

## The Differential in Astronomy: Past to Present

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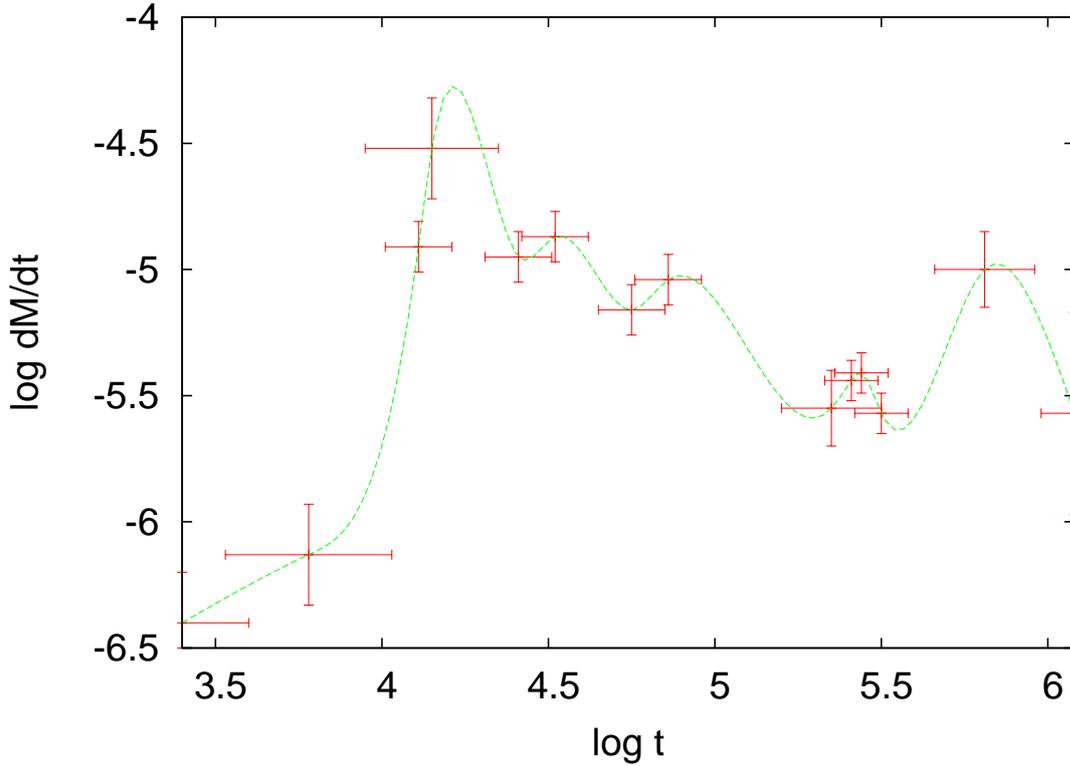
The Antikythera mechanism predates similar geared devices by over one thousand years. Here, we consider the mechanism of epicyclic gearing in the Antikythera along with modern techniques in observational astronomy such as measuring differential delays and dispersions. We can estimate individual source parameters with the former and combine them with turbulence indicators such as the latter to monitor physical processes such as star formation.

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†A footnote may follow.



**Figure 1:** Evolution of accretion.  $\dot{M}_{acc}$  in  $M_{\odot} \text{yr}^{-1}$  as a function of time in years. A critical point at  $\sim 7500$  years marks the onset of an  $\dot{M}_{acc}$  acceleration consistent with the  $\Delta(\sigma^2)$  turnover in Figure 2.

## 1. The Differential and the Antikythera Mechanism

The Antikythera mechanism was a differential device in that output gears reflected varying input velocities. Interlocking component moon gears with offset axes measured orbital eccentricities and phases according to differences between sidereal, anomalistic, and synodic months. Several functions of the device and derivation of the basic speed ratio,  $\sigma_0$ , for an epicyclic train consisting of two three component elements (central, planetary gear, and arm) are shown in slides 1 and 2 (see attachment).

