PROCEEDINGS OF SCIENCE

Confining gauge theories with adjoint scalars on $R^3 \times S^1$

Hiromichi Nishimura[*†]
University of Bielefeld
E-mail: nishimura@physik.uni-bielefeld.de

Michael Ogilvie
Washington University, St. Louis
E-mail: mco@physics.wustl.edu

Recent work on QCD-like gauge theories on $R^3 \times S^1$ has shown that we can study confinement both perturbatively using the effective potential of the Polyakov loop and nonperturbativley using the semi-classical evaluation of monopoles and instantons. We extend the theory with an adjoint scalar field and use a deformation potential inspired by two-dimensional fermions with periodic boundary conditions, which unlike the previous models give a second-order phase transition. The model shows a rich phase structure, including a new confined phase where the Polyakov loop mixes with the scalar field. This new phase in turn shows that the confined phase is incompatible with the Higgs phase. Moreover, the mixing gives rise to topological objects that generalize the instanton constituents of BPS and KK monopoles in Euclidean space, which are then related to infinite sum of Julia-Zee dyons in Minkowski space by Poisson duality. All phases in the model are connected by a dilute monopole gas, and the string tension associated with Wilson loops orthogonal to the compact direction can be computed using Abelian duality.

Xth Quark Confinement and the Hadron Spectrum,
October 8-12, 2012
TUM Campus Garching, Munich, Germany

[*]Speaker.

[†]The author acknowledges the support by the Sofja Kovalevskaja program of the Alexander von Humboldt foundation.

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1. Introduction

Recent work on confining gauge theories on $R^3 \times S^1$ has revealed that a confined phase can exist at small circumference of S^1 if certain deformations or fields are added to pure gauge theories; see [1] for a review. The use of $R^3 \times S^1$ with a small circumference, as opposed to R^4, makes the gauge coupling small. Thus we now have four-dimensional field theories in which we can study confinement using semiclassical methods.

With confinement in the pure gauge theory on $R^3 \times S^1$ under analytic control, we extend these results by introducing scalar fields [2]. Together with a deformation term, the scalar potential added to the model will allow us to examine the relationship between confinement and the Higgs mechanism and to explore what turns out to be a very rich phase structure.

2. The effective potential

We consider deformed $SU(2)$ gauge theory with a scalar field in the adjoint representation. The phase diagram of our model can be constructed from an approximate form of the one-loop effective potential, including the deformation term. The effective potential can be calculated in background field gauge, with the background fields for scalar field, $\phi = (0,0,v)$ and the Polyakov loop, $P = \exp\left(ig\int_0^L dx_4 A_4\right) = \mathrm{diag}\left[\exp(i\theta), \exp(-i\theta)\right]$, where g is the gauge coupling. The total one-loop effective potential, U, constitutes of three parts when the circumference of S^1, L, is small:

$$U = V_c + V_L + V_d \tag{2.1}$$

where V_c is the classical contribution, which is the sum of the kinetic term and the scalar potential

$$V_c(\phi) = g^2 \mathrm{Tr}_F \left[A_4, \phi\right]^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4} \lambda \left(\phi^2\right)^2. \tag{2.2}$$

The positivity of the kinetic term for the adjoint representation implies that the effective potential will be minimized if $[A_4, \phi] = 0$. The finite-L effective potential, V_L, of both the gauge fields and adjoint scalar field can be written as

