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Nuclear physics from lattice QCD and effective field theory

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I present an overview of recent progress in the study of nuclear physics in lattice QCD and provide a perspective of the role of effective field theory in this context. Emphasis is placed on the physics of systems of multiple baryons, including scattering systems and light nuclei and hypernuclei. After briefly introducing relevant aspects of lattice QCD methodology, I review the recent developments that are enabling nuclear physics to be studied from the underlying theory of the Standard Model and discuss the difficulties particular to these lattice QCD calculations.

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1. Introduction

Nuclear physics entails the study of the properties and interactions of hadrons, such as the proton and neutron, and atomic nuclei and it is central to our understanding of our world at the smallest scales. At its core, nuclear physics arises from the Standard Model of particle physics which describes how matter interacts through the strong force (quantum chromodynamics, QCD), and the electromagnetic and weak (electroweak) forces. This theory was developed in the 1970s and provides an extremely successful description of our world at the most fundamental level to which it has been probed. The Standard Model has been, and continues to be, subject to stringent tests at particle accelerators around the world, so far passing without blemish. However, at the relatively low energies that are relevant for nuclear physics, calculations involving the strong interaction are enormously challenging, and to date, the only systematic tool with which to perform them is lattice QCD (LQCD). In this approach, space-time is discretised and the equations of QCD are numerically integrated on a space-time lattice; for realistic calculations, this requires highly optimised algorithms and cutting-edge supercomputing resources.

LQCD calculations have led to important insights in particle physics and are critical ingredients in the determination of the parameters of the Standard Model. In recent years the application of LQCD to the intrinsically more complex realm of nuclear physics has begun in earnest. This is a challenging task as nuclei are complicated systems with many important energy scales, from the total energy of up to a few hundred GeV, through hadronic energies and excitations $\sim \Lambda_{\rm QCD} \sim 300$ MeV, to nuclear bindings of a few MeV per nucleon, and to nuclear excitations that can be just a few tens of keV. Nevertheless, this is an important endeavour as it will ultimately place nuclear physics on the firm foundation of the Standard Model. Since parameters such as the light quark masses are tuneable in LQCD, it also offers the opportunity to study nuclear physics in the broader context of universes other than our own and to explore the apparent fine tunings that occur in nuclear physics such as the deuteron binding energy and the energy-level structure of the triple-α process. Progress over the last few years has been significant, and the goal of this contribution is to highlight this, but also to survey the aspects that require further attention.

After introducing the LQCD approach, I begin with a general discussion of the role that effective field theory has played and will continue to play in the context of connecting lattice QCD to the physical world. The discussion is made concrete by looking at recent developments in the understanding of two- and three- hadron systems in finite volumes which make use of effective field theory. Following this, I discuss recent numerical studies of multi-hadron systems, starting with two-body scattering systems and moving to two-body bound states and to larger baryon number, B. After presenting this overview of the current state of the field, I discuss current issues and future challenges that must be faced in order to provide a truly ab initio approach to nuclear physics.

2. Lattice QCD for Nuclear Physics

Of the various components of the Standard Model, the most challenging piece to deal with in the low energy regime is the strong interaction, described by Quantum Chromodynamics. These difficulties arise because of the non-perturbative nature of strong interactions at long distances ($r > 0.1$ fm). For the majority of this discussion, I shall consequently ignore the electroweak interactions and focus on QCD which is defined in Euclidean space by the partition function

$$\mathscr{Z}_{\rm QCD} = \int \mathscr{D}A_\mu \mathscr{D}\overline{q}\mathscr{D}q \exp\{-S_{\rm QCD}[A,\overline{q},q]\}\,, \tag{2.1}$$

where A_μ and q are the gluon and quark fields respectively and, defining $D_\mu = \partial_\mu - igA_\mu$ and $F_{\mu\nu} = [D_\mu, D_\nu]$,

$$S_{\rm QCD}[A,\overline{q},q] = \int d^4x \left[-\frac{1}{2}{\rm tr}F^{\mu\nu}F_{\mu\nu} + \overline{q}(i\not{D} - m)q\right]\,, \tag{2.2}$$

is the QCD action. The focus of our discussion will be on spectroscopy which is enabled by measurement of two-point correlation functions defined for some set of quantum numbers $\{Q\}$ by

$$C_{\{Q\}}(t) = \frac{1}{\mathscr{Z}_{\rm QCD}} \int \mathscr{D}A_\mu \mathscr{D}\overline{q}\mathscr{D}q \tilde{\mathscr{O}}_Q(t)\mathscr{O}_Q^\dagger(0) \exp\{-S_{\rm QCD}[A,\overline{q},q]\}\,, \tag{2.3}$$

where the composite quark-gluon operators $\mathscr{O}_Q^\dagger$ and $\tilde{\mathscr{O}}_Q$ create and annihilate states with the quantum numbers Q. For brevity, we have suppressed the spatial structure of such operators which may be used to project to a fixed momentum for example. The Euclidean time behaviour of such correlation functions is governed by the energies of QCD eigenstates with the requisite quantum numbers and by the specific forms of the operators and by determined this behaviour, the spectrum can be investigated

