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Positional astronomy and the Parapegma of the Antikythera Mechanism

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Among the surviving inscriptions of the Antikythera Mechanism, there is a portion of a list of star and constellation risings and settings, known as a parapegma. The largest part of this list cites nine consecutive astronomical events. These events take place periodically once a year and their sequence is sensitively dependent on geographic latitude. In order to calculate the dates of these visible risings and settings, a software program has been written, from first principles, using the Mathematica software package. The equatorial coordinates of the Sun and the stars for 150 B.C. were used. They were calculated taking into account the precession of the equinoxes,

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the decrease of the obliquity of the ecliptic, nutation and aberration, as well as, for the stars, the effects of proper motion. The visibility of a star at its position on the sky was determined by comparing its apparent magnitude with the sky background brightness. Atmospheric extinction and atmospheric refraction were also taken into account. The dates of the occurrence of these events were calculated for geographic latitudes between 25°N − 45°N. The sequence of the events and their position among the signs of the zodiac give a best fit between 33.3°N − 37.0°N which includes Rhodes and marginally Syracuse.

1. Introduction

One of the main questions still not answered about the Antikythera Mechanism is where it was built or where it was used. The following research, based on our joint and thorough astronomical analysis of the Parapegma of the Antikythera Mechanism [1], sheds perhaps some light towards this direction.

In the centre of its front plate, the Antikythera Mechanism had two concentric annuli. The outer one displayed the Egyptian calendar and the inner one the signs of the zodiac [2, 3, 4]. Looking carefully at the inner dial, small capital letters are observed above some of the scale divisions of each zodiac sign. These letters follow the sequence of the letters of the Greek alphabet. A similar sequence of the Greek letters is also found on some of the text fragments of the Mechanism describing astronomical phenomena. The largest of these fragments, C1-a, cites 9 consecutive astronomical events (for the type of events, see the following section). Each event is written in a serarate line and at the beginning of each line there is a separate letter of the Greek alphabet. These 'index letters' in sequence almost certainly correspond to the small capital letters found on the zodiac dial [2], thus forming a 'parapegma', a calendar of astronomical events. Parapegmata were known to be widely used in ancient Greece and were not only calendars of astronomical events but also of meteorological ones as well [5].

The fragments of the Mechanism with Parapegma inscriptions are two pieces of flat plate attached to Fragment C (C1-a and C1-b), Fragment 09, Fragment 20, Fragment 22 and Fragment 28 [6]. The inscriptions though read on most of these fragments are quite incomplete. The analysis that follows is thus based on the 9 astronomical events of C1-a. Also the crucial information of the autumn equinox taking place when the Sun enters the zodiac sign of Libra, mentioned on plate C1-b, was used [7, 8]. Fig. 1 shows the Parapegma inscriptions from plates C1-a and C1-b.

2.Types of events mentioned in the Parapegma

The types of the mentioned events on plate C1-a are categorized as follows: (a) there are 7 events that refer to the rising and setting of stars, i.e. their morning rising, their morning setting, their evening rising and their evening setting and (b) there are 2 events that refer to the signs of the zodiac.

Κ	ΕΙΕΣΠΕΡΙΑ	Κ	in the evening
Λ	ΥΑΔΕΣΔΥΟΝΤΑΙΕΣΠΕΡΙΑΙ	Λ	The Hyades set in the evening
Μ	ΤΑΥΡΟΣΑΡΧΕΤΑΙΑΝΑΤΕΛΛΕΙΝ	Μ	Taurus begins to rise
Ν	ΛΥΡΑΕΠΙΤΕΛΛΕΙ ΕΣΠΕΡΙΑ	Ν	Lyra rises in the evening
Ξ	ΠΛΕΙΑΣΕΠΙΤΕΛΛΕΙΕΩΙΑ	Ξ	The Pleiades rise in the morning
Ο	ΥΑΣΕΠΙΤΕΛΛΕΙ ΕΩΙΑ	Ο	The Hyades rise in the morning
Π	ΔΙΔΥΜΟΙΑΡΧΟΝΤΑΙΕΠΙΤΕΛΛΕΙΝ	Π	Gemini begins to rise
Ρ	ΑΕΤΟΣΕΠΙΤΕΛΛΕΙΕΣΠΕΡΙΟΣ	Ρ	Aquila rises in the evening
Σ	ΑΡΚΤΟΥΡΟΣ ΔΥΝΕΙ ΕΩΙΟΣ	Σ	Arcturus sets in the morning