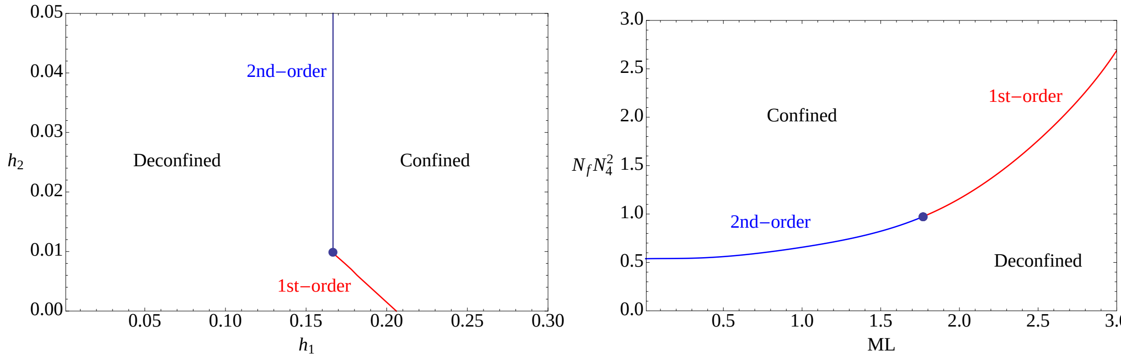
$$V_L = \frac{2\pi^2}{L^4} B_4\left(\frac{\theta}{\pi}\right) + \frac{\left(m^2 + \lambda v^2 + 3g^2 v^2\right)}{2L^2} B_2\left(\frac{\theta}{\pi}\right) + \frac{\lambda v^2}{4L^2} \tag{2.3}$$

where B_k is the Bernoulli polynomial.

In order to realize the confined phase for small L, we will add a double-trace deformation term V_d. This term will be a $Z(N)_C$-invariant function of P, and therefore will be nonlocal in the compact variable x_4. Many forms of V_d may be used, such that the confined phase is favored for some range of parameters. We consider two forms which give rise to the second order phase transition. First one takes the form

$$V_d = h_1 L^{-4} \left(\mathrm{Tr}_F P\right)^2 + h_2 L^{-4} \left(\mathrm{Tr}_F P\right)^4. \tag{2.4}$$

For sufficiently large $h_1 > 0$, the symmetry will be restored. Furthermore, larger values of h_2 make the phase transition continuous. We plot the phase diagram of the deformed $SU(2)$ as shown in Figure 1(a). The second choice, which we found was the most analytically tractable, is based



(a) with Equation 2.4 (b) with Equation 2.5

Figure 1: Deformed $SU(2)$ phase diagrams

on the one-loop potential for N_f adjoint Dirac fermions with periodic boundary conditions in two dimensions. These sheets of two-dimensional fermions can be embedded in four dimensions with a density $(N_4/L)^2$ in the plane orthogonal to the plane of the fermions with mass M

$$V_d = \frac{2MLN_f N_4^2}{\pi L^4} \sum_{n=1}^{\infty} \frac{K_1(nML)\,\mathrm{Tr}_A P^n}{n} \tag{2.5}$$

where K_n is the modified Bessel function. The infinite series can be summed exactly in the limit when the mass goes to zero,

$$\lim_{M\to 0} V_d = \frac{2N_f N_4^2}{\pi L^4} \sum_{n=1}^{\infty} \frac{\mathrm{Tr}_A P^n}{n^2} = \frac{4N_f N_4^2}{\pi L^4} (\theta - \pi/2)^2 \tag{2.6}$$

where $0 \le \theta \le \pi$. This deformation leads to a second-order phase transition at some N_f for sufficiently small M as shown in Figure 1(b). We will use the $M = 0$ form in what follows, thereby obtaining a second-order deconfinement transition.

Finally, the one-loop effective potential becomes

$$\begin{aligned} U \;=\; & \frac{1}{2} m^2(L)\,v^2 + \frac{1}{4}\lambda(L)\,v^4 + \frac{2}{\pi^2 L^4}\left(\theta - \frac{\pi}{2}\right)^4 + \frac{a}{L^4}\left(\theta - \frac{\pi}{2}\right)^2 \\ & + \frac{(m^2 + \lambda v^2 + 3g^2 v^2)}{2\pi^2 L^2}\left(\theta - \frac{\pi}{2}\right)^2 \end{aligned} \tag{2.7}$$

where we have defined the dimensionless parameter

$$a \equiv \frac{4N_f N_4^2}{\pi} - 1. \tag{2.8}$$

In order for us to take the phase diagram predicted by our one-loop effective potential seriously, both the gauge coupling $g(L)$ and the scalar coupling $\lambda(L)$ must be small. The gauge coupling is naturally small at a scale where $\Lambda L \ll 1$ as a consequence of asymptotic freedom, but the scalar coupling must be tuned to make $\lambda(L)$ small.

