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Currents and masses in the QCD_2 flux tube

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We show from the action integral that under the assumption of longitudinal dominance and transverse confinement, QCD_4 in (3+1) dimensional space-time can be approximately compactified into QCD_2 in (1+1) dimensional space-time. In such a process, the relation between the coupling constant g_{2D} in QCD_2 and the coupling constant g_{4D} in QCD_4 is derived. The quark and gluon masses as well as the QCD_2 fermion current acquired in the compactification are obtained. They depend crucially on the excitation of the partons in the transverse degrees of freedom.

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1. Introduction

Previously, t'Hooft showed that in the limit of large N_c with fixed g^2N_c in single-flavor quantum chromodynamics in (3+1) dimensional space-time (QCD$_4$), planar diagrams with quarks at the edges dominate, whereas diagrams with the topology of a fermion loop or a wormhole are associated with suppressing factors of $1/N_c$ and $1/N_c^2$, respectively [1]. In this case a simple-minded perturbation expansion with respect to the coupling constant g cannot describe the spectrum, while the $1/N_c$ expansion may be a reasonable concept, in spite of the fact that N_c is equal to 3 and is not very big. The dominance of the planar diagrams allows one to consider QCD in one space and one time dimensions (QCD$_2$) and the physics resembles those of the dual string or a flux tube, with the physical spectrum of a straight Regge trajectory [2]. The properties of QCD in two-dimensional space-time have been investigated by many workers [1, 2, 3]. The flux tube picture of longitudinal dynamics manifests itself in various aspects of hadron spectroscopy [4].

In the high-energy arena, the flux tube picture finds phenomenological applications in hadron collisions and high-energy e^-e^+ annihilations [5, 6, 7, 8, 9]. In these high-energy processes, the (average) transverse hadron momenta of produced hadrons are observed to be limited, as appropriate for particles confined in a flux tube. The idealization of the three-dimensional flux tube as a one-dimensional string leads to the string fragmentation picture of particle production in (1+1) space-time dimensions. The particle production description of Casher, Kogut, and Susskind [5] in (1+1) dimensional Abelian gauge theory led to results that mimics the dynamics of particle production in hadron collisions and in the annihilation of e^+e^- pairs at high energies. Furthermore, the Lund model of classical string fragmentation has been quite successful in describing quantitatively the process of particle production in these high energy processes [6, 7, 8, 9].

With the successes of lower-dimensional descriptions of high-energy collision processes in QCD, we would like to examine the circumstances in high-energy processes under which QCD$_4$ in (3+1) dimensions can be compactified into QCD$_2$ in (1+1) dimensions, if one starts with the QCD$_4$ action integral. In such a process, we will be able to find out how quantities in the compactified QCD$_2$ can be related to quantities in QCD$_4$. The success of the compactification program will facilitate the examination of some problems in QCD$_4$ in the simpler dynamics of QCD$_2$.

2. $4D \to 2D$ Compactification in the Action Integral

The $4D$-action $\mathscr{A}$ resides in (3+1) dimensional space-time. There are however environments which allow the compactification of the $4D$ action to reside in two-dimensional (1+1) space-time, within which the dynamics can be greatly simplified.

We note that in hadron collisions and e^-e^+ annihilations at high energies, the string fragmentation process occurs when a valence quark pulls apart from a valence antiquark longitudinally at high energies. It is therefore reasonable to conceive that the QCD$_4$ compactification can take place under the dominance of longitudinal dynamics, not only of the leading valence quark and antiquark pair, but also the produced $q\bar{q}$ parton pairs. In the Lorentz gauge, as A_ν is proportional to the current j_ν, the gauge field components A_1^a and A_2^a along the transverse direction are then small in magnitude in comparison with those of A_0^a and A_3^a and can be neglected. The absence of the transverse gauge fields provides a needed simplification for compactification.

The spatially one-dimensional string being an idealization of a more realistic three-dimensional flux-tube, the description of produced $q\bar{q}$ parton pairs within the string presumes the confinement of these produced partons inside the string. Hence, it is reasonable to conceive further that the QCD$_4$ compactification takes place under transverse confinement. We can describe transverse confinement in terms of a confining scalar interaction $S(\mathbf{r}_\perp)$ and the quark mass function is then $m(\mathbf{r}_\perp) = m_0 + S(\mathbf{r}_\perp)$, where m_0 is the quark rest mass.