Since the QCD action is bilinear in the quark fields, they can be integrated exactly, leading to

$$\mathscr{Z}_{\rm QCD} = \int \mathscr{D}A_\mu \det\mathscr{M}[A] \exp\{-S_{\rm g}[A]\}\,, \tag{2.4}$$

$$C_{\{Q\}}(t) = \frac{1}{\mathscr{Z}_{\rm QCD}} \int \mathscr{D}A_\mu \tilde{\mathscr{O}}_Q(t)\mathscr{O}_Q^\dagger(0) \det\mathscr{M}[A] \exp\{-S_{\rm g}[A]\}\,, \tag{2.5}$$

where $S_g[A]$ is the purely gluonic part of the action and $\mathscr{M}[A]$ is the Dirac operator.

To render the calculation finite, we discretise space-time and impose boundary conditions (periodic in spatial directions and periodic (anti-periodic) in time for bosons (fermions)). The gluon degrees of freedom are implemented through SU(3)-valued link variables $U_\mu(x) = \exp[i\,A_\mu(x)]$ and for details of the various discretised forms of the QCD action we refer the reader to the literature, see for example, [1]. In order to perform the requisite functional integrals over the gluon fields, we use importance sampling, recognising that the factor $\mathscr{P}[U] = \det\mathscr{M}[U]\exp\{-S_{\rm g}[U]\}$ can be interpreted as a Boltzmann weight as we work in Euclidean space. By generating an ensemble of configurations of the gluon link variables according to the probability measure that this encodes, we are able to estimate the correlation function $C_{\{Q\}}$ reliably. For N configurations, $\{U_\mu^{[1]},\ \ldots,\ U_\mu^{[N]}\}$,

$$C_{\{Q\}}(t) = \frac{1}{N}\sum_{c=1}^{N} \tilde{\mathscr{O}}_Q(t)\mathscr{O}_Q^\dagger(0)\left[U_\mu^{[c]}\right] + \mathscr{O}\left(N^{-1/2}\right)\,, \tag{2.6}$$

with uncertainties that decrease as the size of the ensemble increases.

To generate such ensembles, requires algorithms that efficiently and effectively explore the space of possible gluon configurations. Because of the 4-dimensional nature of space-time, and the non-locality of $\mathscr{P}[U]$, this is a challenging problem, requiring supercomputing resources. The necessary methods have been developed and improved for many years, culminating in the last

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few years in relatively precise determinations of a number of quantities of importance to particle physics and of the baryon number $B = 0,1$ ground state hadron spectrum with fully controlled statistical and systematic uncertainties [2] (these results have been nicely reviewed recently in Refs. [3, 4]). This is an important achievement as, when compared to experimental measurements, it demonstrates that QCD describes the strongly interacting regime of the strong interaction (a fact that deserves more recognition than it receives) and that lattice QCD provides a systematic tool for the computation of hadronic contributions to Standard Model observables. It also demonstrates that the field of LQCD is at a point where more computationally challenging problems, such as many of those encompassed by nuclear physics, can begin to be tackled.

A number of groups have recently taken up the task of applying LQCD to the nuclear physics of few hadron systems and significant progress has been made. A discussion of this progress, its interconnections to effective field theory, and the issues that still remain is the subject of this review.

3. The role of effective field theory in lattice QCD

In this discussion, I focus on the role of EFT in regard to nuclear physics *and* lattice QCD. The contribution of E. Epelbaum to this conference provided a broader overview of effective field theory in nuclear physics as a whole.

In the last few years, the combination of algorithmic improvements and the increased computational resources that have been available to lattice QCD researchers has meant that calculations of simple quantities are now performed at, or very close to, the physical values of the light quark masses. This has removed the need for, or at least lessened the impact of, extrapolations of results with heavier quark masses to the physical values. Such extrapolations have previously been guided in a model independent manner by chiral perturbation theory (see Ref. [5] for a review) and represent a major point of intersection of the two fields but one that will ultimately not be necessary as calculations are performed at the physical masses. Extrapolations based on generalisations of chiral perturbation theory are even used to take into account the polynomial effects of the lattice discretisation, although some care should be taken as logarithmic dependence of low-energy constants on the lattice spacing arising from short distance contributions is not captured in such theories [6].

