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QCD in terms of gauge-invariant dynamical variables

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For a complete description of the physical properties of low-energy QCD, it might be advantageous to first reformulate QCD in terms of gauge-invariant dynamical variables, before applying any approximation schemes. Using a canonical transformation of the dynamical variables, which Abelianises the non-Abelian Gauss-law constraints to be implemented, such a reformulation can be achieved for QCD. The exact implementation of the Gauss laws reduces the colored spin-1 gluons and spin-1/2 quarks to unconstrained colorless spin-0, spin-1, spin-2 and spin-3 glueball fields and colorless Rarita-Schwinger fields respectively. The obtained physical Hamiltonian can then be rewritten into a form, which separates the rotational from the scalar degrees of freedom, and admits a systematic strong-coupling expansion in powers of $\lambda = g^{-2/3}$, equivalent to an expansion in the number of spatial derivatives. The leading-order term in this expansion corresponds to non-interacting hybrid-glueballs, whose low-lying masses can be calculated with high accuracy by solving the Schrödinger-equation of the Dirac-Yang-Mills quantum mechanics of spatially constant physical fields (at the moment only for the 2-color case). Due to the presence of classical zero-energy valleys of the chromomagnetic potential for two arbitrarily large classical glueball fields (the unconstrained analogs of the well-known constant Abelian fields), practically all glueball excitation energy is expected to go into the increase of the strengths of these two fields. Higher-order terms in λ lead to interactions between the hybrid-glueballs and can be taken into account systematically using perturbation theory in λ.

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1. Introduction

The QCD action

$$\mathscr{S}[A,\psi,\overline{\psi}] = \int d^4x \left[-\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{\psi}\left(i\gamma^\mu D_\mu - m\right)\psi\right] \tag{1.1}$$

is invariant under the $SU(3)$ gauge transformations $U[\omega(x)] \equiv \exp(i\omega_a\tau_a/2)$

$$\psi^\omega(x) = U[\omega(x)]\ \psi(x), \qquad A^\omega_{a\mu}(x)\tau_a/2 = U[\omega(x)]\left(A_{a\mu}(x)\tau_a/2 + \frac{i}{g}\partial_\mu\right)U^{-1}[\omega(x)]. \tag{1.2}$$

Introducing the chromoelectric $E^a_i \equiv F^a_{i0}$ and chromomagnetic $B^a_i \equiv \frac{1}{2}\varepsilon_{ijk}F^a_{jk}$ and noting that the momenta conjugate to the spatial A_{ai} are $\Pi_{ai} = -E_{ai}$, one obtains the canonical Hamiltonian

$$H_C = \int d^3x \left[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}(A) - gA_{ai}\, j_{ia}(\psi) + \overline{\psi}\left(\gamma_i\partial_i + m\right)\psi - gA_{a0}\left(D_i(A)_{ab}E_{bi} - \rho_a(\psi)\right)\right], \tag{1.3}$$

with the covariant derivative $D_i(A)_{ab} \equiv \delta_{ab}\partial_i - gf_{abc}A_{ci}$ in the adjoint representation.

Exploiting the time dependence of the gauge transformations (1.2) to put (see e.g. [1])

$$A_{a0} = 0\,, \qquad a = 1,..,8 \qquad \text{(Weyl gauge)}, \tag{1.4}$$

and quantising the dynamical variables A_{ai}, $-E_{ai}$, $\psi_{\alpha r}$ and $\psi^*_{\alpha r}$ in the Schrödinger functional approach by imposing equal-time (anti-) commutation relations (CR) , e.g. $-E_{ai} = -i\partial/\partial A_{ai}$, the physical states Φ have to satisfy both the Schrödinger equation and the Gauss laws

$$H\Phi = \int d^3x \left[\frac{1}{2}E^2_{ai} + \frac{1}{2}B^2_{ai}[A] - A_{ai}\, j_{ia}(\psi) + \overline{\psi}\left(\gamma_i\partial_i + m\right)\psi\right]\Phi = E\Phi\,, \tag{1.5}$$

$$G_a(x)\Phi = \left[D_i(A)_{ab}E_{bi} - \rho_a(\psi)\right]\Phi = 0\,, \quad a = 1,..,8\,. \tag{1.6}$$