ΛΑΙΑΡΧΟΝΤΑΙΕΠΙΤ	Libra begins to rise
ΜΕΡΙΑΦΘΙΝΟ	Autumn equinox
ΤΕΛΛΟΥΣΙ	... rise ...
ΕΛΛΕ	... rises ...

Fig. 1: The Parapegma inscriptions from the plates C1-a and C1-b. The red letters are dubious letters or letters that can not be read nowadays. *Upper*: The Greek text from plate C1-a and its translation in English. *Lower*: The Greek text from plate C1-b and its translation in English.

2.1 The star events

In this type of events, the subject is always a star, a star cluster or a constellation. The verb used is "ΕΠΙΤΕΛΛΕΙ" (rises) or "ΔΥΝΕΙ" (sets) (placing the <u>star</u> in the east or the west respectively), followed by the adjective "ΕΩΙΟΣ" (in the morning) or "ΕΣΠΕΡΙΟΣ" (in the evening) (placing the <u>Sun</u> in the east or the west respectively). Therefore, we can have four types of events: two with the star and the Sun in conjunction (morning rising and evening setting) and two in opposition (morning setting and evening rising).

In antiquity, the star events were differentiated into true ones (*ἀληθινοί*) and visible ones (*φαινόμενοι*). The true ones occur when the Sun and the star are on the horizon and so they are unobservable. The calculation of the dates of these events is easy using basic spherical trigonometry, yet the results for the events of the plate C1-a show quite large errors both in the sequence of the events as well as the zodiac signs in which they are reported to occur.

The visible events are observable events and these are the ones that were used by farmers, sailors, i.e they are the events of practical interest. The computation of these events is complicated as the depression of the Sun below the horizon and the altitude of the star are not known a priori. More precisely, these visible events are defined as [9]:

- <u>morning rising</u> (*mr*): this is the first day of the year that a star (or a star cluster or a constellation) becomes visible above the eastern horizon in the morning twilight (after a period when it was hidden below the horizon or when it was just above the horizon but hidden by the brightness of the Sun).
- <u>morning setting</u> (*ms*): this is the first day of the year that a star (or a star cluster or a constellation) is seen setting.
- <u>evening rising</u> (*er*): this is the last day of the year that a star (or a star cluster or a constellation) is seen rising.

• <u>evening setting</u> (*es*): this is the last day of the year that a star (or a star cluster or a constellation) is seen setting in the evening twilight.

2.2 The zodiac statements

In this type of events, the subject is always a zodiac constellation/sign, followed by the phrase “ΑΡΧΕΤΑΙ ΑΝΑΤΕΛΛΕΙΝ/ΕΠΙΤΕΛΛΕΙΝ” (begins to rise) (placing the constellation in the east). The use of the zodiac constellations together with the fact that there is always an index letter inscribed at the first scale division of each sign of the zodiac on the inner dial of the Mechanism indicate that these statements are used to mark the position of the Sun, when it crosses the boundaries from one zodiac constellation to the next.

3.Positional Astronomy for the calculation of the visible star events

In order to find a way to calculate the visible events with no free parameters, a new software program was written from first principles, using the Mathematica software package. The main idea was, for a certain depression of the Sun, to calculate the position of the star and to find a criterion to determine whether the star is visible at its position.

3.1 The calculation of the position of a star for a certain depression of the Sun

The basic equations of spherical trigonometry that were used were:

$$\cos A = \frac{\sin\delta - \sin\varphi \sin\upsilon}{\cos\varphi \cos\upsilon} \tag{1}$$

$$\sin\upsilon = \sin\varphi \sin\delta + \cos\varphi \cos\delta \cos H \tag{2}$$

$$ST = \alpha + H \tag{3}$$

where A is the azimuth and υ the altitude (the horizon coordinates), φ the geographic latitude, α the right ascension and δ the declination (the equatorial coordinates), ST the sidereal time and H the hour angle.