Lattice calculations are necessarily performed in a finite volume but we are primarily interested in the infinite volume limit of these calculations. The imposition of boundary conditions to reduce the system to a finite volume is an infrared modification of the theory for a large class of such boundary conditions and so is amenable to treatment in the appropriate effective field theory. For single hadron correlation functions in the low energy regime, chiral perturbation theory therefore allows a model-independent extrapolation to infinite volume and will continue to play this important role in the future. For infinite volume extrapolations of higher energy observables, such as form factors at momentum transfers, $Q \sim 1$ GeV, we must currently rely on more phenomenological approaches.

Multi-hadron systems present a more complex problem, and one in which effective field theory plays an even more important role. Understanding finite volume systems with the quantum numbers of multiple hadrons is in essence a matching of QCD correlations onto an effective hadronic description and this hadronic description is central in defining the observables that can be computed. Hadrons are emergent collective degrees of freedom produced by QCD dynamics but are the degrees of freedom that we are necessarily interested in as they define the asymptotic states of the infinite volume theory. QCD correlations that are amenable to an *effective* hadronic description are those in which it makes sense to consider dominant contributions from a finite, ideally small, number of hadronic degrees of freedom. Chiral perturbation theory provides such a description at low energy, but the concept is somewhat more general. For example, an effective hadronic description of $B = 1$, $I = \frac{1}{2}$, $J^P = \frac{1}{2}^+$ matrix elements of the vector current at multi-GeV momentum transfer exists, but a chiral expansion of the process is not useful because of the large momentum scale. In lattice calculations of the three point functions that probe such a matrix element, the dominant contribution at long times comes from single nucleon states at rest and boosted to the high energy scale (ignoring discretisation artifacts) with contributions from internal excitations of the nucleon and nucleon+pion states suppressed in correlation functions by the energy gap.

For multi-hadron systems, an effective hadronic description allows us to make the connection between Euclidian space lattice calculations and hadronic quantities that we observe in the Minkowski space of the real world. Provided an effective hadronic description exists, analytic continuation is performed by understanding the analytic structure of the hadronic theory. If the hadronic description is too complex to determine cleanly, analytic continuation becomes ambiguous. An example is provided by correlators with the quantum numbers of a minimum of two stablehadrons (for example, $I = 2$, $J = 0$, even parity, corresponding most simply to two pion systems). In a finite volume, and at energies far below the inelastic threshold, the long time behaviour of the hadronic theory is dominated by two-body states of the lightest hadrons that can produce the required quantum numbers. In the absence of bound states, the most important contribution is from the two-hadrons at rest in the corresponding centre-of-mass (CoM) frame, but sub-leading contamination arises from two hadron state moving back-to-back in the CoM frame and from internal excitations of the hadrons. By careful analysis of multiple different correlators with the given quantum numbers, it is possible to extract information about these states. Crucially, the hadronic description allows us to understand the analytic structure of the scattering amplitude in this regime and determinations of the energies of these states translate to extractions of the scattering phase shift. At earlier times, or with more complicated sets of correlation functions, contributions from higher energy states of the given quantum numbers begin to be resolved and eventually more complex $N > 2$ hadron states make significant contributions, resulting in more complicated analytic structure in the correlation function. If we can construct an effective hadronic description in this regime and determine enough information (at high enough precision) about the various contributions by matching to the finite-volume, Euclidean QCD correlators, we can in principle determine infinite volume Minkowski space information in the inelastic regime. If such a hadronic description is ambiguous, then model dependence necessarily arises. It should be noted that the effectiveness of a hadronic description is volume dependent; as the volume increases, the number of finite volume states in a given energy range increases making a hadronic description more cumbersome. If one is interested in information about states other than the ground state, this becomes increasingly difficult to reliably extract – an important part of the obstruction discussed by Maiani and Testa [7].

4. Multi-hadron processes – theoretical understanding

In the last few years, there has been significant activity devoted to generalisations of the so-

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called Lüscher formalism. Following earlier works on quantum mechanical systems, Lüshcer showed [8, 9] that the spectrum of two-particle states in a Euclidean quantum field theory in a finite volume is connected in a calculable way to the phase shift of the two-particle interaction and can thus be used to determine the phase shift at discrete values of energy up to the inelastic threshold. This formalism has been used extensively to study the scattering phase shifts of many different meson-meson, meson-baryon and baryon-baryon systems as will be discussed below.

