Theory status of $b \to s\,\ell^+\ell^-$ decays and their combined analysis

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The experimental information on $b \to s\,\ell^+\ell^-$ decays has grown enormously during the last couple of years due to BaBar, Belle, CDF and LHCb. Especially, exclusive modes $B \to K^{(*)}\ell^+\ell^-$, which have the largest rate and are easily accessible experimentally, provide a variety of observables which constrain non-standard interactions that would affect these flavour-changing neutral current decays beyond the Standard Model. Nowadays, theoretical predictions focus on low- and high dilepton invariant mass regions, where expansions in $\Lambda_{\rm QCD}/m_b$ and form factor symmetries provide means to identify optimised observables. The first experimental results of non-optimised observables have stimulated first global analysis of $b \to s\,\ell^+\ell^-$ decays in combination with $b \to s\gamma$ and $B_s \to \mu^+\mu^-$. Such global analysis are now ready to be applied to include high-statistics results from LHCb and Super-Flavour factories which are about to come within the next years.

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In the past the experimental program of quark flavour physics adressed primarily the exploration of CP-violation in the *B*-system and the tightly related picture of quark flavour mixing in the Standard Model (SM) represented by the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, the ever increasing luminosity allows nowadays also to explore flavour-changing neutral current (FCNC) decays of *b*-hadrons which are loop-suppressed in the SM and therefore have an enhanced sensitivity to non-standard virtual contributions. They test the SM at the loop-level and constitute indirect searches for non-standard effects. Consequently, precise experimental measurements are needed and a good control is required over theoretical uncertainties.

The class of FCNC decays, mediated by $b \to s \ell^+ \ell^-$ ($\ell = e, \mu, \tau$) is a phenomenologically rich sub-class with larger branching fractions of $\sim \mathcal{O}(10^{-6})$ (in the SM), compared to the CKM-suppressed $b \to d \ell^+ \ell^-$ decays $|V_{td}/V_{ts}|^2 \sim \mathcal{O}(10^{-2})$. It comprises inclusive and exclusive semi-leptonic decays $B_{u,d} \to (X_s, K, K^*) \ell^+ \ell^-$, $B_s \to (f_0, \phi) \ell^+ \ell^-$, $\Lambda_b \to \Lambda \ell^+ \ell^-$ as well as the purely leptonic $B_s \to \ell^+ \ell^-$ decay. Further channels with excited K^* have been discussed in the literature. Within the last few years four experimental collaborations analysed some of these channels with number of events in the range of 150 to 250 for BaBar [1], Belle [2] and CDF [3] and about 1000 events at LHCb [4] as listed in Table 1. The results based on the final data set have been released this year by BaBar and announced this summer by CDF, whereas Belle's results are based on a partial data set. By now LHCb dominates statistically, adding about 2 fb^{-1} of data this year 2012 and possibly another 4 fb^{-1} until the year 2018 before a long shut-down. The Super-Flavour factories Belle II [5] and SuperB [6] will be able to collect data sets with about $10000 - 15000$ events [7].

Theoretical predictions of $b \to s \ell^+ \ell^-$ decays are obtained using the effective theory of $\Delta B = 1$ decays of the electroweak interaction of the SM. It provides the universal starting point for the calculation of observables of inclusive and exclusive decays. The short-distance information at the electroweak sale $\mu \sim M_W$ of the order of the mass of the *W*-boson are contained in effective coupling constants C_i (Wilson coefficients) whereas flavour-changing interactions of $b \to s$ are described by one dimension five $b \to s \gamma$ operator $\mathcal{O}_7$ and two dimension six $b \to s \ell^+ \ell^-$ operators $\mathcal{O}_{9,10}$. Due to operator mixing, additional 4-quark operators have to be included, which are the current-current operators $\mathcal{O}_{1,2}^{u;c}$, the QCD-penguin operators $\mathcal{O}_{3,4,5,6}$ and the chromo-magnetic dipole operator $\mathcal{O}_8$. The effective Hamiltonian reads [8, 9]