The Gauss law operators G_a are the generators of the residual time independent gauge transformations in (1.2), satisfying $[G_a(x),H] = 0$ and $[G_a(x),G_b(y)] = if_{abc}G_c(x)\delta(x-y)$.
Furthermore, H commutes with the angular momentum operators

$$J_i = \int d^3x \left[-\varepsilon_{ijk}A_{aj}E_{ak} + \Sigma_i(\psi) + \text{orbital parts}\right]\,, \quad i = 1,2,3\,. \tag{1.7}$$

The matrix element of an operator O is given in the Cartesian form

$$\langle\Phi'|O|\Phi\rangle \propto \int dA\, d\overline{\psi}\, d\psi\, \Phi'^*(A,\overline{\psi},\psi)\, O\, \Phi(A,\overline{\psi},\psi)\,. \tag{1.8}$$

The spectrum of Equ.(1.5)-(1.6) for the case of Yang-Mills quantum mechanics of spatially constant gluon fields, has been found in [2] for $SU(2)$ and in [3] for $SU(3)$, in the context of a weak coupling expansion in $g^{2/3}$, using the variational approach with gauge-invariant wave-functionals automatically satisfying (1.6). The corresponding unconstrained approach, a description in terms of gauge-invariant dynamical variables via an exact implementation of the Gaws laws, has been considered by many authors (o.a. [1],[4]-[10], and references therein) to obtain a non-perturbative description of QCD at low energy, as an alternative to lattice QCD.

I shall first discuss in Section 2 the unphysical, but technically much simpler case of 2-colors, and then show in Section 3 how the results can be generalised to $SU(3)$.

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2. Unconstrained Hamiltonian formulation of 2-color QCD

2.1 Canonical transformation to adapted coordinates

Point transformation from the A_{ai}, ψ_α to a new set of adapted coordinates, the 3 angles q_j of an orthogonal matrix $O(q)$, the 6 elements of a pos. definite symmetric 3×3 matrix S, and new ψ'_β

$$A_{ai}\left(q,S\right) = O_{ak}\left(q\right) S_{ki} - \frac{1}{2g}\varepsilon_{abc}\left(O\left(q\right)\partial_i O^T\left(q\right)\right)_{bc}, \qquad \psi_\alpha\left(q,\psi'\right) = U_{\alpha\beta}\left(q\right)\psi'_\beta\,, \tag{2.1}$$

where the orthogonal $O(q)$ and the unitary $U\left(q\right)$ are related via $O_{ab}(q) = \frac{1}{2}\mathrm{Tr}\left(U^{-1}(q)\tau_a U(q)\tau_b\right)$. Equ. (2.1) is the generalisation of the (unique) polar decomposition of A and corresponds to

$$\chi_i(A) = \varepsilon_{ijk}A_{jk} = 0 \quad \text{("symmetric gauge")}. \tag{2.2}$$

Preserving the CR, we obtain the old canonical momenta in terms of the new variables

$$-E_{ai}(q,S,p,P) = O_{ak}\left(q\right)\left[P_{ki} + \varepsilon_{kil}{}^*D^{-1}_{ls}(S)\left(\Omega^{-1}_{sj}(q)p_j + \rho_s(\psi') + D_n(S)_{sm}P_{mn}\right)\right]. \tag{2.3}$$

In terms of the new canonical variables the Gauss law constraints are Abelianised,

$$G_a\Phi \equiv O_{ak}(q)\Omega^{-1}_{ki}(q)p_i\Phi = 0 \quad \Leftrightarrow \quad \frac{\delta}{\delta q_i}\Phi = 0 \quad \text{(Abelianisation)}, \tag{2.4}$$

and the angular momenta become

$$J_i = \int d^3x\left[-2\varepsilon_{ijk}S_{mj}P_{mk} + \Sigma_i(\psi') + \rho_i(\psi') + \text{orbital parts}\right]. \tag{2.5}$$

Equ.(2.4) identifies the q_i with the gauge angles and S and ψ' as the physical fields. Furthermore, from Equ.(2.5) follows that the S are colorless spin-0 and spin-2 glueball fields, and the ψ' colorless reduced quark fields of spin-0 and spin-1. Hence the gauge reduction corresponds to the conversion "color $\to$ spin". The obtained unusual spin-statistics relation is specific to SU(2).