Significant complications are introduced by the breaking of rotational symmetry that is caused by the lattice regulator and the finite volume spatial boundary conditions. Eigenstates of the lattice calculation are classified by the irreducible representations of the appropriate symmetry group (for a cubic spatial volume and a spatially isotropic discretisation, this is the octahedral group and its double covering) and hence determine a combination of the infinite volume partial wave phase shifts. The symmetries are further reduced when the eigenstates that are studied are boosted relative to the lattice boundary conditions or when a rectangular rather than cubic spatial volume is considered. The case of moving systems was first considered by Rummukainen and Gottlieb [10] and has been significantly clarified and further generalised in recent years [11, 12, 13, 14, 15, 16, 17, 18]. Nevertheless, by boosting the system, many more energy eigenstates can be accessed in a single lattice calculation, resulting in a more detailed extraction of phases shifts Boosted systems of unequal mass have been treated in Refs. [19, 20] and boosted bound states have been investigated in Refs. [21, 22]. Scattering in the case of multiple channels has been considered in a number of contexts in Refs. [23, 16, 17, 15] and a detailed investigation of higher partial waves is presented in Ref. [24] .

A particularly simple way to arrive at the eigenvalue equation that defines the Lüscher formalism is to consider the scattering process in terms of a hadronic effective field theory as first proposed in [25], and this approach has been taken up in the analysis of various two-particle channels in Refs. [26, 27, 28, 29, 30, 31, 32]. A number of other interesting developments have also been reported recently [33, 34, 35, 36, 37].

For three body systems, there have been interesting investigations based on effective field theory presented in Refs. [38, 39, 40, 41, 42, 43, 44]. The program developed in these works in principle allows multi-body interactions to be extracted from a detailed analysis of the spectra of two- and three-hadron systems at finite volume. However, such an analysis has only been attempted in the case of pion systems [45, 46, 47, 48, 49, 50]. Further work in this direction is required to be able to make full use of numerical calculations of $N > 3$ hadron systems where we can presently only interpret the bound states of the system.

5. Hadron-hadron scattering

The simplest multi-hadron systems to address are two-body systems, and numerical investigations encounter such systems in two contexts, in direct studies of systems corresponding to scattering states and resonances, and in multi-hadron contributions to the excited meson and baryon spectrum (see Ref. [51] for a recent investigation). In what follows we will overview recent direct studies of scattering

There have been a number of recent investigations of various meson-meson scattering channels. For $\pi\pi$ scattering, the isospin $I = 2$ channel is numerically the simplest to study and has received significant attention [52, 53, 54, 48, 49, 55], and an important achievement has been an extraction of the d-wave phase shift [56]. The $I = 0$ channel is technically more demanding as it involves quark-line disconnected contractions but it is phenomenologically interesting and is also important in the analysis of $\Delta I = 1/2\ K \to \pi\pi$ decays and has also been studied recently [57, 58, 54]. The $I = 1$ P-wave $\pi\pi$ channel contains the ρ, the prototypical resonance (see Ref. [59] for a review of resonance studies), and many groups have recently presented investigations [60, 61, 62, 63, 64]. A particularly clean study on anisotropic lattices, showing the structure of the resonance has appeared very recently [51]. Systems involving kaons and heavy mesons have also been investigated, with studies of $K\pi$ scattering in $I = 1/2,\ 3/2$ [65, 66, 48, 67, 68], KK scattering in $I = 1$ [48], $D\pi$ scattering [69, 70], DD scattering [71], $J\psi$–ϕ scattering [72] and finally of Υ–π and η_b–π scattering using lattice non-relativistic QCD for bottom quarks [50].

Meson-baryon systems are of significant phenomenological interest as, at least for πN, they can be studied experimentally. It is also possible that the Kn interaction plays an important role in the interior of dense stars where a kaon condensed phase may appear [73] depending on the strength of various interactions amongst kaons and nucleons (KK, Kn, Knn,...). In the last few years, not much attention has been focused on numerical studies of meson-baryon interactions. The only recent study has been of negative parity πN scattering [74] (see also Ref. [75]) although there have been a number of phenomenological investigations referred to above.

Baryon-baryon scattering is also of great phenomenological interest, and for the nucleon-nucleon system, the results of many decades of experimental investigation offer the possibility of precision tests of lattice methods for two-hadron systems once calculations can be performed at the physical quark masses and the systematics of the lattice method are accounted for. Other baryon-baryon scattering channels are difficult to access experimentally and LQCD offers the prospect of providing more precise determinations of phase shifts than can be made experimentally. Such determinations would also materially improve our understanding of various aspects of nuclear astrophysics. There have been a number of recent investigations of the baryon-baryon scattering parameters and phase shifts. In Ref. [76], hyperon–nucleon scattering and the consequences for dense nuclear matter were investigated, while Ref. [77] presents a study of the NN scattering lengths and effective ranges in $N_f = 3$ LQCD at the physical strange quark mass, while all octet-baryon–octet-baryon channels are investigated by HALQCD in $N_f = 3$ [78, 79] and $N_f = 2+1$ [80] LQCD using a method based on determining energy- and sink-dependent potentials at one energy and constructing phase shifts by ignoring the energy-dependence. LQCD also allows the investigation of more exotic scattering processes such as Ω–Ω, as investigated in Ref. [81].