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left(\mathcal{H}_{\text{eff}}^{(t)} + \hat{\lambda}_u \mathcal{H}_{\text{eff}}^{(u)} \right) + \text{h.c.}, \qquad \hat{\lambda}_u = V_{ub} V_{us}^* / V_{tb} V_{ts}^*, \qquad (1)$$

$$\mathcal{H}_{\text{eff}}^{(t)} = C_1 \mathcal{O}_1^c + C_2 \mathcal{O}_2^c + \sum_{3 \leq i} C_i \mathcal{O}_i, \qquad \mathcal{H}_{\text{eff}}^{(u)} = C_1 (\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2 (\mathcal{O}_2^c - \mathcal{O}_2^u) \qquad (2)$$

with

$$\mathcal{O}_7 = \frac{e}{(4\pi)^2} m_b (\bar{s} \sigma^{\mu\nu} P_R b) F_{\mu\nu}, \qquad \mathcal{O}_{9\,(10)} = \frac{e^2}{(4\pi)^2} (\bar{s} \gamma^\mu P_L b) (\bar{\ell} \gamma_\mu (\gamma_5) \ell) \qquad (3)$$

where the Wilson coefficients are renormalised in the $\overline{\text{MS}}$-scheme and evaluated at the renormalisation scale $\mu \sim m_b$ of the order of the *b*-quark mass. They have been calculated in the SM up to the next-to-next-to-leading order (NNLO) in QCD. At higher order in the electromagnetic coupling also QED-penguin operators have been considered for the inclusive decay [10]. Within extensions

# of evts	BaBar 2012 471 M $\bar{B}B$	Belle 2009 605 fb^{-1}	CDF 2011 6.8 fb^{-1}	LHCb 2011 1 fb^{-1}
$B^0 \to K^{*0} \ell^+\ell^-$	$137 \pm 44^\dagger$	$247 \pm 54^\dagger$	164 ± 15	673 ± 30
$B^+ \to K^{*+} \ell^+\ell^-$			20 ± 6	76 ± 16
$B^+ \to K^+ \ell^+\ell^-$	$153 \pm 41^\dagger$	$162 \pm 38^\dagger$	234 ± 19	1250 ± 42
$B^0 \to K_S^0 \ell^+\ell^-$			28 ± 9	60 ± 19
$B_s \to \phi \ell^+\ell^-$			49 ± 7	77 ± 10
$\Lambda_b \to \Lambda \ell^+\ell^-$			24 ± 5	

Table 1: The number of events for various exclusive $b \to s\ell^+\ell^-$ decays (CP- and lepton-flavour ($\ell = e, \mu$) averaged) obtained by BaBar [1], Belle [2], CDF [3] and LHCb [4]. The q^2 region around J/ψ and ψ' have been vetoed. † An unknown mixture of B^0 and $B^\pm$.

of the SM, new contributions arise $C_i \to C_i^{\text{SM}} + C_i^{\text{NP}}$ and additional $(\bar{s}\ldots b)(\bar{\ell}\ldots \ell)$ operators can contribute which have zero or negligible Wilson coefficients in the SM. For example right-handed currents give rise to chirality-flipped $\mathcal{O}_{7',9',10'}$ obtained by the interchange $P_L \leftrightarrow P_R$. Scalar and pseudo-scalar operators $\mathcal{O}_{S,S',P,P'}$ can have enhanced contributions from neutral Higgs-penguin or box-type diagrams, where the latter give also rise to tensor operators $\mathcal{O}_{T,T5}$. There are also non-standard scenarios which give rise to FCNC's at tree-level, such as LeptoQuark's or extensions with non-unitary quark mixing matrices. CP violation is suppressed in the SM in $b \to s$ transitions due to the smallness of $\text{Im}[\hat{\lambda}_u]$ which is doubly Cabibbo-suppressed.

The Wilson coefficients of the loop-induced $b \to s\gamma$ and $b \to s\ell^+\ell^-$ SM operators $\mathcal{O}_{7,9,10}$ – and potentially non-standard operators $\mathcal{O}_i$ with $i = 7', 9', 10', S, S', P, P', T, T5$ – are of great interest for indirect searches of signatures beyond the SM. They constitute the numerically leading contribution for large parts in the kinematic region of the dilepton invariant mass q^2 in most of the observables. However, when q^2 approaches production thresholds of $q\bar{q}$-resonances, 4-quark operators $b \to sq\bar{q}$ induce an additional interfering amplitude $b \to s(q\bar{q}) \to s\ell^+\ell^-$ which involves nonperturbative dynamics that are theoretically not well under control. Especially the current-current operators $\mathcal{O}_{1,2}^q$ with $q = c$ result in large peaking backgrounds $b \to sJ/\psi$ and $b \to s\psi'$ with branching fractions of $\mathcal{O}(10^2)$ larger than the once from $\mathcal{O}_{7,9,10}$ and are vetoed in the experimental analysis. Analogous contributions for $q = u$ are suppressed by $\hat{\lambda}_u$ whereas QCD-penguin operators have tiny Wilson coefficients.