2.2 Physical quantum Hamiltonian

According to the general scheme [1], the correctly ordered physical quantum Hamiltonian in terms of the physical variables $S_{ik}(\mathbf{x})$ and the canonically conjugate $P_{ik}(\mathbf{x}) \equiv -i\delta/\delta S_{ik}(\mathbf{x})$ reads [8]

$$\begin{aligned} H(S,P) =& \frac{1}{2}\mathscr{J}^{-1}\int d^3\mathbf{x}\, P_{ai}\,\mathscr{J} P_{ai} + \frac{1}{2}\int d^3\mathbf{x}\left[B^2_{ai}(S) - S_{ai}\,j_{ia}(\psi') + \overline{\psi}'\left(\gamma_i\partial_i + m\right)\psi'\right] \\ & - \mathscr{J}^{-1}\int d^3\mathbf{x}\int d^3\mathbf{y}\Big\{\left(D_i(S)_{ma}P_{im} + \rho_a(\psi')\right)(\mathbf{x})\,\mathscr{J} \\ & \qquad \langle \mathbf{x}\, a|{}^*D^{-2}(S)|\mathbf{y}\, b\rangle\,\left(D_j(S)_{bn}P_{nj} + \rho_b(\psi')\right)(\mathbf{y})\Big\}, \end{aligned} \tag{2.6}$$

with the Faddeev-Popov (FP) operator

$${}^*D_{kl}(S) \equiv \varepsilon_{kmi}D_i(S)_{ml} = \varepsilon_{kli}\partial_i - g(S_{kl} - \delta_{kl}\mathrm{tr}S), \tag{2.7}$$

and the Jacobian $\mathscr{J} \equiv \det|{}^*D|$. The matrix element of a physical operator O is given by

$$\langle\Psi'|O|\Psi\rangle \propto \int_{S\ \text{pos.def.}}\int_{\overline{\psi}',\psi'}\prod_{\mathbf{x}}\left[dS(\mathbf{x})d\overline{\psi}'(\mathbf{x})d\psi'(\mathbf{x})\right]\mathscr{J}\,\Psi'^*[S,\overline{\psi}',\psi']O\Psi[S,\overline{\psi}',\psi']. \tag{2.8}$$

The inverse of the FP operator and hence the physical Hamiltonian can be expanded in the number of spatial derivatives, equivalent to a strong coupling expansion in $\lambda = g^{-2/3}$.

2.3 Coarse-graining and strong coupling expansion of the physical Hamiltonian in $\lambda = g^{-2/3}$

Introducing an UV cutoff a by considering an infinite spatial lattice of granulas $G(\mathbf{n},a)$ at $\mathbf{x} = a\mathbf{n}$ ($\mathbf{n} \in Z^3$) and averaged variables

$$S(\mathbf{n}) := \frac{1}{a^3}\int_{G(\mathbf{n},a)} d\mathbf{x}\, S(\mathbf{x}) \;, \tag{2.9}$$

and discretised spatial derivatives, the expansion of the Hamiltonian in $\lambda = g^{-2/3}$ can be written

$$H = \frac{g^{2/3}}{a}\left[\mathscr{H}_0 + \lambda\sum_\alpha \mathscr{V}_\alpha^{(\partial)} + \lambda^2\left(\sum_\beta \mathscr{V}_\beta^{(\Delta)} + \sum_\gamma \mathscr{V}_\gamma^{(\partial\partial\neq\Delta)}\right) + \mathscr{O}(\lambda^3)\right]. \tag{2.10}$$

The "free" Hamiltonian $H_0 = (g^{2/3}/a)\mathscr{H}_0 + H_m = \sum_{\mathbf{n}} H_0^{QM}(\mathbf{n})$ is the sum of the Hamiltonians of Dirac-Yang-Mills quantum mechanics of constant fields in each box, and the interaction terms $\mathscr{V}^{(\partial)}, \mathscr{V}^{(\Delta)}, ..$ leading to interactions between the granulas.