More precisely, using the equatorial coordinates of the Sun, the certain altitude that the Sun was set and a certain latitude, we calculate the sidereal time, for two of course cases, at sunrise and at sunset, as all 4 events have to be investigated. And as the Sun’s equatorial coordinates ($\alpha_{i\text{-Sun}}$, $\delta_{i\text{-Sun}}$, i = 1 to 365) need to be set on a daily basis, we have 2 tables of 365 values of sidereal time. Using the sidereal times found, the equatorial coordinates of the star and the same latitude, we calculate the azimuth and the altitude of the star, again for two cases, with the Sun in the east and in the west.

So for a certain latitude and a certain depression of the Sun, we have found the position of the star. In order for the depression of the Sun to no longer be a free parameter, all calculations were repeated for all values of Sun’s depression j in the range between -15° and -1°, in steps of 0.1°, thus for 141 values of Sun’s depression. Consequently 2 tables with star’s position were formed: a table of 141×365 values of ($A_{i,j\text{-Star-SunEast}}$, $\upsilon_{i,j\text{-Star-SunEast}}$), i.e. all possible star’s horizon coordinates with the Sun in the east and a table of 141×365 values of ($A_{i,j\text{-Star-SunWest}}$, $\upsilon_{i,j\text{-Star-SunWest}}$),), i.e. all possible star's horizon coordinates with the Sun in the west.

3.2 The determination of the visibility of a star

So knowing the position of the Sun and the star, the question left to be answered was: Is the star visible at its position?

Whether a star is visible at its position depends on the sky background brightness. Two sources of measured values of sky brightness were used: (a) Nawar [10] published numerical tables of the sky twilight brightness B as a function of the depression of the Sun below the horizon, the azimuthal distance of the sky patch away from the Sun and the altitude of the sky patch above the horizon and (b) Koomen *et al*. [11] also published independent measurements of sky brightness B, in the form of tables at several altitudes of the place in the sky above the horizon and several azimuthal distances from the Sun, for several depressions of the Sun.

Using the Mathematica software package, we interpolated these values, to any position on the sky, again, for any altitude of the Sun between –15.0° and –1.0° (in steps of 0.1°). So we now know, for each position of the star, the sky brightness and we need a function that determines whether the star of magnitude *m* will be visible on the given sky brightness. This function was investigated by Tousey and Koomen [12] who published values of the maximum apparent magnitude *m* of a star in order to be visible at a given sky brightness *B*. Using these values, another interpolating function was formed in order for the visibility of a star at its position on the sky ($A_{i,j\text{-Star-SunEast}}$, $\upsilon_{i,j\text{-Star-SunEast}}$) or ($A_{i,j\text{-Star-SunWest}}$, $\upsilon_{i,j\text{-Star-SunWest}}$) to be determined. Using this function, we calculated whether the star of magnitude *m* was visible or not at each of these 141 x 365 positions in the eastern and in the western sky, ending up with 2 tables of 141×365 'visible/not visible' values, again for the eastern and the western sky.

Looking through the 141×365 values of 'visible/not visible' values in the eastern sky, we record for each depression of the Sun the first and the last day of the year that the star is visible (the star rises and sets in the morning). Looking through the 141×365 values of 'visible/not visible' values in the western sky, we record again for each depression of the Sun the first and the last day of the year of the star's period of visibility (the star rises and sets in the evening). The dates of the visible *mr*, *ms*, *er* and *es* are located by examining these recordings at all values of Sun's depression.

Table 1 shows two examples of such recordings, one of the visible *mr* and one of the visible *es* of Vega in Olympia, in 150 B.C. The Sun's depression changes per 1° in these examples. The day of the visible *mr* of Vega (11/11), the first day of the year that Vega is visible and is thus at minimum altitude difference from the Sun, is easily located. The day of the visible *es* of Vega (21/1) is the last day of the year that Vega is visible and is again at minimum altitude difference from the Sun.