Exclusive decays are currently available with high-statistics for the two most prominent decays $B^+ \to K^+\ell^+\ell^-$ and $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$. In comparison, the current experimental precision of inclusive decays can not compete and future measurements at Super-Flavour factories have to be awaited. In view of this, the theoretical status will be discussed only for exclusive decays in the following.

1. Exclusive decays

Two distinct theoretical methods have been applied in order to calculate observables of exclusive decays $B \to M\ell^+\ell^-$, where M denotes light mesons K, K^*, for the two regions of dilepton

invariant mass below and above the two narrow $c\bar{c}$-resonances J/ψ and ψ'. These methods are based on the different kinematical limits of large hadronic recoil (of M) at low-q^2 and low hadronic recoil at high-q^2 which allow for expansions in $\lambda \equiv \Lambda_{\mathrm{QCD}}/m_b$.

At low-q^2, contributions due to $b \to s\,q\bar{q}$ ($q = u,d,s,c$) 4-quark operators and the $b \to s\,gluon$ operator $\mathcal{O}_8$ are treated within QCD factorization (QCDF) [11] using the large energy limit of the recoiling meson M. It allows to include NLO corrections in the strong coupling α_s and effects of spectator-quark scattering. The amplitudes factorise schematically into perturbatively calculable quantities C and T and nonperturbative objects, form factors ξ and meson-distribution amplitudes $\phi_{B,M}$,

$$A \sim C\xi + \phi_B \otimes T \otimes \phi_M + \mathcal{O}(\lambda) \tag{1.1}$$

where $\otimes$ implies a convolution over the according momentum fractions. Notably, the large energy symmetry relations for heavy-to-light form factors allow to reduce seven $B \to V$ form factors to two universal $\xi_{\perp,\parallel}$ and the three $B \to P$ form factors to one ξ_P [12] which are valid up to order λ corrections and constitute the first part of lacking terms in Eq. (1.1). The corresponding α_s corrections at leading order in λ have been included in C and T [13]. For example the three K^*-transversity amplitudes in $B \to K^*\ell^+\ell^-$ decays have a simple form at leading order in λ and α_s [14]

$$A_{\perp,\parallel}^{L,R} \sim \pm C_{\perp}^{L,R} \times \xi_{\perp} + \mathcal{O}(\alpha_s,\lambda), \qquad A_0^{L,R} \sim C_{\parallel}^{L,R} \times \xi_{\parallel} + \mathcal{O}(\alpha_s,\lambda), \tag{1.2}$$

where the two short-distance coefficients are a linear combination of $C_{7,9,10}$

$$C_{\perp}^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b M_B}{q^2}C_7, \qquad C_{\parallel}^{L,R} = (C_9 \mp C_{10}) + \frac{2m_b}{M_B}C_7. \tag{1.3}$$

Besides the form factor symmetries, a second source of lacking sub-leading corrections in λ are due to the expansions of the amplitude itself. They involve also divergent contributions of distribution amplitudes ϕ_M at the sub-leading order which introduce a model-dependence. This affects especially the isospin-asymmetry which arises due to differences in spectator interactions [15]. Additionally, soft-gluon effects from $c\bar{c}$-resonances due to current-current operators $\mathcal{O}_{1,2}^c$ have been calculated within a non-local OPE [16] for the tails at q^2 below the resonances. They can change the rate up to $(10-20)\%$ for q^2 values of interest $\sim 6\,\mathrm{GeV}^2$, rising further for values even closer to the resonances.