Therefore, under the assumption of longitudinal dominance and transverse confinement, the SU(N) gauge invariant action integral in (3+1) Minkowski space-time is given by [10]:

$$\mathscr{A} = \int d^4x \ \Big\{ Tr\left[\bar{\Psi}(x)\left(\gamma^\mu_{4\mathrm{D}}(i\partial_\mu + g_{4\mathrm{D}}\, T_a A^a_\mu(x)) - m(\mathbf{r}_\perp)\right)\,\Psi(x)\right] - \frac{1}{4}F^a_{\mu\nu}(x)\,F^{\mu\nu}_a(x)\Big\}, \tag{2.1}$$

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g_{4\mathrm{D}}\ f^a_{bc}\, A^b_\mu(x)\, A^c_\nu(x), \tag{2.2}$$

where $A^a_\mu(x)$ and $\Psi(x)$ are gauge and fermion fields respectively with coordinates $x \equiv x^\mu = (x^0, \mathbf{x}) = (x^0, x^1, x^2, x^3)$ and transverse coordinates $\mathbf{r}_\perp = (x^1, x^2)$, $g_{4\mathrm{D}}$ is the coupling constant, $\gamma^\nu_{4\mathrm{D}}$ are the stanadard Dirac matrices, and T_a are the generators of the SU(N) group.

2.1 Fermion part of the action integral

We first examine $\mathscr{A}_F$, the fermion part of the $4D$ action integral in Eq. (2.1) that involves the fermion field. To carry out the compactification, we write the Dirac fermion field $\Psi(x)$ in terms of functions $G_\pm(\vec{r}_\perp)$ and $f_\pm(x^0, x^3)$ [7]:

$$\Psi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} G_1(\mathbf{r}_\perp)\left(f_+(x^0;x^3) + f_-(x^0;x^3)\right) \\ -G_2(\mathbf{r}_\perp)\left(f_+(x^0;x^3) - f_-(x^0;x^3)\right) \\ G_1(\mathbf{r}_\perp)\left(f_+(x^0;x^3) - f_-(x^0;x^3)\right) \\ G_2(\mathbf{r}_\perp)\left(f_+(x^0;x^3) + f_-(x^0;x^3)\right) \end{pmatrix}. \tag{2.3}$$

Using this explicit form of the Dirac fermion field $\Psi(x)$, we carry out the simplifications and integrations over x^1 and x^2 that eventually lead from $\mathscr{A}_F$ to $\mathscr{A}_F(2D)$,

$$\mathscr{A}_F(2D) = Tr\int d^2X\bar{\Psi}(X)\left[\left(i\gamma^\mu\partial_\mu + g_{2\mathrm{D}}\gamma^\mu T_a A^a_\mu(X)\right) - m_{qT}\right]\Psi(X), \tag{2.4}$$

where $\mu = 0, 3$, and we have introduced the Dirac fermion field $\Psi(X)$, γ-matrices, and metric tensor $g_{\mu\nu}$, according to the following specifications in (1+1)-dimensional space-time in QCD$_2$,

$$\Psi(X) = \begin{pmatrix} f_+(X) \\ f_-(X) \end{pmatrix}, \qquad X = (x^0, x^3), \tag{2.5}$$

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.6}$$

The $2D$ coupling constant $g_{\rm 2D}$ is related to $4D$ coupling constant $g_{\rm 4D}$ by

$$g_{\rm 2D} = \int dx^1 dx^2 g_{\rm 4D}[|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{3/2}, \tag{2.7}$$

where the transverse wave functions $G_{1,2}(\mathbf{r}_\perp)$ are normalized according to

$$\int dx^1 dx^2 \left(|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2\right) = 1. \tag{2.8}$$

The transverse quark mass m_{qT} in Eq. (2.4) is given by

$$m_{qT} = \int dx^1 dx^2 \left\{ m(\mathbf{r}_\perp)\left(|G_1(\mathbf{r}_\perp)|^2 - |G_2(\mathbf{r}_\perp)|^2\right) + [(G_1^*(\mathbf{r}_\perp)(p_1 - ip_2)G_2(\mathbf{r}_\perp)) - h.c.] \right\}. \tag{2.9}$$