2.4 Zeroth-order: Dirac-Yang-Mills Quantum mechanics of spatially constant fields

Transforming to the intrinsic system of the symmetric tensor S, with Jacobian $\sin\beta\prod_{i<j}(\phi_i - \phi_j)$,

$$S = R^T(\alpha,\beta,\gamma)\ \mathrm{diag}(\phi_1,\phi_2,\phi_3)\ R(\alpha,\beta,\gamma), \qquad \psi'^{(i)}_{L,R} = R^T_{ij}\widetilde{\psi}^{(j)}_{L,R}, \qquad \psi'^{(0)}_{L,R} = \widetilde{\psi}^{(0)}_{L,R}, \tag{2.11}$$

the "free" Hamiltonian in each box (volume V) takes the form [9]

$$H_0^{QM} = \frac{g^{2/3}}{V^{1/3}}\left[\mathscr{H}^G + \mathscr{H}^D + \mathscr{H}^C\right] + \frac{1}{2}m\left[\left(\widetilde{\psi}_L^{(0)\dagger}\widetilde{\psi}_R^{(0)} + \sum_{i=1}^3 \widetilde{\psi}_L^{(i)\dagger}\widetilde{\psi}_R^{(i)}\right) + h.c.\right], \tag{2.12}$$

with the glueball part $\mathscr{H}^G$, the minimal-coupling $\mathscr{H}^D$, and the Coulomb-potential-type part $\mathscr{H}^C$

$$\mathscr{H}^G = \frac{1}{2}\sum_{ijk}^{\text{cyclic}}\left(-\frac{\partial^2}{\partial\phi_i^2} - \frac{2}{\phi_i^2-\phi_j^2}\left(\phi_i\frac{\partial}{\partial\phi_i} - \phi_j\frac{\partial}{\partial\phi_j}\right) + (\xi_i - \widetilde{J}_i^Q)^2\frac{\phi_j^2+\phi_k^2}{(\phi_j^2-\phi_k^2)^2} + \phi_j^2\phi_k^2\right), \tag{2.13}$$

$$\mathscr{H}^D = \frac{1}{2}(\phi_1+\phi_2+\phi_3)\left(\widetilde{N}_L^{(0)} - \widetilde{N}_R^{(0)}\right) + \frac{1}{2}\sum_{ijk}^{\text{cyclic}}(\phi_i - (\phi_j+\phi_k))\left(\widetilde{N}_L^{(i)} - \widetilde{N}_R^{(i)}\right), \tag{2.14}$$

$$\mathscr{H}^C = \sum_{ijk}^{\text{cyclic}}\frac{\widetilde{\rho}_i(\xi_i - \widetilde{J}_i^Q + \widetilde{\rho}_i)}{(\phi_j+\phi_k)^2}\;, \tag{2.15}$$

and the total spin $\qquad J_i = R_{ij}(\chi)\ \xi_j\;, \qquad [J_i, H] = 0\;. \qquad$ (2.16)

The matrix elements become

$$\langle\Phi_1|\mathscr{O}|\Phi_2\rangle = \int d\alpha\sin\beta d\beta d\gamma\int_{0<\phi_1<\phi_2<\phi_3} d\phi_1 d\phi_2 d\phi_3\ (\phi_1^2-\phi_2^2)(\phi_2^2-\phi_3^2)(\phi_3^2-\phi_1^2)\int d\overline{\psi}' d\psi' \Phi_1^*\mathscr{O}\Phi_2\;.$$

The l.h.s. of Fig.1 shows the 0^+ energy spectrum of the lowest pure-gluon (G) and quark-gluon (QG) cases for one quark-flavor which can be calculated with high accuracy using the variational approach. The energies of the quark-gluon ground state and the sigma-antisigma excitation are lower than that of the lowest pure-gluon state. This is due to a large negative contribution from $\langle\mathscr{H}^D\rangle$, in addition to the large positive $\langle\mathscr{H}^G\rangle$, while $\langle\mathscr{H}^C\rangle \simeq 0$ (see [9] for details).