At high-q^2, a local operator product expansion (OPE) can be applied to the contributions of 4-quark operators due to the hard momentum $\Lambda_{\mathrm{QCD}} \ll \sqrt{q^2} \sim m_b$ [17, 18] which is passing through the $q\bar{q}$-resonance. Now the K^*-transversity amplitudes depend only on one coefficient $C^{L,R}$ [19]

$$A_i^{L,R} \sim C^{L,R} \times f_i + C_7 \times \mathcal{O}(\lambda) + \mathcal{O}(\lambda^2), \qquad C^{L,R} = (C_9 \mp C_{10}) + \kappa\frac{2m_b^2}{q^2}C_7. \tag{1.4}$$

The well-known Isgur-Wise form factor relations [20], improved by the inclusion of QCD corrections κ [17], can be used to eliminate the three tensor form factors by the vector and axial-vector form factors. The according linear combinations are denoted by f_i ($i = 0,\perp,\parallel$) [19]. Due to the local OPE $B \to M$ form factors arise as the only nonperturbative objects, which are at the lowest order (dimension 3) the usual QCD form factors [17, 18]. The use of the form factor relations introduces an uncertainties of order λ which is $\propto C_7$. However, since the numerically leading term is

dominated by $|C_{9,10}| \sim 4.2$ in comparison to $|C_7| \sim 0.3$ (in the SM), an additional numerical suppression of $|C_7/C_{9,10}| \sim 0.1$ arises. In the OPE dimension four terms are absent such that sub-leading contributions to the amplitude are suppressed by λ^2. At higher orders in the OPE new form factors of the higher dimensional operators enter. All such form factors can be calculated at high-q^2 in principle on the lattice due to the low recoil of M. The NLO α_s corrections to the dimension three term are known as well [21] and lead to small renormalisation scale dependences. Finally, duality violating contributions to the OPE have been estimated based on a model and found to be of a few percent at the level of the rate when integrating over sufficiently large q^2-bins [18].

The main uncertainties in predictions of exclusive decays are due to form factors and lacking sub-leading contributions in the power expansions in λ. At high-q^2 sub-leading contributions are calculable and their omission less problematic due to the stronger suppression Eq. (1.4) compared to Eq. (1.2). Moreover, the according form factors of higher-dimensional operators can be calculated in principle on the lattice. Contrary, at low-q^2, no approach is known to the arising divergences in convolutions of distribution amplitudes at sub-leading order in QCDF, which results in a larger theoretical uncertainty in this region. Currently, the form factors are known mainly from light-cone sum rule (LCSR) calculations which are restricted to low-q^2 [22, 16]. At high-q^2 form factors have been calculated for $B \to K$ on the lattice in quenched approximation [23, 24], whereas only preliminary unquenched results for $B \to K^{(*)}$ are reported without final error estimates [25]. Therefore, currently all predictions at high-q^2 rely on extrapolations of the LCSR form factor results from the low-q^2. Unquenched lattice results should become available in the close future for $B \to K$ and $B \to K^*$.

2. Optimised Observables

Both decays, $B^+ \to K^+\ell^+\ell^-$ and $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$, allow to measure several observables in their angular distributions. For the 3-body final state in $B^+ \to K^+\ell^+\ell^-$ this is the angle θ_ℓ between the ℓ^- momentum and the direction of flight of the meson M in the $(\ell^+\ell^-)$ center of mass (CMS) frame. In the 4-body final state $B^0 \to K^{*0}(\to K^+\pi^-)\ell^+\ell^-$ two additional angles exist. These are an analogous angle θ_K between the Kaon momentum and the K^* momentum in the $(K\pi)$ CMS frame and the angle ϕ spanned by the two decay planes of the $(\ell^+\ell^-)$ and $(K\pi)$ systems. The intermediate K^* is assumed to be on-shell, such that the $(K\pi)$ invariant mass is fixed to the mass of the K^* and the narrow width approximation is used frequently. A very recent work addressed the latter issue [26] by including a finite width for the K^* and two additional scalar resonances finding a non-negligible impact at low-q^2 depending on the observable and the value of q^2.

Currently, for $B^+ \to K^+\ell^+\ell^-$ the branching ratio Br, the lepton forward-backward asymmetry A_{FB} and the isospin asymmetry A_I have been measured in q^2-bins covered by the theoretical methods described above, whereas for $B^0 \to K^{*0}\ell^+\ell^-$ these are Br, A_{FB}, A_I, the longitudinal K^* polarization fraction F_L, and further observables in the ϕ-distribution by LHCb and CDF: $S_{3,9}$ and $A_T^{(2)}$, A_{im}.