The depression of the Sun and the altitude of the star when an event takes place are read in the corresponding columns and, they are not, therefore, free parameters. The dates of the *ms* (the first day the star is seen setting) and the *er* (the last day it is seen rising) are similarly located.

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Table 1. Results using Nawar's sky brightness values for the visible *mr* and *es* of Vega in Olympia, in 150 B.C. ($k = 0.23$).

	Visible *mr* of Vega				Visible *es* of Vega		
Sun's altitude (°)	Date in the Julian calendar	Star's apparent altitude (°)	Difference between the Sun's and the star's altitude	Sun's altitude (°)	Date in the Julian calendar	Star's apparent altitude (°)	Difference between the Sun's and the star's altitude
-15	17/11	1.74	16.74	-15	15/1	1.35	16.35
-14	16/11	1.77	15.77	-14	15/1	1.95	15.95
-13	15/11	1.79	14.79	-13	16/1	2.01	15.01
-12	14/11	1.82	13.82	-12	17/1	2.07	14.07
-11	13/11	1.84	12.84	-11	18/1	2.12	13.12
-10	12/11	1.86	11.86	-10	19/1	2.18	12.18
-9	12/11	2.48	11.48	-9	20/1	2.24	11.24
-8	11/11	2.50	10.50	-8	20/1	2.88	10.88
-7	**11/11**	**3.14**	**10.14**	-7	21/1	2.95	9.95
-6	12/11	4.45	10.45	**-6**	**21/1**	**3.62**	**9.62**
-5	13/11	5.81	10.81	-5	19/1	5.61	10.61
-4	18/11	10.10	14.10	-4	15/1	9.09	13.09
-3	28/11	18.67	21.67	-3	5/1	17.23	20.23
-2	21/12	38.74	40.74	-2	7/12	40.04	42.04
-1	30/1	69.31	70.31	-1	17/10	71.16	72.16

4.Parameters used

As the basic spherical trigonometric equations for the computation of risings and settings are functions of the geographic latitude of the observer, the above described procedures were repeated for the location of the dates of all star events on the parapegma of the Antikythera Mechanism for all geographic latitudes in the range 25°N – 45°N (in steps of 0.5°). The calculations were done for 150 B.C. as the construction of the Mechanism has been placed around 150-100 B.C.

4.1 Stars and constellations

The equatorial coordinates of both the Sun and the stars were calculated using the procedures described by Peter Duffett-Smith [13], taking into account changes due to the precession of the equinoxes, the decrease in the obliquity of the ecliptic, nutation and aberration, as well as, for the stars, the effects of proper motion.

From the star events mentioned on Fragment C1-a, one refers to a star (Arcturus), two refer to constellations (Lyra and Aquila) and three refer to two open star clusters (Pleiades and Hyades). From the two constellations, we identify Lyra with the star Vega and Aquila with the star Altair. Concerning the two open clusters, the Hyades were identified with Aldebaran while for the Pleiades, a total apparent magnitude and mean equatorial coordinates were calculated from the ten brightest stars of the cluster (results hardly differ if one uses the total cluster magnitude).

4.2 Atmospheric refraction

Atmospheric refraction *R* causes astronomical objects to appear higher in the sky than they are in reality. Near the horizon, the altitude of a star increases significantly (by up to about 0.5°). This effect was taken into account, using the following empirical equation [14, 15]:

$$R = \frac{1.02'}{\tan\left(h + \frac{10.3}{h+5.11}\right)} \tag{4}$$

where *h* (°) is the true altitude.

4.3 Atmospheric extinction

Atmospheric extinction reduces the brightness of a star as seen from Earth. The corrected apparent magnitude *m'* of a star was calculated using the following formula [14]:

$$\Delta m = m' - m = k \cdot X = k \cdot \left[\cos Z + 0.025 \cdot e^{-11 \cdot \cos Z}\right]^{-1} \tag{5}$$

where *k* is the extinction coefficient, *X* the air mass and *Z* (°) the apparent zenith distance of the star, due to atmospheric refraction. As the extinction coefficient *k* varies with location, three different cases with *k* = 0.17, 0.23, 0.30 - covering reasonable values for ancient sites - were examined. More accurate formulas for the extinction were also tested, with insignificantly different results.