Furthermore, as a consequence of the zero-energy valleys "$\phi_1 = \phi_2 = 0$, ϕ_3 arbitrary" of the classical magnetic potential $B^2 = \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2 + \phi_1^2\phi_2^2$, practically all glueball excitation-energy results from an increase of expectation value of the "constant Abelian field" ϕ_3 as shown for the pure-gluon case on the r.h.s. of Fig.1 (see [7] for details).

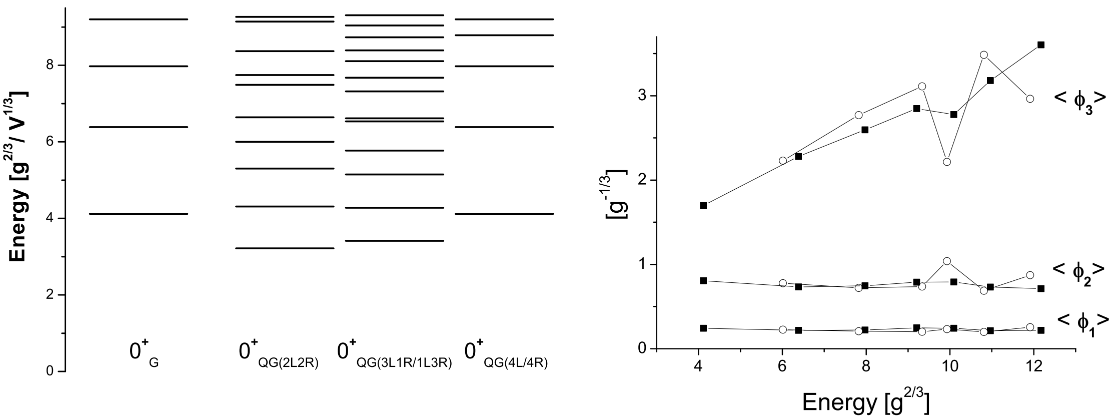

Figure 1: L.h.s.: Lowest energy levels for the pure-gluon (G) and the quark-gluon case (QG) for 2-colors and one quark flavor. The energies of the quark-gluon ground state and the sigma-antisigma excitation are lower than that of the lowest pure-gluon state. R.h.s. (for pure-gluon case and setting $V \equiv 1$): $\langle \phi_3 \rangle$ is raising with increasing excitation, whereas $\langle \phi_1 \rangle$ and $\langle \phi_2 \rangle$ are practically constant, independent of whether spin-0 (dark boxes) or spin-2 states (open circles).

2.5 Perturbation theory in λ and coupling constant renormalisation in the IR

Including the interactions $\mathscr{V}^{(\partial)}, \mathscr{V}^{(\Delta)}$ using 1st and 2nd order perturbation theory in $\lambda = g^{-2/3}$ give the result [8] (for pure-gluon case and only including spin-0 fields in a first approximation)

$$E^{+}_{\rm vac} = \mathscr{N}\frac{g^{2/3}}{a}\left[4.1167 + 29.894\lambda^2 + \mathscr{O}(\lambda^3)\right], \tag{2.17}$$

$$E_1^{(0)+}(k) - E^{+}_{\rm vac} = \left[\,2.270 + 13.511\lambda^2 + \mathscr{O}(\lambda^3)\right]\frac{g^{2/3}}{a} + 0.488\frac{a}{g^{2/3}}k^2 + \mathscr{O}((a^2k^2)^2), \tag{2.18}$$

for the energy of the interacting glueball vacuum and the spectrum of the interacting spin-0 glueball. Lorentz invariance demands $E = \sqrt{M^2+k^2} \simeq M + \frac{1}{2M}\,k^2$, which is violated in this 1st approximation by a factor of 2. In order to get a Lorentz invariant result, $J = L+S$ states should be considered including also spin-2 states and the general $\mathscr{V}^{(\partial\partial)}$.