In $B^+ \to K^+\ell^+\ell^-$ the angular distribution w.r.t. $\cos\theta_\ell$ allows to measure the lepton A_{FB} and the observable F_H [27, 36]. The first is very sensitive to scalar and tensor $(\bar{s}\ldots b)(\bar{\ell}\ldots\ell)$ operators which are absent in the SM and similarly for F_H [27]. In the presence of chirality-flipped operators

$\mathcal{O}_{7',9',10',S',P'}$ the combination $(C_i + C_{i'})$ enters all observables, contrary to $B_s \to \ell^+\ell^-$ which depends on $(C_i - C_{i'})$ (for $i = 10, S, P$) and $B \to K^*\ell^+\ell^-$, which depends on both combinations.

The structure of the K^*-transversity amplitudes (1.2) and (1.4) has phenomenological interesting consequences for the 4-body final state in $B \to K^*(\to K\pi)\ell^+\ell^-$ decays. Its angular analysis offers a large number of angular observables $J_i(q^2)$ ($i = 1s, \dots 9$) [28], such that suitable combinations of $J_i(q^2)$ could be identified which exhibit a reduced hadronic uncertainty and enhanced sensitivity to short-distance couplings of the SM and scenarios beyond. These "optimized observables" are at low-q^2 $A_T^{(2,3,4,5,\text{re,im})}$ and $P_{4,5,6}$ [14, 29, 30] whereas at high-q^2 $H_T^{(2,3,4,5)}$ [19]. Further observables have been identified in the presence of scalar operators at low-q^2 [30]. Additionally, at high-q^2 also combinations are known which do not depend on the short-distance couplings (mostly in the SM operator basis) [19] and allow to probe the form factor shapes with data [31, 32]. CP asymmetric combinations with reduced hadronic uncertainties have been also found at low-q^2 [14, 29] and high-q^2 [33]. The sensitivity to B_s-mixing parameters ϕ_s and $\Delta\Gamma_s$ in time-integrated CP asymmetries of $B_s \to \phi(\to K^+K^-)\ell^+\ell^-$ turns out to be small [33, 34]. The $J_i(q^2)$ normalised to the decay rate and the associated CP-asymmetries have been also studied model-independently and model-dependently in great detail [34]. At the moment no experimental measurements are available for the optimised observables, except for $A_T^{(2)}$ from CDF [3].

3. Global analysis

Currently, global analysis of radiative, semi-leptonic and leptonic decays combine available data for inclusive $B \to X_s(\gamma, \ell^+\ell^-)$, exclusive $B \to K\ell^+\ell^-$, $B \to K^*(\gamma, \ell^+\ell^-)$ and the leptonic $B_s \to \mu^+\mu^-$ modes. There are model-independent studies which determine the constraints on the Wilson coefficients for varying sets of operators or model-dependent studies which derive bounds on the parameters of extensions of the SM.

The determination of confidence or probability regions varies in all analysis. Most simple approaches determine allowed regions of parameter space by combining $n-\sigma$ ($n = 1,2,3$) experimental and theoretical errors [35, 24]. Others calculate χ^2 values by combining experimental and theoretical errors following the R-fit scheme [33, 36] or different definitions [37, 38, 39]. A third approach includes parameters associated with theoretical uncertainties as nuisance parameters in the fit [32]. Once more precise data will be available, the more sophisticated methods should be used and further experimental correlations among observables have to be included as well, a task requiring a close collaboration between experimental and theory sides. First dedicated software tools have been developed for exclusive $b \to s\,\ell^+\ell^-$ decays [40] or extended [41].

