^3 \times 20$	180
				$24^3 \times 20$	60

Table 3: Lattice parameters in Ref. [3]. They adjust parameters, β and κ to fix the values of $m_\pi/m_\rho \sim 0.74$. See Table 4.

Group	Gauge action	β	a [fm]	κ	$N_L^3 \times N_T$	# Conf.
Gupta et al. [9]	Plaquette	6.000	0.0762(8)[10]	0.1540	$16^3 \times 80$	35
Fukugita et al. [8]	Plaquette	5.700	0.162(6) [10]	0.1640	$12^3 \times 20$	70
Alford et al.	Lüscher & Weisz	1.719	0.249(5)	—	$6^3 \times 40$	—
					$8^3 \times 40$	—
Alford et al.	Lüscher & Weisz	1.157	0.400(4)	—	$5^3 \times 40$	—
					$6^3 \times 40$	—
					$8^3 \times 40$	—

Table 4: The values of m_π/m_ρ and m_π . In the upper table results of Ref. [3] are shown. In the lower table results of our calculation are shown.

data	m_π/m_ρ	m_π [MeV]
Gupta	—	940(30)
Fukugita	0.740(8)	620(90)
Alford ($a = 0.25$ [fm])	0.756(5)	840(11)
Alford ($a = 0.40$ [fm])	0.756(4)	790(6)
Iwasaki ($\beta = 2.1, \kappa = 0.1666$)	0.736(3)	451(6)
Iwasaki ($\beta = 2.3, \kappa = 0.1565$)	0.746(6)	593(16)
Plaquette ($\beta = 5.7, \kappa = 0.1640$)	0.748(4)	754(12)
Iwasaki ($\beta = 2.1, \kappa = 0.1690$)	0.646(4)	368(5)
Iwasaki ($\beta = 2.3, \kappa = 0.1594$)	0.545(7)	374(11)