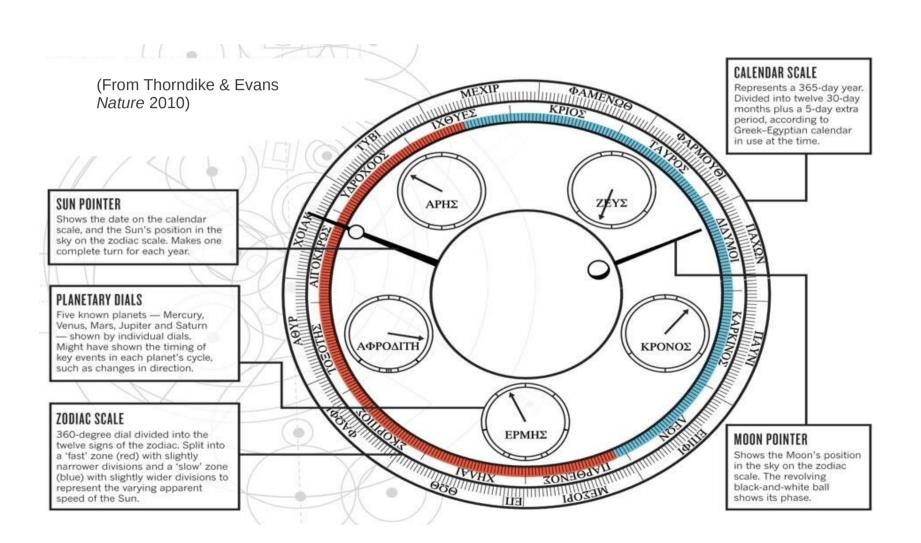
ΜΗΧΑΝΙΣΜΟΣ ΑΝΤΙΚΥΘΗΡΩΝ

M.Saul, UNSW Kerastari 2012



Epicyclic Gear Train

Planetary Gear(p)

External Central Gear: $m R_a = R_c + R_p$

$$v_p = v_a + v_{pa}, \ \omega_c R_c = \omega_a R_a$$
 - $\omega_p R_p$

$$\mathrm{F_{ au}} = -rac{ au_{\mathbf{c}}}{\mathrm{R_c}} = -rac{ au_{\mathbf{p}}}{\mathrm{R_p}} = rac{ au_{\mathbf{a}}}{\mathrm{R_a}}$$

Internal Central Gear:
$$R_a = R_c - R_p$$

$$\mathbf{v_p} = \mathbf{v_a} + \mathbf{v_{pa}}, \ \omega_c \mathbf{R_c} = \omega_a \mathbf{R_a} + \omega_p \mathbf{R_p}$$

$$\mathrm{F_{ au}} = -rac{ au_{\mathrm{c}}}{\mathrm{R_{\mathrm{c}}}} = rac{ au_{\mathrm{p}}}{\mathrm{R_{\mathrm{p}}}} = rac{ au_{\mathrm{a}}}{\mathrm{R_{\mathrm{a}}}}$$

Tooth Number Ratio:
$$\frac{Z_c}{Z_p} = \frac{R_c}{R_p}$$

3 Component Element: $(\omega_c - \omega_a)Z_c + (\omega_p - \omega_a)Z_p = 0$ (1)

$$rac{ au_{
m c}}{
m Z_c} = rac{ au_{
m p}}{
m Z_p} = -rac{ au_{
m a}}{
m Z_c + Z_p} \,\, (2) \,\, ({
m negative} \,\, {
m Z_c} \,\, {
m for} \,\, {
m Internal} \,\, {
m Gear})$$

Net External Torque:
$$\Sigma \tau = \tau_{\rm c} + \tau_{\rm p} + \tau_{\rm a}$$

$$au_{\mathrm{c}}(1+rac{\mathbf{Z_{p}}}{\mathbf{Z_{c}}}-rac{(\mathbf{Z_{c}}+\mathbf{Z_{p}})}{\mathbf{Z_{c}}})=\mathbf{0}$$

Total External Power: $\Sigma P = \omega_c \tau_c + \omega_p \tau_p + \omega_a \tau_a$

$$\tau_{\rm c}(\omega_{\rm c} + \omega_{\rm p} \frac{{
m Z_p}}{{
m Z_c}} - \omega_{\rm a} \frac{{
m Z_c + Z_p}}{{
m Z_c}}) = \tau_{\rm c} \frac{(\omega_{\rm c} - \omega_{\rm a}){
m Z_c} + (\omega_{\rm p} - \omega_{\rm a}){
m Z_p}}{{
m Z_c}} = 0$$

First Element: External Sun Gear and Planet 2

Second Element: Planet 3 and Internal Gear 4

$$\begin{vmatrix} \mathbf{a} \\ \mathbf{u} \end{vmatrix} (\omega_1 - \omega_\mathbf{a}) \mathbf{Z}_1 + (\omega_2 - \omega_\mathbf{a}) \mathbf{Z}_2 = \mathbf{0} \text{ from (1)}$$

$$rac{ au_1}{ ext{Z}_1} = rac{ au_2}{ ext{Z}_2} = -rac{ au_{ ext{a2}}}{ ext{Z}_1 + ext{Z}_2} ext{ from } (2)$$

3-4-arm:
$$(\omega_4 - \omega_a)(-\mathbf{Z_4}) + (\omega_3 - \omega_a)\mathbf{Z_3} = \mathbf{0}$$

$$(\omega_4 - \omega_a)(-\mathbf{Z_4}) + (\omega_3 - \omega_a)\mathbf{Z_3} = \mathbf{0}$$

$$\frac{\tau_4}{-\mathbf{Z}_4} = \frac{\tau_3}{\mathbf{Z}_3} = -\frac{\tau_{\mathbf{a}3}}{-\mathbf{Z}_4 + \mathbf{Z}_3}$$
 (internal central gear)

Planets 2,3 Coupled:

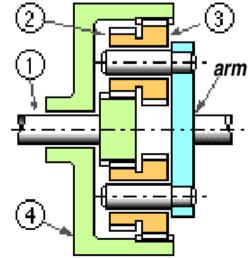
$$\omega_3 = \omega_2$$

$$\tau_3 = -\tau_2$$

$$\tau_{\mathbf{a}} = \tau_{\mathbf{a}\mathbf{2}} + \tau_{\mathbf{a}\mathbf{3}}$$

Thus
$$\frac{\omega_1 - \omega_a}{\omega_4 - \omega_a} = \sigma_0$$
,

$$\tau_1 = -\frac{\tau_4}{\sigma_0} = \frac{\tau_a}{\sigma_0 - 1},$$



where
$$\sigma_0 = -\frac{\mathbf{Z_2Z_4}}{\mathbf{Z_1Z_3}} \equiv \text{basic speed ratio} = \frac{\omega_1}{\omega_4}|_{\omega_{\mathbf{a}=0}}$$

Differential devices have other applications, from analog computers to $\mu arcsec$ astrometry and imaging, nm laser metrology...

Tracking Differential Delay

- •Measure Group Delay with varying baseline, source position, atmosphere
- •Complex Visibility whose argument is varying phase is a function of this delay
- •Phase referenced to calibrator by differential delay
- •Phase referenced to target at different frequency
- •Multical optimization, correlation techniques
- •e.g. zenith path delay error \rightarrow systematic delay difference $\Delta \tau$ (cal-tar)
- •ATCA, EVLA, GMRT, WRST, VLBA, ALMA, SKA...