Independence of the physical glueball mass

$$M = \frac{g_0^{2/3}}{a}\left[\mu + c g_0^{-4/3}\right] \tag{2.19}$$

of box size a, one obtains

$$\gamma(g_0) \equiv a\frac{d}{da}g_0(a) = \frac{3}{2}g_0\frac{\mu + c g_0^{-4/3}}{\mu - c g_0^{-4/3}} \tag{2.20}$$

which vanishes for $g_0 = 0$ (pert. fixed point) or $g_0^{4/3} = -c/\mu$ (IR fixed point, if $c < 0$). For $c > 0$

$$\text{for } c > 0: \qquad g_0^{2/3}(Ma) = \frac{Ma}{2\mu} + \sqrt{\left(\frac{Ma}{2\mu}\right)^2 - \frac{c}{\mu}}\,, \qquad a > a_c := 2\sqrt{c\mu}/M \tag{2.21}$$

My (incomplete) result $c_1^{(0)}/\mu_1^{(0)} = 5.95$ suggests, that no IR fixed points exist. critical coupling $g_0^2|_c = 14.52$ and $a_c \sim 1.4$ fm for $M \sim 1.6$ GeV.

3. Symmetric gauge for SU(3)

Using the idea of *minimal embedding* of $su(2)$ in $su(3)$ by Kihlberg and Marnelius [4]

$$\tau_1 := \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \tau_2 := -\lambda_5 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix} \quad \tau_3 := \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\tau_4 := \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \tau_5 := \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \tau_6 := \lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\tau_7 := \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \tau_8 := \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{3.1}$$

such that the corresponding non-trivial non-vanishing structure constants, $[\frac{\tau_a}{2}, \frac{\tau_b}{2}] = i c_{abc} \frac{\tau_c}{2}$, have at least one index $\in \{1,2,3\}$, the symmetric gauge, Equ.(2.2), can be generalised to $SU(3)$ [5, 10],

$$\chi_a(A) = \sum_{b=1}^{8} \sum_{i=1}^{3} c_{abi} A_{bi} = 0\,, \quad a = 1,\ldots,8 \qquad (\text{"symmetric gauge" for SU(3)}). \tag{3.2}$$

Carrying out the coordinate transformation [10]

$$A_{ak}\left(q_1,..,q_8,\widehat{S}\right) = O_{a\hat{a}}(q)\,\widehat{S}_{\hat{a}k} - \frac{1}{2g} c_{abc}\left(O(q)\,\partial_k O^T(q)\right)_{bc}, \quad \psi_\alpha\left(q_1,..,q_8,\psi^{RS}\right) = U_{\alpha\hat{\beta}}(q)\,\psi^{RS}_{\hat{\beta}}$$

$$\widehat{S}_{\hat{a}k} \equiv \begin{pmatrix} S_{ik} \\ S_{Ak} \end{pmatrix} = \left(\begin{array}{ccc} & S_{ik} \text{ pos. def.} & \\ \hline W_0 & X_3 - W_3 & X_2 + W_2 \\ X_3 + W_3 & W_0 & X_1 - W_1 \\ X_2 - W_2 & X_1 + W_1 & W_0 \\ -\frac{\sqrt{3}}{2} Y_1 - \frac{1}{2} W_1 & \frac{\sqrt{3}}{2} Y_2 - \frac{1}{2} W_2 & W_3 \\ -\frac{\sqrt{3}}{2} W_1 - \frac{1}{2} Y_1 & \frac{\sqrt{3}}{2} W_2 - \frac{1}{2} Y_2 & Y_3 \end{array}\right), \quad c_{\hat{a}\hat{b}k} \widehat{S}_{\hat{b}k} = 0\,, \tag{3.3}$$

an unconstrained Hamiltonian formulation of QCD can be obtained. The existence and uniqueness of (3.3) can be investigated by solving the 16 equs.

$$\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{a}j} = A_{ai}A_{aj} \;(6 \text{ equs.}) \quad \wedge \quad d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{a}i}\widehat{S}_{\hat{b}j}\widehat{S}_{\hat{c}k} = d_{abc}A_{ai}A_{bj}A_{ck} \;(10 \text{ equs.}) \tag{3.4}$$

for the 16 components of $\widehat{S}$ in terms of 24 given components A.