In aperture synthesis observations, we multiply then integrate input voltages over time (frequency cycles) to obtain an output Fourier Transform (amplitude and phase) proportional to the cross-correlation of the electric field detected by component telescopes, i.e. according to photon path differences between components. For a two element interferometer, if the path length is  $\Delta_0$ , the multiplier output is  $AB^* = E_A E_B^* e^{ik\Delta_0}$ , where  $A$  and  $B$  are the component signals, and the path delay is the ratio of  $\Delta_0$  to  $c$ . For high resolution imaging through phase referencing target with calibration sources, we can combine synthesis astrometry with internal metrology, monitoring path differences inside the telescope as well (slide 3).

## 2. The Differential in Observational Astronomy

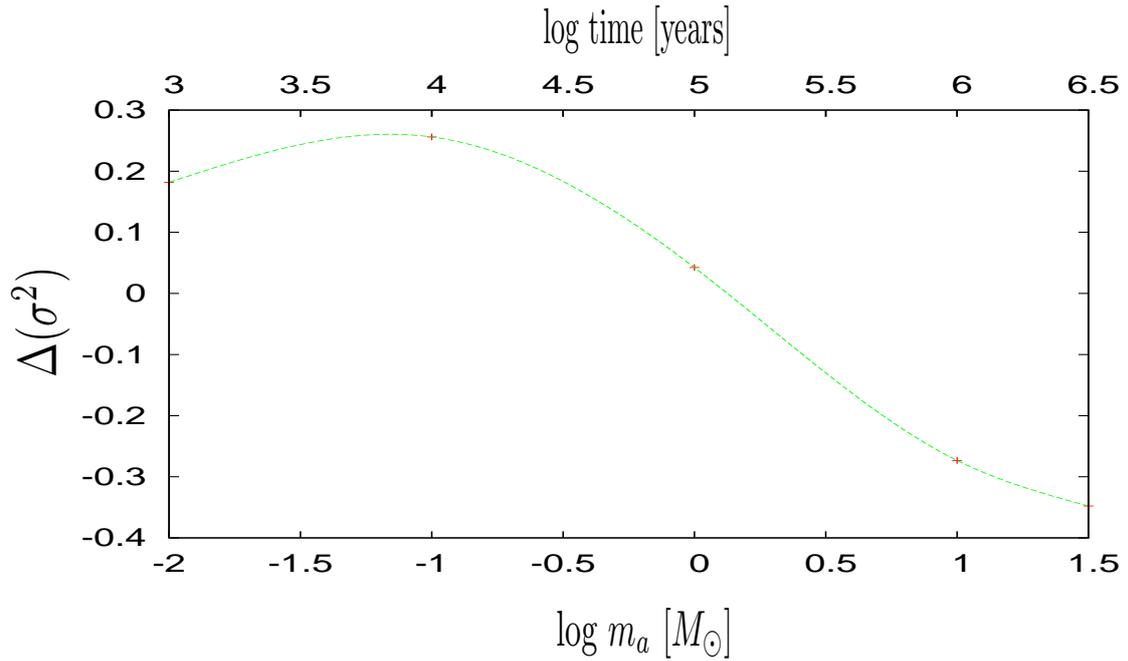
In the past, we measured the basic speed ratio of input to output speeds in an epicyclic train to estimate astronomical positions and phases. Today, we measure the differential ratio of outer to inner core dispersions in a set of protostars to trace turbulence.