In obtaining these results, we have considered $2D$ gauge fields $A_\mu^a(2D, x^0, x^3) \equiv A_\mu^a(2D) \equiv A_\mu^a(X)$ related to the $4D$-field gauge fields $A_\mu^a(x^0, x^3, \mathbf{r}_\perp)$ by

$$A_\mu^a(x^0, x^3, \mathbf{r}_\perp) = \sqrt{|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2}\, A_\mu^a(X), \quad \mu = 0, 3. \tag{2.10}$$

The above equation means that along with the confinement of the fermions, for which the wave function $G_{1,2}(\mathbf{r}_\perp)$ is confined within a finite region of transverse coordinates $\mathbf{r}_\perp$, we also consider the confinement of the gauge field $A_\mu^a(X)$, $\mu = 0, 3$, within the same finite region of transverse coordinates, as in the case for a flux tube.

The result of Eq. (2.7) reveals that as a result of the compactification of QCD$_4$, the coupling constant $g(2D)$ in lower dimensional space in QCD$_2$ acquires the dimension of a mass, and is related to the confining wave functions of the fermions. Fermions in different excited states inside the tube will have different coupling constants as indicated in Eq. (2.7). The effective quark mass m_{qT} also depends on the transverse fermion wave functions, as indicated in Eq. (2.9). In the lower two-dimensional space-time, fermions in excited transverse states have a quark mass different from those in the ground transverse states.

2.2 Gauge field part of the action integral

To go from $\mathscr{A}_F$ to $\mathscr{A}_F(2D)$ we have assumed that the currents in the x^0 and x^3 directions are much larger in magnitude than the currents in the transverse directions so that A_1^a and A_2^a are small in comparison and can be neglected. As a consequence, $F_{12} = 0$ (we omit the superscript symbol a (color) for simplicity). The evaluation of all other components of $F_{\mu\nu}$ give for the gauge field part of the action integral

$$\begin{aligned}\int \frac{d^4x}{4} \; F^a_{\mu\nu} F_a^{\mu\nu} &= \int \frac{dx^0 dx^3}{4} \int dx^1 dx^2 (|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2) F^a_{03}(2D) F_a^{03}(2D) \\ &- \int \frac{dx^0 dx^3}{4} \int dx^1 dx^2 \Big(\{\partial_1 [|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 + \{\partial_2 [|G_1(\mathbf{r}_\perp)|^2 + |G_2(\mathbf{r}_\perp)|^2]^{1/2}\}^2 \Big) \\ &\times [A_0(2D, x^0, x^3) A^0(2D, x^0, x^3) + A_3(2D, x^0, x^3) A^3(2D, x^0, x^3)]. \end{aligned} \tag{2.11}$$

It is useful to introduce the gluon mass m_{gT} that arises from the confinement of the gluons in the transverse direction,

$$m_{gT}^2 = \frac{1}{2} \int dx^1 dx^2 \left[\left\{ \partial_1 \left(\sum_{i=1}^{2} |G_i(\mathbf{r}_\perp)|^2 \right)^{1/2} \right\}^2 + \left\{ \partial_2 \left(\sum_{i=1}^{2} |G_i(\mathbf{r}_\perp)|^2 \right)^{1/2} \right\}^2 \right]. \tag{2.12}$$

As this gluon mass m_{gT} arises from the confinement compactification of the gluon within the flux tube, we can call such a mass the compactification mass of the gluon. Equation (2.11) becomes

$$\int \frac{d^4x}{4} F^a_{\mu\nu}(4D)\, F_a^{\mu\nu}(4D) \;=\; \int \frac{dx^0dx^3}{4}\left\{F^a_{03}(2D)F_a^{03}(2D) - 2m^2_{gT}\,[A^a_\mu(X)A^\mu_a(X)]\right\} \quad (2.13)$$