Analysing the Gauss law operators and the unconstrained angular momentum operators in terms of the new variables in analogy to the 2-color case, it can be shown that the original constrained 24 colored spin-1 gluon fields A and the 12 colored spin-1/2 quark fields ψ (per flavor) reduce to 16 physical colorless spin-0, spin-1, spin-2, and spin-3 glueball fields (the 16 components of $\widehat{S}$) and a colorless spin-3/2 Rarita-Schwinger field ψ^{RS} (per flavor), respectively. As for the 2-color case, the gauge reduction converts color $\rightarrow$ spin, which might have important consequences for low energy Spin-Physics. In terms of the colorless Rarita-Schwinger fields the $\Delta^{++}(3/2)$ could have the spin content $(+3/2, +1/2, -1/2)$ in accordance with the Spin-Statistics-Theorem.

Transforming to the intrinsic system of the embedded upper part S of $\widehat{S}$ (see [10] for details)

$$S = R^T(\alpha,\beta,\gamma)\,\mathrm{diag}(\phi_1,\phi_2,\phi_3)\,R(\alpha,\beta,\gamma),\ \wedge\ X_i \to x_i,\ Y_i \to y_i,..,\ \wedge\ \psi^{RS} \to \widetilde{\psi}^{RS}, \quad (3.5)$$

one finds that the magnetic potential B^2 has the zero-energy valleys ("constant Abelian fields")

$$B^2 = 0\ :\ \phi_3 \text{ and } y_3 \text{ arbitrary} \quad \wedge \quad \text{all others zero} \quad (3.6)$$

Hence, practically all glueball excitation-energy should result from an increase of expectation values of these two "constant Abelian fields", in analogy to $SU(2)$. Furthermore, at the bottom of the valleys the important minimal-coupling-interaction of $\widetilde{\psi}^{RS}$ (analogous to (2.14)) becomes diagonal

$$\mathscr{H}^D_{\text{diag}} = \frac{1}{2}\widetilde{\psi}_L^{(1,\frac{1}{2})\dagger}\left[(\phi_3\lambda_3 + y_3\lambda_8)\otimes\sigma_3\right]\widetilde{\psi}_L^{(1,\frac{1}{2})} - \frac{1}{2}\widetilde{\psi}_R^{(\frac{1}{2},1)\dagger}\left[\sigma_3\otimes(\phi_3\lambda_3 + y_3\lambda_8)\right]\widetilde{\psi}_R^{(\frac{1}{2},1)}\ . \quad (3.7)$$

Due to the difficulty of the FP-determinant (see [10]), precise calculations are not possible yet. Note, however, that in one spatial dimension the symmetric gauge for SU(3) reduces to

$$A^{(1d)} = \begin{pmatrix} 0 & 0 & A_{13} \\ 0 & 0 & A_{23} \\ 0 & 0 & A_{33} \\ 0 & 0 & A_{43} \\ 0 & 0 & A_{53} \\ 0 & 0 & A_{63} \\ 0 & 0 & A_{73} \\ 0 & 0 & A_{83} \end{pmatrix} \quad \to \quad \widehat{S}^{(1d)} = \widehat{S}^{(1d)}_{\text{intrinsic}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \phi_3 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3 \end{pmatrix} \quad (3.8)$$

which consistently reduces the Equ.(3.4) for given A_3 to

$$\phi_3^2 + y_3^2 = A_{a3}A_{a3} \quad \wedge \quad \phi_3^2 y_3 - 3y_3^3 = d_{abc}\,A_{a3}A_{b3}A_{c3} \quad (3.9)$$

with 6 solutions separated by zero-lines of the FP-determinant ("Gribov-horizons"). Exactly one solution exists in the "fundamental domain" $0 < \phi_3 < \infty \wedge \phi_3/\sqrt{3} < y_3 < \infty$, and we can replace

$$\int_{-\infty}^{+\infty}\prod_{a=1}^{8} dA_{a3} \to \int_0^\infty d\phi_3 \int_{\phi_3/\sqrt{3}}^\infty dy_3\ \phi_3^2\left(\phi_3^2 - 3y_3^2\right)^2 \propto \int_0^\infty r dr \int_{\pi/6}^{\pi/2} d\psi \cos^2(3\psi). \quad (3.10)$$