6. Nuclei

6.1 Dibaryons

The last few years have seen remarkable progress in lattice calculations of baryon number $B = 2$ systems (dibaryons). In Ref [82], the first calculation of a QCD bound-state with $B > 1$ was presented, albeit at unphysical values of the quark masses corresponding to $m_\pi \sim 390$ MeV. That calculation concerned the so-called H-dibaryon postulated many years ago by R. Jaffe [83]. Subsequent works have considered the H-dibaryon further and have also looked at the deuteron,

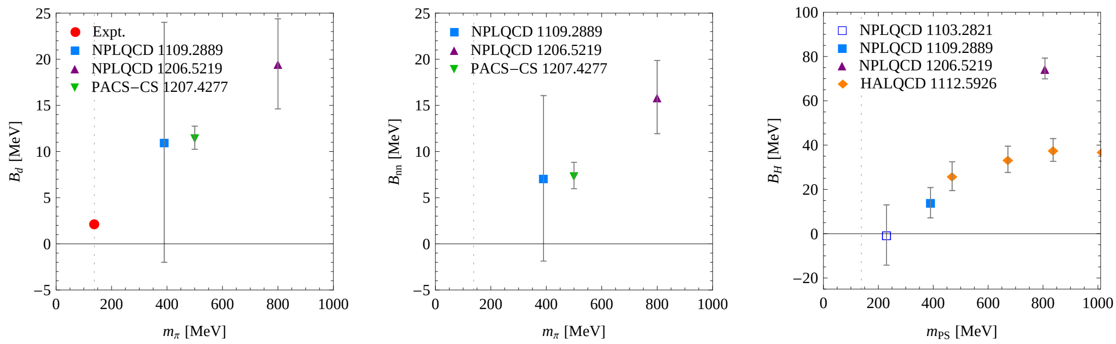

Figure 1: Summary of the results obtained in lattice QCD ($n_f = 2+1$ or 3) calculations of the binding energies of the deuteron, di-neutron and H-dibaryon. In the case of the deuteron, the red circle corresponds to the physical binding energy. For the H-dibaryon, the results labelled HALQCD and NPLQCD 1206.5219 use three degenerate flavours of quarks and the point at $m_\pi = 230$ MeV (NPLQCD 1103.2821) should be treated with caution as no infinite volume extrapolation has been performed.

di-neutron and other more exotic channels [84, 85, 86, 79, 87, 88]. The results of these calculations are summarised in Figure 1 for the deuteron, di-neutron and H-dibaryon (for other channels, the reader is referred to the original works). In both the two nucleon channels, it is apparent that these systems become more bound as the quark masses increase and a naive linear fit suggests consistency with the bound deuteron and near threshold di-neutron system at the physical mass. The H-dibaryon is predicted to be very close to threshold at the physical quark masses, but further calculations at lighter quark masses are required to directly ascertain its nature and, as it appears to be a finely-tuned system, care must be taken to ensure that the effects of discretisation, isospin breaking and electroweak contributions are correctly accounted for. In the case of the H-dibaryon, it is also apparent that there is a significant discrepancy between the $SU(3)_f$ symmetric NPLQCD and HALQCD calculations at $m_\pi \sim 800$ MeV which is further exacerbated by the non-observation of bound deuteron and di-neutron states at this mass by the HALQCD collaboration [79]. These two sets of calculations have fairly similar lattice discretisations and volumes with minor differences that are unlikely to account for the discrepancy. These calculations also differ in methodology, with NPLQCD (and PACS-CS) essentially performing spectroscopy (see Figure 2 for representative effective mass plots from the study in Ref. [87]) and HALQCD using the "potential method" (see Ref. [89] for a review). This suggests that there may be systematic effects that are underestimated in one or both approaches and it is important to resolve this discrepancy.

6.2 Many-baryon contractions

The issue of how to efficiently compute the many Wick contractions required in systems of many baryons has received attention recently. Using interpolating operators built from only the lowest possible Fock component, a two point correlation function of atomic number A has $3A$ quark creation and annihilation operators that must be contracted in all possible ways, leading to a factorially difficult problem that scales as $\prod_f N_f!$ where N_f is the number of quarks of flavours, and the product runs over all such flavours, f. The presence of symmetries, Pauli blocking and cancelations amongst contractions means that this counting can be a vast overestimate, but determining the minimal set of contractions is a non-trivial task. In Refs. [90, 91], algorithms have been