5.Results

After calculating the dates of the events, two types of analysis were done: (a) an analysis according to the sequence of the star events and (b) an analysis according to the zodiac sign to which the star events are reported to occur.

5.1 Sequence analysis

The sequence of the events of plate C1-a is compared to the calculated sequence by the program. 'Sequence errors' were determined: when two events in the parapegma are listed in the time order that agrees with the actual phenomena an error of 0 days is assigned, while in the opposite case the assigned error equals the number of days that the first event lies back of the following event. For each latitude (φ), the total sequence error (e_s) was calculated (Fig. 2).

5.2 Zodiac analysis

For the zodiac analysis, the inscription of C1-b was used. This inscription determines the relation of the days of the year to the signs of the zodiac as on this plate it was read that the autumn equinox takes place when the Sun enters the sign of Libra. Using the duration of the

Sun's passage through each one of the zodiac signs, described by Geminos [16], the calendrical boundaries of each of the signs of the zodiac were defined. 'Zodiac errors' for each of the six star events were estimated: when the event takes place within the boundaries of the correct (as mentioned on plate C1-a) zodiac sign, we assign an error of 0 days while in the opposite case the assigned error is set equal to the number of days by which the event deviates from the correct sign. For each latitude (φ), the total zodiac error (e_z) was calculated (Fig. 2).