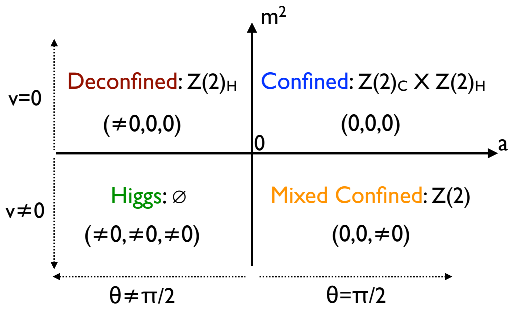


Figure 2: Phase diagram of $SU(2)$ Higgs model as a function of a and m^2. The values of the order parameters are shown in parenthesis as ($\langle \mathrm{Tr}_F P \rangle$, $\langle \mathrm{Tr}_F \left[P^2 \phi \right] \rangle$, $\langle \mathrm{Tr}_F \left[P \phi \right] \rangle$).

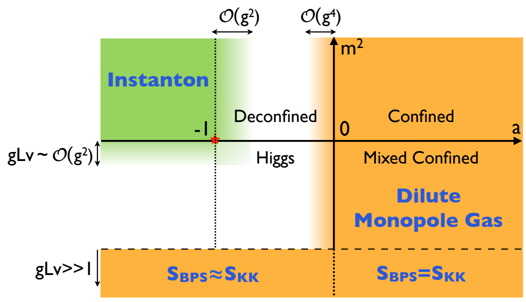


Figure 3: The phase diagram showing the region where the dilute monopole gas approximation is valid and $S_{BPS} \approx S_{KK}$. The dilute gas region itself is somewhat larger than the shaded region.

3. The phase diagram

An understanding of the overall phase structure can be based on the global symmetries of this class of models. The action is invariant under two global $Z(2)$ symmetries. The first symmetry, $Z(2)_H$ is the invariance of the action under a transformation of the scalar field $\phi \to -\phi$. The other global symmetry, $Z(2)_C$, is associated with the center symmetry of the $SU(2)$ gauge group, and is present because all fields have 0 N-ality. Under this global symmetry, the action is invariant, but the Polyakov loop P transforms as $P \to -P$. It is useful to consider three distinct gauge-invariant order parameters associated with the $Z(2)_C \times Z(2)_H$ symmetry. Although these order parameters are nonlocal in the compact direction, they are local in the three noncompact directions. The first of these is the trace in the fundamental representation of the Polyakov loop P itself, $\langle \mathrm{Tr}_F P(x) \rangle$, which is independent of x_4. It transforms nontrivially under $Z(2)_C$ but is invariant under $Z(2)_H$. The second is $\langle \mathrm{Tr}_F \left[P^2(x) \phi(x) \right] \rangle$ which is invariant under $Z(2)_C$, but transforms nontrivially under $Z(2)_H$. Finally, there is $\langle \mathrm{Tr}_F \left[P(x) \phi(x) \right] \rangle$, which transforms nontrivially under both groups.

The phase diagram as a function of a and m^2, which are the controlling parameters of the effective potential in Equation 2.7, is shown in Figure 2 with the values of three order parameters and the residual symmetries. There is a phase that is unique in this model, where $Z(2)_C \times Z(2)_H$ spontaneously breaks to $Z(2)$. We will refer to this phase as the mixed confined phase. The mixed confined phase in some sense takes the place of a phase where $Z(2)_H$ is broken but $Z(2)_C$ is unbroken, which would be a phase where both the Higgs mechanism and confinement hold. The fact that the Higgs and confined phases are not compatible in our model is consistent with the arguments made by 't Hooft [3, 4].