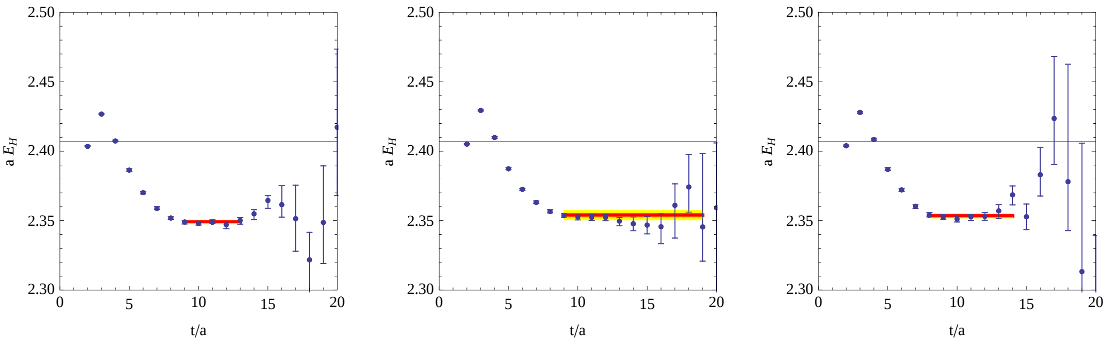

Figure 2: Effective mass plots for one of the H-dibaryon correlators studied in Ref. [87] on three different volumes (3.4, 4.5 and 6.7 fm from left to right, respectively) that lead to the results indicated by the upright triangle in the right panel of Fig. 1. The horizontal line indicates the two hadron threshold.

developed that confront this issue and aim to allow for calculations of systems with large numbers of baryons. The fundamental approach used in both of these works is to perform contractions by iterating over a simplified list of indices of quark fields and corresponding weights. Where they differ is in the method used to construct such lists; Ref. [90] iterates over the full, factorially-large set of possible index values, whereas Ref. [91] constructs the index lists recursively by building up a multi-baryon system a single baryon at a time. The former algorithm suffers from a scalability issue and requires supercomputing resources [90] to construct lists even for $A = 4$, and, given this, it is unlikely that the approach can be usefully applied far beyond that point. In contrast, the recursive approach of Ref. [91], running on a laptop, has been used to produce contraction code for a large range of nuclei including ^{4}He, ^{8}Be, ^{12}C, ^{16}O and ^{28}Si and these codes have been used to compute correlation functions with the quantum numbers of these systems for the first time. Very recently, a recursion-based multi-baryon algorithm has appeared in Ref. [92] that additionally performs the multiplications of terms in the index lists recursively.

6.3 Larger nuclei and hypernuclei

The study of baryon number $B > 2$ multi-baryon states began a few years ago with a high statistics study of the $\Xi^0\Xi^0 n$ and triton systems by the NPLQCD collaboration [93] at light quark masses corresponding to $m_\pi \sim 390$ MeV. This study was followed up by the PACS-CS collaboration who have investigated ^{3}He and ^{4}He first in quenched QCD [94] and very recently in QCD with quark masses corresponding to $m_\pi \sim 500$ MeV [88]. In both latter studies, the binding energies of these states were found to be remarkably close to those measured in experiment despite the unphysical nature of the calculations. After developing new contraction methods discussed above [91], the NPLQCD collaboration have performed a comprehensive calculation of a large number of phenomenologically relevant nuclei and hyper-nuclei for $A < 5$, albeit at a heavy quark mass corresponding to $m_\pi \sim 800$ MeV [87]. Figure 3 shows the binding energies of the various nuclei and hypernuclei up to baryon number $B = 4$ that were extracted in Ref. [87].

The improved contraction methods discussed above also enabled the construction of correlation functions with the quantum numbers of significantly larger nuclei such as ^{8}Be, ^{12}C, ^{16}O and ^{28}Si [91], opening the way for studies of these systems. Examples of these correlations are shown in Fig. 4, and while the correlators for $A < 20$ show signs of the expected approach to single expo-

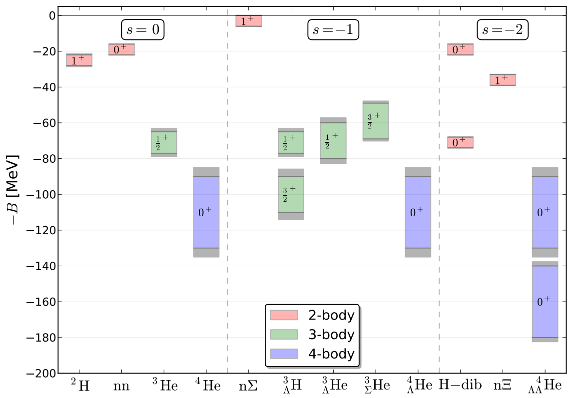

Figure 3: Spectrum of light nuclei and hypernuclei studied in Ref. [87].