5.3 Results and geographic latitude

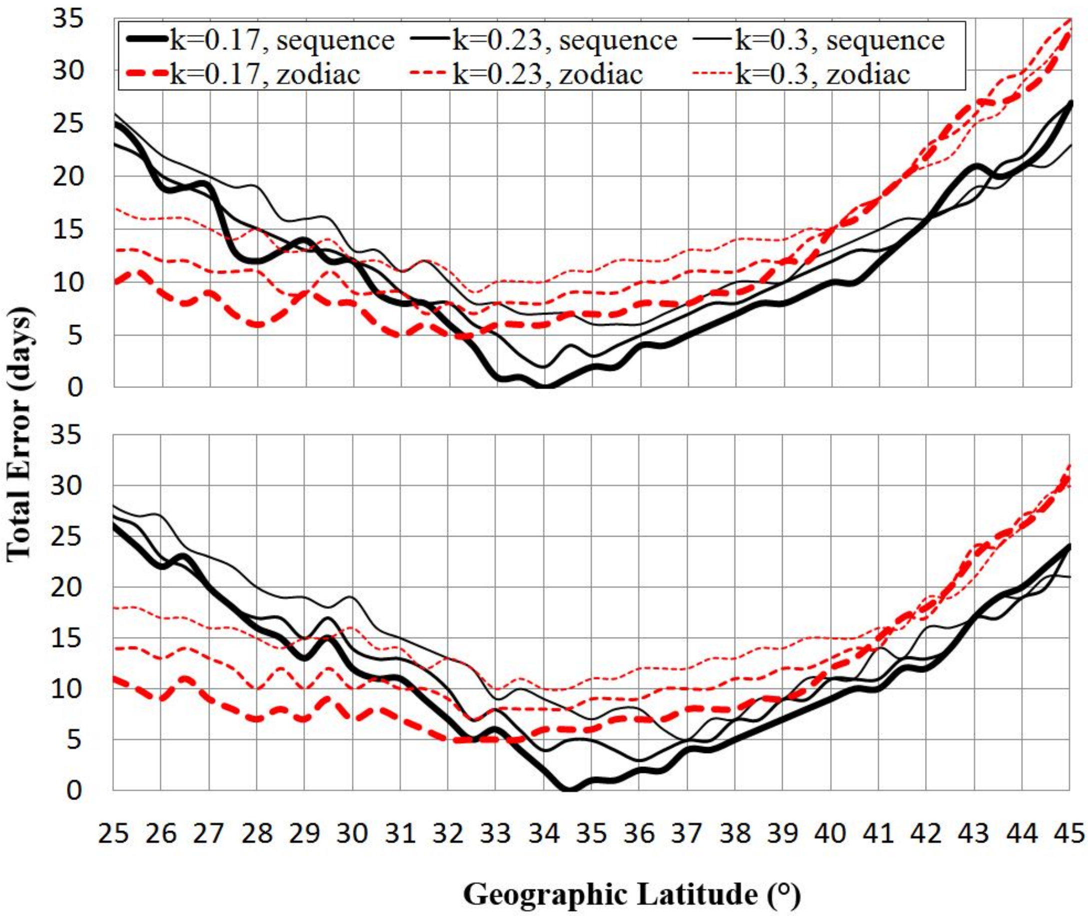

Fig. 2. The total sequence error (e_s black solid curves) and the total zodiac error (e_z red dashed curves) vs. geographic latitude for the events on plate C1-a using the Nawar (*top*) and the Koomen (*bottom*) sky brightness values. k = 0.17, 0.23, 0.30.

The errors in both analyses are much smaller than the ones found for the true events, confirming thus what it was rather expected, i.e. that the Parapegma of the Antikythera Mechanism was made of visible risings and settings.

Taking into account both the sequence and the zodiac analysis, the errors get minimized in the geographic zone between 33.3ºN – 37.0ºN. Rhodes and Syracuse lie within this zone or really close to it and they have been reported in the past as places of origin and/or use of the Antikythera Mechanism. The cities of Corcyra, Dodona and Bouthrotos, whose month names well match the luni-solar calendar of the Mechanism [17], lie between latitudes of 39ºN and 40ºN and seem thus to be less favored.

Yet, the preserved events of the parapegma are too small in number to decisively establish the latitude for which the Antikythera parapegma was composed. Furthermore, we do not really know whether the parapegma was created from direct observations or under the influence of an already existing 'body of knowledge' (which must ultimately have rested on observations of some kind) or even from rules of thumb or schematic patterns or questionable mathematical calculations. However, the fact that the Antikythera Mechanism data fit reasonably well with our numerical solutions of the real events strongly supports that most probably (a) the parapegma was accurate and (b) its data very likely originated from real observations.

6.Other parapegmata

Three other parapegmata were examined in order to check our method of calculation: Ptolemy's parapegma and Geminos' parapegmata of Euctemon and Eudoxus.

In the *Phaseis* [18, 19], Ptolemy reported a very detailed parapegma, for five different geographic latitudes. We investigated the star events referring to the *clima* of 14 hours. In the tables of ascensions in the *Almagest*, Ptolemy identifies the *clima* of 14 hours with lower Egypt and a latitude of 30°22′. Using spherical trigonometry, the latitude where the longest day was 14 hours in Ptolemy's day is found to be 30°33′. And the actual latitude of Alexandria, where Ptolemy lived and worked, is 31°13′.

Using the method described above, the latitude of best fit obtained for the *clima* of 14 hours was found to be in the range 27.8° – 31.5°N. This is a very good confirmation that our method of analysis yields sensible results.

The analysis for Geminos' parapegma of Euctemon ended up with quite large sequence error values while for Eudoxus' parapegma, false citations were detected. These two analyses marked the ability of our method to detect a problematical parapegma.

7.Conclusions

A small part of the parapegma, with seven star events and two zodiac statements, is preserved on Fragment C of the Antikythera Mechanism. A new method of astronomical analysis of these events and statements reveals that the parapegma works best for geographic latitudes in the range 33.3°N – 37.0°N. The method was also applied to other parapegmata and in the case of Ptolemy, the results corroborate very well the geographic region that Ptolemy describes. It should be pointed out, however, that (a) the limited number of preserved events and (b) the historical uncertainties of the way the parapegma was actually formed do not allow a strong statement about the geographic association of the Antikythera Mechanism to be made. Rather, the star phases can be taken as suggesting a likely band of latitudes 33.3°N – 37.0°N; and they do make it appear unlikely that the parapegma could have been designed for the northern cities of Epirus that might have been acceptable based on the calendrical data.

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