We collect all the fermion and gauge field parts of the action integral in $\mathscr{A}(4D)$ in Eq. (2.1). All terms in the action integral $\mathscr{A}(4D)$ (including matrices and coefficients) are in the Minkowski $(1+1)$ dimensional space-time. We can rename the action integral $\mathscr{A}(4D)$ to be $\mathscr{A}(2D)$ given explicitly by

$$\mathscr{A}(2D) = \int d^2X \quad \left\{ Tr\left[\bar{\Psi}(X)\left[\gamma^k\left(i\partial_\mu + g_{2\mathrm{D}}\; T_a A^a_\mu(X)\right) - m_{qT}\right]\Psi(X)\right.\right.$$
$$\left.\left. -\frac{1}{4}F^a_{\mu\nu}(2D)\; F_a^{\mu\nu}(2D) + \frac{1}{2}m^2_{gT}\,[A^\mu_a(X)A^a_\mu(X)]\right\}. \qquad (2.14)$$

Thus, in the presence of longitudinal dominance and transverse confinement, we succeed in compactifying the action integral from $\mathscr{A}(4D)$ in QCD$_4$ to $\mathscr{A}(2D)$ in QCD$_2$, by introducing g_{2D}, m_{qT}, and m_{gT} that contain information about the transverse profile. All the transverse flux tube information is subsumed under these quantities. In this way, the $2D$ gauge field appears to be massive where m_{gT} and m_{gT} arise as a consequence of the transverse confining motion of both fermion and gauge fields. The physical explanation of such effect consists in the decrease of the number of trajectories in moving from one point of the space to another point, as a direct consequence of compactification. Such constraints in movement manifest themselves as masses of field particles. The magnitudes of m_{qT} and m_{gT} depend strongly on the kind of the compactification that is dictated by the transverse functions $G_1(\mathbf{r}_\perp)$ and $G_2(\mathbf{r}_\perp)$ (see Eqs.(2.9) and (2.12)).

3. Solution of the Dirac fields in (1+1) space-time

Having completed the program of compactification of QCD$_4$ to QCD$_2$, we shall employ the new notation henceforth that all field quantities and gamma matrices are in two-dimensional space-time with $\mu = 0,3$, unless specified otherwise. We can use the QCD$_2$ action integral to get the equation of motion for the field. We adopt now the new notation that all field quantities and gamma matrices are in (1+1) dimensional spaxce-time unless indicated otherwise, Varying the action integral $\mathscr{A}(2D)$ given by Eq.(2.14) with respect to $\bar{\Psi}$, we derive the 2D Dirac equation,

$$\left\{i\gamma^\mu\left(\partial_\mu - ig_{2D}\cdot A^a_\mu(X)\; T_a\right) - m_{qT}\right\}\Psi(X) = 0, \qquad (3.1)$$

where $2D$ Dirac matrices are those given in Eq. (2.6). The gauge field A^a_μ written in component form is $A^a_\mu(X) = (A^a_0, -A^a_3)$. The general solution of Eq.(3.1) can be formally written in a form[11]:

$$\Psi(X) \;=\; \int_{-\infty}^{+\infty}\frac{d\omega}{2\sqrt{L}}\sum_p \frac{m_{qT}}{\sqrt{\omega^2+p\omega}}\;\exp(-iP_\mu X^\mu)\; a(p,\omega)\,[\delta(\omega-\varepsilon(p))+\delta(\omega+\varepsilon(p))]\;\begin{pmatrix}\frac{\omega+p}{m_{qT}}+1\\ \\ \frac{\omega+p}{m_{qT}}-1\end{pmatrix}$$
$$\times\;\{T_{l(M_0;M)}\exp\}\left\{ig_{2D}T_a\int dX' A^a_\mu(X')\right\}, \qquad (3.2)$$

where $a(p,\omega)$ are coefficients related to either particles or anti-particles under the field quantization. We have not deliberately separated out positive and negative frequency terms in Eq. (3.2) because the structure of the fermion vacuum is strongly dependent on the explicit form of the external field $A^a_\mu(X)$. The factor $\{T_{l(M_0;M)}\exp\}$ means that the integration is to be carried out along the line on the light cone from the point M_0 to the point M such that the factors in exponent expansion are chronologically ordered from M_0 to M.