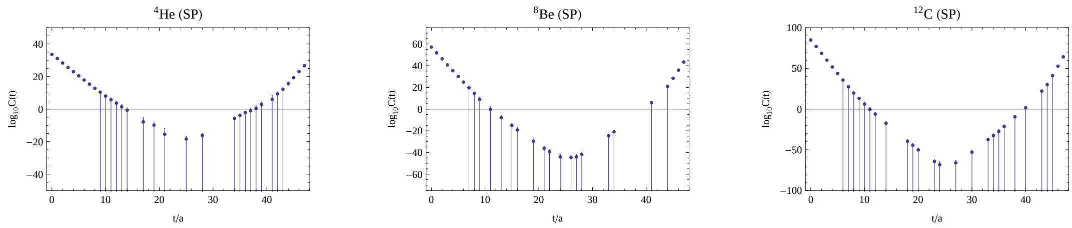

Figure 4: Correlators for larger nuclei studied in Ref. [91].

nential behaviour, no statistically meaningful binding energies could be extracted at the statistical precision used in this preliminary investigation. Indeed, even with high statistics, the presence of very closely spaced excitations in these complex systems will make extraction of the ground state energy a challenge as will be discussed below.

7. Current issues and future challenges

7.1 Statistical precision

Since nuclear physics entails small energies on the scale of QCD, high precision calculations are important, requiring precise statistical sampling of correlation functions The choice of interpolating operators that are used for a particular set of quantum numbers is important and can be used to some extent to delay the onset of the noisiest contributions to a given correlation function [75].

The noise phenomenon is related, at least indirectly, to the so-called *sign problem* [95, 96] that plagues LQCD calculations at non-zero quark chemical potential. A recent study of the 2+1 dimensional Nambu–Jona-Lasinio model [97] (a model similar in some regards to QCD) has suggested that the noise problem is intimately related to spontaneous chiral symmetry breaking and that the sign problem in QCD at non-zero baryon chemical potential may be ameliorated by formulating the calculations in terms of different degrees of freedom that treat pions explicitly (no specific method to do this is proposed). This is an intriguing analysis that also has a bearing on the issue of noise.

7.2 Beyond spectroscopy and scattering

While most effort currently focuses on the spectroscopy of multi-hadron systems, these systems also present a rich set of more complicated observables that are of phenomenological interest. For example, a precise determination of the matrix elements of the axial current in two-nucleon systems would impact our understanding of the pp fusion process that powers the sun and the $\nu\, d \longrightarrow n\, p$ breakup process used as a neutrino detection mode in the Sudbury Neutrino Observatory. In Refs. [98, 99], the problem of determining such matrix elements has begun to be addressed. It is also of some interest to determine the matrix elements of unstable states such as the Δ or ρ resonances (although care needs to be applied in their definition) and in Ref. [100] a possible approach to the problem was discussed. In Ref. [101], a first numerical investigation of matrix elements of multi-hadron systems has been presented although it is still a work in progress. Here, the focus has been on determining the first moment of the parton distribution of systems with the quantum numbers of $n = 1,\ldots,12$ pions. Since the operator insertion is local, these results can be interpreted as the modification of the single pion parton distribution in a medium with varying isospin charge and is a direct analogue of the famous EMC effect in nuclei. Clearly, many improvements and advances in our theoretical understanding are required to allow QCD calculations of a large range of multi-hadron properties and transitions that are of interest to the experimental and phenomenological nuclear physics communities

7.3 How large is a large volume?

In the analysis of two particle systems in finite volume, a requirement for the validity of the Lüscher approach discussed above is that the system size is large compared to the range of the interaction which is typically set by the Compton wavelength of the pion (at least for light pions; at large quark masses, other scales become important [77]). For bound states, the lattice volume must additionally be large compared to the size of the bound state, the scale of which is set by the binding momentum of the system, γ; this second constraint is more stringent for shallow bound states which can be large even on the scale of the pion Compton wavelength. As discussed in Ref. [102], the Lüscher method also requires a volumes large enough that the density of states is such that finite volume sums provide a good approximation to infinite volume integrals. There remains a question as to what large means: does $m_\pi L$, $\gamma L > 4$ suffice? Or are the requirements more stringent? Precise calculations are required to address this question, both at "large" volumes in the region where asymptotic behaviour can be confirmed, but also in smaller volumes where deviations from theoretical expectations can be demonstrated. For precise results, it is important to mark out the region in which systematics such as these are well controlled.

It is likely that a number of earlier calculations with $m_\pi L \sim 4$ have additional systematic uncertainties from volume dependence that is not controlled by the Lüscher approach (particularly is cases where there is a shallow bound state). For example, in the NPLQCD calculations of bound states at $m_\pi \sim 390$ MeV [82, 85], data for $L = 2.0$ and 2.5 fm (corresponding to $m_\pi L = 3.9$ and 4.9, respectively) was dropped and similar exclusions may need to be made elsewhere.