In the context of model-independent analysis, the simplest scenario assumes new physics in $C_{7,9,10}$ which is real, i.e. involves the same CP-violation as in the SM. The according results can be seen in Fig. 1 from exclusive decays only when combining $B \to K^*\gamma$, $B \to K\ell^+\ell^-$ and $B \to K^*\ell^+\ell^-$ as well as from single processes only. Two solutions remain, which are related by a simultaneous sign-flip of all three Wilson coefficients compared to the SM signs. The solution with only C_7 flipped is now excluded [32, 37], mainly due to the measurements of A_{FB}. It can be also seen, that the high-q^2 region plays currently a crucial role. Overall, the SM is in good agreement with the data, but large deviations are still not excluded.
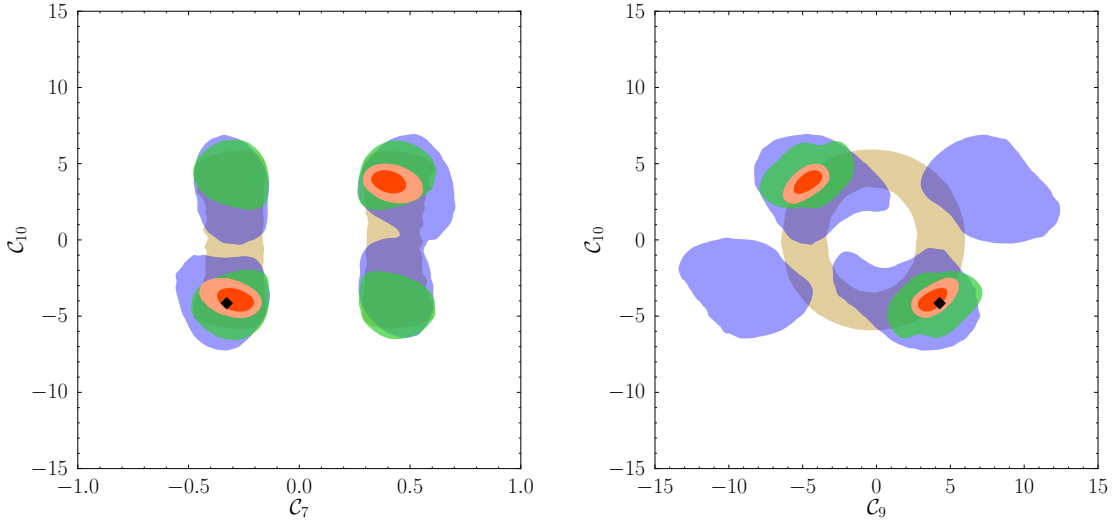
Figure 1: The marginalized 2-dimensional 95% credibility regions of the Wilson coefficients $C_{7,9,10}$ for $\mu = 4.2$ GeV are shown when applying the $B \to K^*\gamma$ constraints in combination with *i)* only low- and high-q^2 data from $B \to K\ell^+\ell^-$ [brown]; *ii)* only low-q^2 data from $B \to K^*\ell^+\ell^-$ [blue]; *iii)* only high-q^2 data from $B \to K^*\ell^+\ell^-$ [green]; and *iv)* all the data, including also $B_s \to \mu^+\mu^-$ [light red], showing as well the 68% credibility interval [red]. The SM values $C_{7,9,10}^{\text{SM}}$ are indicated by ◆ [32].

More general scenarios have been analysed too, including chirality-flipped Wilson coefficients [35, 37, 42, 39] assuming them to be real or complex. Especially the measurement of the optimised observables will be important in order to efficiently constrain these scenarios which currently still allow for large deviations from SM predictions, especially in optimized observables and CP asymmetries.

In the framework of the minimal supersymmetric SM (MSSM) the $b \to s\,\ell^+\ell^-$ transitions provide constraints on flavour-changing left-right mixing in the up-squark-sector $(\delta_{23}^u)_{LR}$, which in turn place constraints on top-quark FCNC decays $t \to c\gamma$, $t \to cg$ and $t \to cZ$ [43]. The interplay of $B_s \to \mu^+\mu^-$ at large $\tan\beta$ and angular observables in $B \to K^*\ell^+\ell^-$ at moderate $\tan\beta$ has been investigated in constrained scenarios such as the CMSSM and NUHM [44]. LeptoQuark interactions, which induce scalar and pseudo-scalar operators $\mathcal{O}_{S,S',P,P'}$, have been constraint with recent data from $B \to (X_s, K)\,\ell^+\ell^-$, and $B_s \to \mu^+\mu^-$ [45].