3.1 Fermion current and gauge fields

The fermion field solution in Eq. (3.2) leads to a 2D fermion current J^μ_a

$$J^\mu_a = g_{2D}\ Tr\left\{\bar{\Psi}(X)\gamma^\mu T_a \Psi(X')\right\}, \quad X' \to X. \tag{3.3}$$

Owing to the operation of trace calculation in the last formula, the current (3.3) contains the factor, we expand the operator exponent in the last equation as a series with respect to $(X'-X) \to 0$. As a result, we derive

$$J^\mu_a(X) = \frac{g^2_{2D}\,\mathscr{S}}{4}\, A^\mu_a(X) \equiv m^2_{gf\,T}\, A^\mu_a(X),$$
$$\mathscr{S} = \frac{1}{2\pi}\sum_f \int d^2P \frac{\partial}{\partial P^\mu}\left\{[\delta(\omega+\varepsilon(p)) \,+\, \delta(\omega-\varepsilon(p))]\ P^\mu\ < a^\dagger(p,\omega)_f\, a_f(p,\omega) >\right\}; \tag{3.4}$$

where $P^\mu = (\omega; p)$ is the 2-momentum introduced.

Calculation of the quantity $\mathscr{S}$ gives [11]: In the cases of a flux tube ($m_{qT} \gg p$) and the massless QED$_2$, we obtain

$$\mathscr{S}_{\text{fluxtube}} = \frac{2}{\pi}\, N_f, \qquad \mathscr{S}_{\text{QED}_2} = \frac{4}{\pi}, \ \ (N_f = 1), \tag{3.5}$$

where N_f is the number of flavors. We note in passing that in the special case of the massless QED$_2$ [12] the obtained effective fermion mass, $m^2_{gf\ T}(\text{QED}_2) = \frac{g^2}{\pi}$, agrees with the Schwinger massless QED$_2$ result [12].

4. Equation of motion for the 2D Gauge Fields

The action integral $\mathscr{A}$ allows us to obtain the equation of motion for the 2D gauge field. We rewrite the action integral (2.14) by expressing explicitly the term corresponding to the interaction between the fermion and the gauge field which is proportional to the current $J^\mu_a(x)$. Substituting $J^\mu_a(x)$ given by Eq.(3.4) into the 2D action integral (2.14), we obtain

$$\mathscr{A}(2D) = \int d^2X\ \left\{\ \frac{i}{2}\left[\bar{\Psi}\gamma^k\partial_k\Psi - 2\bar{\Psi}m_{qT}\Psi - \bar{\Psi}\gamma^k\overleftarrow{\partial}_k\,\Psi\right] - \frac{1}{4}F^a_{\mu\nu}\,F^{\mu\nu}_a + \frac{1}{2}M^2_{gT}A^a_\nu A^\nu_a\right\}. \tag{4.1}$$

Here the constant M_{gT} is given by

$$M^2_{gT} = \frac{1}{2}\int dx^1dx^2\left[\sum_{a=1}^{2}\left\{\partial_a\left(\sum_{i=1}^{2}|G_i(\mathbf{r}_\perp)|^2\right)^{1/2}\right\}^2\right] + \frac{g^2_{2D}\mathscr{S}}{2} \equiv m^2_{gT} + m^2_{gf\,T} \geq 0. \tag{4.2}$$

To find out the meaning of M_{gT}, we consider the variation of the action integral (4.1) with respect to a variation of the gauge field $A_a^\nu(X)$. We obtain equation of motion for the variation $A_a^\nu(x)$. As a result, we derive the Klein-Gordon-like equation:

$$\Box A_a^\nu \,=\, M_{gT}^2\, A_a^\nu. \tag{4.3}$$

The solution of is

$$A_a^\nu(X) \,=\, \sum_k \frac{e_a^\nu\, M_{gT}}{\sqrt{\left(\mathbf{k}^2 + M_{gT}^2\right)^3}} \left\{ \exp(-ikX)\, b_a(k,\nu) \,+ \exp(+ikX)\, \overline{b}_a^\dagger(k,\nu) \right\}, \tag{4.4}$$

where the symbols $b_a(k,\nu)$ and $\overline{b}_a^\dagger(k,\nu)$ are the operators of annihilation and creation of a boson with the mass M_{gT}. In this way, M_{gT} corresponds to the mass of the boson responding to the space-time variation of the gauge field variation. The symbols $k^\mu = (k^0;\mathbf{k})$ and e_a^ν denote a 2-momentum and a pair orthogonal vectors given by the formulae

$$k^\mu = (k^0;\mathbf{k}),\;\; e_a^0 = \frac{|\mathbf{k}|}{M_{gT}}\,(1,0),\quad e_a^3 = \frac{|k^0|}{M_{gT}}(0,1),\quad (k^0)^2 = \mathbf{k}^2 + M_{gT}^2. \tag{4.5}$$