7.4 Lattice Potentials

The HALQCD collaboration has pursued a method of extracting inter-hadron potentials for two-baryon and three-baryon systems (see Ref. [89] for a review). For two hadrons, the approach proceeds by constructing a Nambu-Bethe-Salpeter wavefunction from correlators that separate the two hadrons at the sink by a distance $\mathbf{r}$ (which is defined up to an uncertainty of $\mathcal{O}(\Lambda_{\rm QCD}^{-1})$).[1] Since the potentials that have been extracted are local, they are by definition, energy dependent and, from an ab initio point of view, contain exactly the same information as the phase shift evaluated at the energy of the two hadron system in the lattice calculation. With the assumption of slowly varying behaviour of the phase shift, small extrapolations in energy may be justified; however such assumptions become invalid when the system becomes interesting because of resonance structures and threshold effects. However, extrapolations of phase shifts to $p \sim 300$ MeV from two-baryon systems calculated essentially at rest (such as those presented in Refs. [79, 89, 80]) should be viewed with caution. The potentials extracted also depend on the sink-interpolating operators used in the calculation [105, 106, 107] with significant modifications seen at short hadron separations from different smearings of the quark fields [108] for example. This is expected as potentials are not observable quantities. Indeed, the use of different interpolating operators results in the construction of different potentials that should be phase-shift equivalent at the given energy, but will produce different phase shifts at other energies. In Ref. [107], M. Birse highlighted the ambiguities associated with potentials through the simple example of an attractive square well potential with a repulsive delta function coupling to an excited state at short distance.

7.5 Spectral gaps, large volumes and the approach to the chiral limit

At finite volume, multi-hadron two-point correlation functions for large Euclidean times are dominated by exponentials corresponding to a series of poles in energy arising from states is which the two hadrons move with back-to-back momenta in their CoM frame (as discussed above, four- and higher-particle contributions are expected to dominate eigenstates of higher energy). For weakly interacting states, the gaps between these states are typically given by the difference between E_n and E_{n+1} where $E_n = 2\{m^2 - \frac{4\pi^2}{L^2}n\}^{1/2}$ for two identical hadrons (since $n = |\mathbf{n}|$ with integer triplets $\mathbf{n}$, there are some levels that are not allowed, for example $n = 7$). As L increases, these gaps shrink rapidly (quadratically in the case of two hadrons) and it becomes hard to isolate these levels. Variational approaches can help to some degree, but as the states collapse towards each other, they are by definition becoming more and more similar, so diagonalisation of correlator matrices will become an almost degenerate problem. Examples of this increasing level density for increasing volume are given in the appendices of Ref. [87] for ^{4}He. As pointed out in Ref. [77], in the two-body sector this is further manifest in that the poles in the Lüscher eigenvalue equation accumulate at threshold with extracted energy levels of a given precision starting to straddle these singularities making extraction of phase shifts difficult.

This problem is not specific to the large volume limit. As the quark masses are decreased toward the chiral limit, pions become lighter and lighter. Consequently, the spectrum becomes denser as, for a given choice of quantum numbers, the energy gap between the ground state and states that include additional pions (this is very loose terminology as the ground state should not be thought of as a bare object to which pions are added to get an excited state) tends to zero. The problem is compounded by the fact that one needs to take the infinite volume limit before, or at

[1] The method follows that of Refs. [103, 102, 104], applied for hadron separations inside the range of the interaction.

least in combination with, the chiral limit so that finite-volume distortion of individual hadrons remains exponentially small.

This issue is one of the major challenges that must be addressed to open a path towards nuclear physics at the physical quark masses and seems to require a significant conceptual advance.

7.6 Electroweak effects

None of the calculations discussed above include the effects of the electroweak interaction. For simple quantities such at the ratio of pion and kaon decay constants, f_π/f_K, lattice calculations are attaining the level of precision where electromagnetic effects are important and attempts are being made to include them (see the contribution of L. Lellouch for a discussion of precision lattice calculations and EM effects and Ref. [109] for a recent review). In the future such effects must also be included in calculations relevant for nuclear physics. Indeed as the number of protons increases, so does the charge and the importance of electromagnetic effects, eventually overcoming the smallness of $\alpha_{em} \sim \frac{1}{137}$, making inclusion of electromagnetism even more important in nuclei.

8. Conclusions

Over the last decade, lattice QCD has realised its potential and become a precision tool for the calculation of hadronic contributions in particle physics, thereby becoming a crucial part of our understanding of the Standard Model and of the search for physics beyond it. Nuclear physics presents a new frontier for LQCD as it involves intrinsically more complex systems with multiple length scales. As well as providing tests of the Standard Model in a new regime, the ab initio approach to nuclear physics from the underlying Standard Model offers exciting opportunities to make reliable predictions in cases that are difficult or even impossible to access experimentally. It also presents new challenges, both conceptually and numerically, that we are just beginning to uncover.

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