The transition $b \to s\,\tau^+\tau^-$ is experimentally not constrained except for the recent upper bound $Br(B^+ \to K^+\tau^+\tau^-) < 3.3 \cdot 10^{-3}$ from BaBar [46]. Indirect constraints might be derived using recent measurements of life-time ratio τ_{B_d}/τ_{B_s} which however involve some assumptions. The resulting constraints on the Wilson coefficients of $(\bar{s}b)(\bar{\tau}\tau)$ operators have been discussed in [47]. They do not allow for larger modifications of the decay width difference $\Delta\Gamma_s$ of the B_s-meson in the SM than 35% assuming single operator dominance. The interplay of several operators allows for larger effects [48].

With the higher statistics in experimental results and form factor predictions from lattice the currently proposed observables and strategies of their combination will allow to derive stronger constraints on new physics scenarios beyond the Standard Model which will be complementary to

other particle physics sectors. Currently the SM describes the data, but in the future it can be tested more stringently, especially with the first measurements of optimized observables.

References

[1] [BABAR Collaboration], *Phys. Rev. D* **86** (2012) 032012 [arXiv:1204.3933].

[2] BELLE Collaboration, *Phys. Rev. Lett.* **103** (2009) 171801 [0904.0770].

[3] CDF Collaboration, *Phys. Rev. Lett.* **107** (2011) 201802 [1107.3753]; *Phys. Rev. Lett.* **108** (2012) 081807 [1108.0695].

[4] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. Lett.* **108** (2012) 181806 [1112.3515]; *JHEP* **1207** (2012) 133 [1205.3422].

[5] T. Aushev, W. Bartel, A. Bondar, J. Brodzicka, T. E. Browder, *et al.*, 1002.5012.

[6] B. O'Leary *et al.* [SuperB Collaboration], 1008.1541.

[7] A. J. Bevan, *J. Phys. G* **39** (2012) 023001 [1110.3901].

[8] K. G. Chetyrkin, M. Misiak and M. Munz, *Phys. Lett. B* **400** (1997) 206 [*Erratum-ibid. B* **425** (1998) 414] [hep-ph/9612313].

[9] C. Bobeth, M. Misiak and J. Urban, *Nucl. Phys. B* **574** (2000) 291 [hep-ph/9910220].

[10] C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, *JHEP* **0404** (2004) 071 [hep-ph/0312090]; T. Huber, E. Lunghi, M. Misiak and D. Wyler, *Nucl. Phys. B* **740** (2006) 105 [hep-ph/0512066].

[11] M. Beneke, T. Feldmann, D. Seidel, *Nucl. Phys.* **B612** (2001) 25 [hep-ph/0106067]; *Eur. Phys. J.* **C41** (2005) 173 [hep-ph/0412400].

[12] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, *Phys. Rev. D* **60** (1999) 014001 [hep-ph/9812358].

[13] M. Beneke and T. Feldmann, *Nucl. Phys. B* **592** (2001) 3 [hep-ph/0008255].

[14] F. Kruger, J. Matias, *Phys. Rev.* **D71** (2005) 094009 [hep-ph/0502060].

[15] T. Feldmann and J. Matias, *JHEP* **0301** (2003) 074 [hep-ph/0212158].

[16] A. Khodjamirian, T. Mannel, A. A. Pivovarov, Y. M. Wang, *JHEP* **1009** (2010) 089 [1006.4945].

[17] B. Grinstein, D. Pirjol, *Phys. Rev.* **D70** (2004) 114005 [hep-ph/0404250].

[18] M. Beylich, G. Buchalla, T. Feldmann, *Eur. Phys. J.* **C71** (2011) 1635 [1101.5118].

[19] C. Bobeth, G. Hiller, D. van Dyk, *JHEP* **1007** (2010) 098 [1006.5013]; and in preparation.

[20] N. Isgur and M. B. Wise, *Phys. Rev. D* **42** (1990) 2388.

[21] D. Seidel, *Phys. Rev. D* **70** (2004) 094038 [hep-ph/0403185]; C. Greub, V. Pilipp and C. Schupbach, *JHEP* **0812** (2008) 040 [0810.4077].

[22] P. Ball and R. Zwicky, *Phys. Rev. D* **71** (2005) 014015 [hep-ph/0406232]; *Phys. Rev. D* **71** (2005) 014029 [hep-ph/0412079].