4. Nonperturbative effects

4.1 Classical monopole solutions

The nonperturbative dynamics of gauge theories on $R^3 \times S^1$ are all based on Polyakov's analysis of the Georgi-Glashow model in three dimensions. This is an $SU(2)$ gauge model coupled to an adjoint Higgs scalar. The model we are considering thus differs by the addition of a fourth compact dimension and a suitable deformation added to the action. The four-dimensional Georgi-Glashow model is the standard example of a gauge theory with classical monopole solutions when the Higgs expectation value is nonzero. They are topologically stable because $\Pi_2(SU(2)/U(1)) = \Pi_1(U(1)) = \mathbb{Z}$, and make a nonperturbative contribution to the partition function Z. In three dimensions, these monopoles are instantons. Polyakov showed that a gas of such three-dimensional monopoles gives rise to nonperturbative confinement in three dimensions, even though the theory appears to be in a Higgs phase perturbatively [5].

In the model at hand, both A_4 and ϕ play roles in the monopole solutions. The solutions for all these monopoles can be found explicitly in the BPS limit; when A_4 is nontrivial, the $N-1$ BPS monopoles are joined by a Kaluza-Klein (KK) monopole [6, 7, 8]. Their actions are [2]

$$S_{BPS} = \frac{4\pi}{g^2}\sqrt{4\theta^2 + g^2L^2v^2} \tag{4.1}$$

for BPS monopoles and

$$S_{KK} = \frac{4\pi}{g^2}\sqrt{(2\pi - 2\theta)^2 + g^2L^2v^2} \tag{4.2}$$

for KK monopoles. The KK solution is topologically distinct from the BPS solution because it carries instanton number 1. KK monopoles also have the opposite monopole charge from BPS monopoles. This is consistent with the KvBLL decomposition of instantons in the pure gauge theory with non-trivial Polyakov loop behavior, where $SU(2)$ instantons can be decomposed into a BPS monopole and a KK monopole. Our picture of the confined and mixed confined phases is one where instantons and anti-instantons have "melted" into their constituent monopoles and anti-monopoles, which effectively form a three-dimensional gas of magnetic monopoles.

4.2 Abelian duality

The contribution to the partition function of a single monopole is

$$Z_a = \xi_a \exp[-S_a] \int d^3x \tag{4.3}$$

where a denotes the type of monopoles, $a = \{BPS, KK, \overline{BPS}, \overline{KK}\}$ and the factor of d^3x represents the integration over the location of the monopole. The factor ξ_a together with $\exp[-S_a]$ is called the fugacity, and the one-loop contribution for the case of an adjoint scalar gives [9, 10, 2]

$$\xi_a = c\mu^{7/2}(2L)^{1/2}S_a^2 \tag{4.4}$$

where μ is a Pauli-Villars regulator and c is a numerical constant. From the construction of the KK monopole, we have $\xi_{KK}(\theta) = \xi_{BPS}(\pi - \theta)$. The interaction of the monopoles is essentially the one described by Polyakov in his original treatment of the Georgi-Glashow model in three dimensions [5], slightly generalized to include both the BPS and KK monopoles. The generating functional in terms of a scalar field σ

$$Z_\sigma = \int [d\sigma] \exp\left[-\int d^3x \left(\frac{g^2}{32\pi^2 L}(\partial_j\sigma)^2 - \sum_a \xi_a e^{-S_a + iq_a\sigma}\right)\right] \tag{4.5}$$

is precisely equivalent to the generating function of the monopole gas. Note that each species of monopole has its own magnetic charge sign $q_a = \pm$ as well as its own action S_a. This equivalence is a generalization of the equivalence of a sine-Gordon model to a Coulomb gas, and may be proved by expanding Z_σ in a power series in the ξ_a's, and doing the functional integral over σ for each term of the expansion.