For two spatial dimensions, one can show that (putting in Equ.(3.3) $W_1 \equiv X_1, W_2 \equiv -X_2$)

$$A^{(2d)} = \begin{pmatrix} A_{11} & A_{12} & 0 \\ A_{21} & A_{22} & 0 \\ A_{31} & A_{32} & 0 \\ A_{41} & A_{42} & 0 \\ A_{51} & A_{52} & 0 \\ A_{61} & A_{62} & 0 \\ A_{71} & A_{72} & 0 \\ A_{81} & A_{82} & 0 \end{pmatrix} \to \widehat{S}^{(2d)}_{\text{intrinsic}} = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & 0 \\ \hline 0 & x_3 & 0 \\ x_3 & 0 & 0 \\ 2x_2 & 2x_1 & 0 \\ -\frac{\sqrt{3}}{2}y_1 - \frac{1}{2}x_1 & \frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \\ -\frac{\sqrt{3}}{2}y_1 + \frac{1}{2}x_1 & -\frac{\sqrt{3}}{2}y_2 + \frac{1}{2}x_2 & 0 \end{pmatrix} \quad (3.11)$$

consistently reduces (3.4) to a system of 7 equs. for 8 physical fields (incl. rot.-angle γ), which, adding as an 8th equ. $(d_{\hat{a}\hat{b}\hat{c}}\widehat{S}_{\hat{b}1}\widehat{S}_{\hat{c}2})^2 = (d_{abc}A_{b1}A_{c2})^2$, can be solved numerically for randomly generated $A^{(2d)}$, again yielding solutions separated by horizons. Restricting to a fundamental domain

$$\int_{-\infty}^{+\infty}\prod_{a,b=1}^{8} dA_{a1}dA_{b2} \to \int d\gamma \int_{0<\phi_1<\phi_2<\infty} d\phi_1 d\phi_2 (\phi_1 - \phi_2) \int_{R_1(\phi_1,\phi_2)} dx_1 dx_2 dx_3 \int_{R_2(x_1,x_2,x_3,\phi_1,\phi_2)} dy_1 dy_2\ \mathscr{J}$$

Due to the difficulty of the FP-determinant, I have, however, not yet succeeded in a satisfactory description of the regions R_1 and R_2. For the general case of three dimensions, I have found several solutions of the Equ.(3.4) numerically for a randomly generated A, but to write the corresponding unconstrained integral over a fundamental domain is a difficult, but I think solvable, future task.

4. Conclusions

Using a canonical transformation of the dynamical variables, which Abelianises the non-Abelian Gauss-law constraints to be implemented, a reformulation of QCD in terms of gauge invariant dynamical variables can be achieved. The exact implementation of the Gauss laws reduces the colored spin-1 gluons and spin-1/2 quarks to unconstrained colorless spin-0, spin-1, spin-2 and spin-3 glueball fields and colorless Rarita-Schwinger fields respectively. The obtained physical Hamiltonian admits a systematic strong-coupling expansion in powers of $\lambda = g^{-2/3}$, equivalent to an expansion in the number of spatial derivatives. The leading-order term in this expansion corresponds to non-interacting hybrid-glueballs, whose low-lying masses can be calculated with high accuracy (at the moment only for the unphysical, but technically much simpler 2-color case) by solving the Schrödinger-equation of Dirac-Yang-Mills quantum mechanics of spatially constant fields. Higher-order terms in λ lead to interactions between the hybrid-glueballs and can be taken into account systematically, using perturbation theory in λ, allowing for the study of the difficult questions of Lorentz invariance and coupling constant renormalisation in the IR.

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