[23] D. Becirevic, V. Lubicz and F. Mescia, *Nucl. Phys. B* **769** (2007) 31 [`hep-ph/0611295`];
     A. Al-Haydari *et al.* [QCDSF Collaboration], *Eur. Phys. J. A* **43** (2010) 107 [`0903.1664`].

[24] D. Becirevic, N. Kosnik, F. Mescia and E. Schneider, *Phys. Rev. D* **86** (2012) 034034 [`1205.5811`].

[25] Z. Liu, S. Meinel, A. Hart, R. R. Horgan, E. H. Muller, M. Wingate, `1101.2726`; R. Zhou *et al.*
     [Fermilab Lattice and MILC Collaborations], *PoS LATTICE* **2011** (2011) 298 [`1111.0981`].

[26] D. Becirevic and A. Tayduganov, `1207.4004`.

[27] C. Bobeth, G. Hiller, G. Piranishvili, *JHEP* **0712** (2007) 040 [`0709.4174`].

[28] F. Kruger, L. M. Sehgal, N. Sinha, R. Sinha, *Phys. Rev.* **D61** (2000) 114028 [`hep-ph/9907386`].

[29] U. Egede, T. Hurth, J. Matias, M. Ramon, W. Reece, *JHEP* **0811** (2008) 032 [`0807.2589`], *JHEP*
     1010 (2010) 056 [`1005.0571`]; D. Becirevic, E. Schneider, *Nucl. Phys.* **B854** (2012) 321
     [`1106.3283`].

[30] J. Matias, F. Mescia, M. Ramon and J. Virto, *JHEP* **1204** (2012) 104 [`1202.4266`].

[31] C. Hambrock and G. Hiller, *Phys. Rev. Lett.* **109** (2012) 091802 [`1204.4444`].

[32] F. Beaujean, C. Bobeth, D. van Dyk and C. Wacker, *JHEP* **1208** (2012) 030 [`1205.1838`].

[33] C. Bobeth, G. Hiller, D. van Dyk, *JHEP* **1107** (2011) 067 [`1105.0376`].

[34] C. Bobeth, G. Hiller, G. Piranishvili, *JHEP* **0807** (2008) 106 [`0805.2525`]; W. Altmannshofer,
     P. Ball, A. Bharucha, A. J. Buras, D. M. Straub, M. Wick, *JHEP* **0901** (2009) 019 [`0811.1214`].

[35] S. Descotes-Genon, D. Ghosh, J. Matias, M. Ramon, *JHEP* **1106** (2011) 099 [`1104.3342`].

[36] C. Bobeth, G. Hiller, D. van Dyk and C. Wacker, *JHEP* **1201** (2012) 107 [`1111.2558`].

[37] W. Altmannshofer, P. Paradisi and D. M. Straub, *JHEP* **1204** (2012) 008 [`1111.1257`];
     W. Altmannshofer and D. M. Straub, *JHEP* **1208** (2012) 121 [`1206.0273`].

[38] T. Hurth and F. Mahmoudi, *Nucl. Phys. B* **865** (2012) 461 [`1207.0688`].

[39] S. Descotes-Genon, J. Matias, M. Ramon and J. Virto, `1207.2753`.

[40] EOS Collaboration, http://project.het.physik.tu-dortmund.de/eos/

[41] F. Mahmoudi, *Comput. Phys. Commun.* **178** (2008) 745 [`0710.2067`], *Comput. Phys. Commun.* **180**
     (2009) 1579 [`0808.314`].

[42] D. Becirevic, E. Kou, A. Le Yaouanc and A. Tayduganov, *JHEP* **1208** (2012) 090 [`1206.1502`].

[43] A. Behring, C. Gross, G. Hiller and S. Schacht, *JHEP* **1208** (2012) 152 [`1205.1500`].

[44] F. Mahmoudi, S. Neshatpour and J. Orloff, *JHEP* **1208** (2012) 092 [`1205.1845`].

[45] N. Kosnik, *Phys. Rev. D* **86** (2012) 055004 [`1206.2970`].

[46] K. Flood [BaBar Collaboration], *PoS* **ICHEP2010**, 234 (2010).

[47] C. Bobeth, U. Haisch, `1109.1826`.

[48] A. Dighe and D. Ghosh, *Phys. Rev. D* **86** (2012) 054023 [`1207.1324`].