### 2.1 Accretion mass parameter $m_a$

The total mass infall rate onto the equatorial plane given by  $\dot{M}_{acc}$  is determined by the equilibrium cloud structure at a large distance scale [1]. Average values of  $\dot{M}_{acc}$  in a sample of 21 southern star formation core regions decrease episodically after an initial rise (Figure 1). However, infall and outflow velocities may change significantly with mass and environment, setting the stage for an accretion mass parameter,  $m_a \equiv (\dot{M}_{acc})(t_{out})$ .  $m_a$  can account for composition and configuration variability in each region, particularly with several observed transitions. If accretion is a continuous function, then a smoothly evolving  $m_a$  may be useful in correlating physical variations with evolutionary shifts on the prestellar-PMS timeline. In particular, a star formation timeline analog of turbulence dissipation and instability induced collapse in a single core [2] may be tracked by correlating outer and inner core dispersion with a mass accounting for both outflow and infall. Estimation of time  $t_{out}$  is made by dividing outflow spatial extent by the corresponding deviation from line center;  $\dot{M}_{acc} = 4\pi r^2 \mu m_H n V_{in}$ .

### 2.2 Differential dispersion $\Delta\sigma$

The utility of  $m_a$  becomes evident in the evolution of differential dispersion  $\Delta\sigma$ , defined as the logarithmic inner core relative velocity dispersion,  $\Delta\sigma = \log \sigma_v(\text{outer}) - \log \sigma_v(\text{inner})$ .  $\Delta(\sigma^2)$  over a prestellar-PMS timeline is shown in Figure 2. The position of maximum  $\Delta(\sigma^2)$  at  $m_a \sim 0.07 M_\odot$  corresponds to an inflection point in accretion rate evolution at  $\sim 7500$  years, marking an acceleration in  $\dot{M}_{acc}$ . The coincidence of these critical points with possible disk formation ( $t_{out}$  for C2 is  $\sim 6000$  years; [3]) is indicative of the generation of supersonic turbulence at the accretion plane in molecular clouds. In this scenario, the turnover marks the birth of a protostar and onset of inner core collapse [4]; in a given set of protostars, *a falling  $\Delta(\sigma^2)$  profile indicates birthline approach*. A critical point at  $m_a \sim 1.0 M_\odot$  is consistent with the PMS transition at  $\sim 10^6$  years ( $t_{out}$  for a GMC core in NGC 3576 is  $\sim 1.2 \times 10^6$  years) where exponential growth in core turbulence ceases. Individual, beam averaged dispersion measures can have as little as  $\sim 5\%$  uncertainty; uncertainties in  $L_{bol}$  can be  $\sim 50\%$ . Assuming virialization,  $L_{acc} \propto \sigma^2$ , and the curve in Figure 2 may track a functional relationship between mass accretion and turbulence shifts over time, i.e. an evolving protostar.



**Figure 2:** Evolution of turbulence in star formation.  $\Delta(\sigma^2)$  as a function of binned  $\dot{M}_{acc}$  and  $t_{out}$  over a prestellar-PMS timeline. Values at  $\log m_a/M_\odot = -2$  correspond to seven Coalsack cores; -1: C2, CrA 359.8-17.6,  $\rho$  Oph F, LDN 162, Lup 234.9-34.8, CrA 359.9-17.9, and Sco 346.4-00.5; 0: LDN 483, Vel 267.7-07.1, Lup 242.3-39.1, and IRAS 17377-3109; 1: Car 291.6-01.9, NGC 3576; 1.5: NGC 3576, IRAS 08563-4225. Note the oldest prestellar and youngest protostar regions, C2 and CrA 359.8-17.6, have an average mass  $m_a = 0.1 M_\odot$  and time  $t_{out} \sim 10^4$  years; at this point, surface/envelope, envelope/core dispersion ratios in the two regimes are equal to one percent (1.33), allowing a smooth transition between prestellar and protostar regimes consistent with a turbulence continuum. A turnover in  $\Delta(\sigma^2)$  coincides with the onset of  $\dot{M}_{acc}$  acceleration where disks may be forming, indicating generation of turbulence at the accretion plane. In this scenario, the turnover in  $\Delta(\sigma^2)$  marks the onset of inner core collapse.

## References

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- [4] R. Larson, *Numerical calculations of the dynamics of collapsing proto-star*, *MNRAS*, 1969