Because of both the positivity of M_{gT}^2 and Eq. (4.3), M_{gT} can be interpreted as a mass of the particle whose energy is

$$E(k) \equiv k^0 = +\sqrt{\mathbf{k}^2 + M_{gT}^2}. \tag{4.6}$$

5. Equations of a transverse motion in a tube and Fermion effective mass

To obtain the equations of motion for the functions $G_1(\vec r_\perp)$ and $G_2(\vec r_\perp)$, we vary the action integral $\mathscr{A}(4D)$ (2.1) with the fermion fields $\Psi(4D,x)$ given by Eq.(2.1), under the constraint of the normalization condition, (2.8). To do this we construct a new functional $\mathscr{F}$

$$\mathscr{F} = \mathscr{A}(4D) + \frac{\lambda}{2}\int dx^1 dx^2 \left(\sum_{i=1}^{2} |G_i(\mathbf{r}_\perp)|^2\right) \int dx^0 dx^3 \left(\bar\Psi(X) m_{qT} \Psi(X)\right), \tag{5.1}$$

where λ is the Lagrange multiplier. The last term in Eq. (5.1) takes into account the unitarity of a fermion field in the 4D space-time. Varying the last equation with respect to the functions $G_1(\mathbf{r}_\perp)$ and $G_2(\mathbf{r}_\perp)$ we derive

$$\begin{aligned}&(p_1 + ip_2)G_1(\mathbf{r}_\perp) = (m(\mathbf{r}_\perp) + \lambda)G_2(\mathbf{r}_\perp), \qquad (p_1 - ip_2)G_2(\mathbf{r}_\perp) = (\lambda - m(\mathbf{r}_\perp))G_1(\mathbf{r}),\\ &(p_1 + ip_2)G_2^*(\mathbf{r}_\perp) = (m(\mathbf{r}_\perp) - \lambda)G_1^*(\mathbf{r}_\perp), \qquad (p_1 - ip_2)G_1^*(\mathbf{r}_\perp) = -(m(\mathbf{r}_\perp) + \lambda)G_2^*(\mathbf{r}).\end{aligned} \tag{5.2}$$

Carrying out complex conjugation in the last two equations we obtain $\lambda = \lambda^*$. Substituting the equations (5.2) for $G_{1,2}(\mathbf{r}_\perp)$ functions into the formula (2.9) for m_{qT} we get

$$m_{qT} \,=\, \lambda. \tag{5.3}$$

Thus, the effective mass of the compactified 2D fermion field is equal to the energy of the transverse motion of the 4D fermion, which is an eigenvalue of Eq. (5.2). We should note here that the 2D fermion can generally gain a mass even when the initial 4D fermion appears to be massless. The explanation of such phenomenon is the same as before. The compactification effectively leads to constraints in moving a fermion from one point of a space-time to another one due to the decrease of the number of trajectories in the 2D space-time, as compared to the 4D space-time.

6. Conclusion

Under the assumption of longitudinal dominance and transverse confinement, the SU(N) gauge invariant field theory of QCD$_4$ can be compactified into QCD$_2$ in Minkowski $(1+1)$ dimensional space-time from the consideration of the action integral. The compactified 2D action integral $\mathscr{A}(2D)$ depends only on 2D-fields. The corresponding coupling constants, effective quark, and gluon masses in two-dimensional space-time are derived. Such compactification leads to strong changes in physics of the 2D Lagrangian that is manifested in both the renormalization of coupling constant and fermions as well as gauge field bosons acquiring masses whioch depends on the transverse states of the fermions. Further investigations on the transverse fermion states are therefore of great interest.

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