It is well known that the magnetic monopole plasma leads to confinement in three dimensions. For our effective three-dimensional theory, any Wilson loop in a hyperplane of fixed x_4, for example a Wilson loop in the $x_1 - x_2$ plane, will show an area law. It can be obtained from the kink solution connecting the two vacua of the dual field σ [10]. We write the potential term in the dual effective lagrangian as

$$-\sum_a \xi_a e^{-S_a + iq_a\sigma} \to 2\left(\xi_{BPS}(\theta)\, e^{-S_{BPS}(\theta)} + \xi_{KK}(\theta)\, e^{-S_{KK}(\theta)}\right)[1-\cos(\sigma)] \tag{4.6}$$

which has minima at $\sigma = 0$ and $\sigma = 2\pi$; we have added a constant for convenience such that the potential is positive everywhere and zero at the minima. A one-dimensional soliton solution $\sigma_s(z)$ connects the two vacua, and the string tension σ_{3d} for Wilson loops in the three noncompact directions is given by

$$\sigma_{3d} = \int_{-\infty}^{+\infty} dz L_{eff}(\sigma_z(z)) \tag{4.7}$$

which can be calculated via yet another Bogomol'nyi inequality to be

$$\sigma_{3d} = \frac{4g}{\pi}\sqrt{\frac{1}{2L}\left(\xi_{BPS}(\theta)\, e^{-S_{BPS}(\theta)} + \xi_{KK}(\theta)\, e^{-S_{KK}(\theta)}\right)}. \tag{4.8}$$

The apparent renormalization group-dependence of the final result is discussed in [2]. It is notable that in the confined phase σ_{3d} can be written in a form independent of the renormalization group scale [10].

In Figure 3, we show a final version of the phase diagram. The figure shows the large region where the dilute monopole gas description should be valid, and either $S_{BPS} = S_{KK}$ or $S_{BPS} \simeq S_{KK}$. Note that this region includes all of the confined and mixed confined regions, a large part of the Higgs phase, and a small part of the deconfined phase. The region where the dilute gas approximation is valid is somewhat larger. However, we have also indicated the region where the dilute gas approximation breaks down, because $S_{BPS} \approx 0$ and $S_{KK} \approx 8\pi^2/g^2$. For obvious reasons, we have labeled this region as an instanton region, although the correct treatment of topological excitations in this region is no clearer in the Higgs system than in the pure gauge case.

4.3 Poisson duality

We can understand the role of topological excitations from a different point of view by invoking duality in a form similar to that used by Poppitz and Unsal in their analysis of the Seiberg-Witten model [11]; their work also serves as an introduction to duality in this context. We begin with an easy variant of the Poisson summation formula associated with $Z(N)_C$. Let $f(\theta)$ be a function defined on the interval $-\pi < \theta < \pi$. We define the Fourier series in the usual way:

$$f(\theta) = \sum_{n\in Z} \tilde{f}(n)\, e^{in\theta} \tag{4.9}$$

$$\tilde{f}(n) = \int_{-\pi}^{\pi} \frac{d\theta}{2\pi} f(\theta)\, e^{-in\theta}. \tag{4.10}$$

Then we have

$$\sum_{k=0}^{N-1} f\left(\theta - \frac{2\pi k}{N}\right) = \sum_{n\in Z} N\tilde{f}(nN)\,e^{inN\theta} \tag{4.11}$$

so that for $N = 2$ only the even coefficients $\tilde{f}(2n)$ contribute. Let us apply this identity to the combination

$$\xi_{BPS}(\theta)\,e^{-S_{BPS}(\theta)} + \xi_{KK}(\theta)\,e^{-S_{KK}(\theta)} = \xi_{BPS}(\theta)\,e^{-S_{BPS}(\theta)} + \xi_{BPS}(\pi-\theta)\,e^{-S_{BPS}(\pi-\theta)} \tag{4.12}$$

which occurs in the dual Lagrangian and in the formula for σ_{3d} so we have

$$f(\theta) = \xi_{BPS}(\theta)\,e^{-S_{BPS}(\theta)}. \tag{4.13}$$

For small g^2, $S_{BPS}(\theta)$ is strongly peaked at $\theta = 0$, so we can make the approximation

$$\tilde{f}(n) \simeq \int_{-\infty}^{\infty} \frac{d\theta}{2\pi}\xi_{BPS}(0)\,e^{-S_{BPS}(\theta)}e^{in\theta}. \tag{4.14}$$

Although this integral, with the limits taken to infinity, can be evaluated in a saddle point approximation, it can also be evaluated exactly [11], giving

$$\tilde{f}(2n) \simeq \xi_{BPS}(0)\,\frac{gLv}{2\pi}\cdot\frac{\frac{4\pi}{g^2}}{\sqrt{\left(\frac{4\pi}{g^2}\right)^2+n^2}}K_1\left[gLv\sqrt{\left(\frac{4\pi}{g^2}\right)^2+n^2}\right]. \tag{4.15}$$

The Higgs phase represents the most general domain of applicability of the duality transformation, because in the Higgs phase $v \neq 0$ and $0 \leq \theta < \pi/2$. It is natural to introduce $M(n)$ the mass of a Minkowski-space Julia-Zee dyon [12] of magnetic charge $4\pi/g$ and electric charge ng

$$M(n) = v\sqrt{\left(\frac{4\pi}{g}\right)^2 + (ng)^2}. \tag{4.16}$$

The asymptotic expansion of the Bessel function for large argument gives a factor of $\exp[-LM(n)]$:

$$\tilde{f}(2n) \simeq \xi_{BPS}(0)\,\frac{LM(0)}{2\pi}\cdot\frac{1}{\sqrt{\left(\frac{4\pi}{g^2}\right)^2+n^2}}\sqrt{\frac{\pi}{2LM(n)}}\exp[-LM(n)]. \tag{4.17}$$

Thus each term in the sum carries a factor of $\exp[-LM(n)+i2n\theta]$. This suggests an obvious interpretation of the finite sum over BPS and KK monopoles, which are constituents of instantons, as being equivalent to a gas of Julia-Zee dyons, each carrying a Polyakov loop factor appropriate to its charge. This interpretation is valid throughout most of the Higgs and mixed confined phases, except in the region near $m^2 = 0$ where the mass of the lightest dyon $M(0) = 4\pi v/g$, which is a Minkowski-space monopole, becomes light. Within this framework, the only significant difference between the mixed confined and Higgs phases is that in the mixed confined phase, θ is restricted to $\pi/2$.

5. Conclusions

We have shown that the deformed $SU(2)$ adjoint Higgs model on $R^3 \times S^1$ have four different phases distinguished by the behavior of the three gauge-invariant order parameters associated with $Z(2)_C \times Z(2)_H$. We have calculated the area-law behavior of Wilson loops orthogonal to the compact S^1 direction in at least part of all four phases where a picture of a dilute magnetic monopole gas is valid. Furthermore, we show that the monopole gas picture, arrived at using Euclidean instanton methods, can be interpreted as a gas of finite-energy dyons using Poisson duality.

For $SU(N)$ gauge theories on $R^3 \times S^1$, the natural set of order parameters is $\mathrm{Tr}_F P^k$, and the $Z(N)$ center symmetry can break to a subgroup $Z(p)$ [13, 14]. With the addition of an adjoint scalar, there is the additional set of order parameters of the form form $\mathrm{Tr}_F P^k \phi$ available. This suggests a very rich phase structure is possible. Finally, the overall phase structure we have predicted in our four-dimensional model should be relatively easy to test with lattice simulations.

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